

*Solutions Manual for Calculus Volume
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Calculus

Volume 3

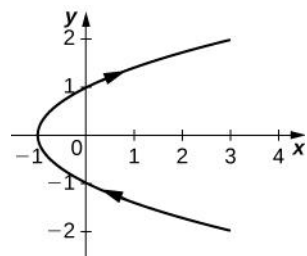
Chapter 1
Parametric Equations and Polar Coordinates
1.1 Parametric Equations

Section Exercises

For the following exercises, sketch the curves below by eliminating the parameter t . Give the orientation of the curve.

1. $x = t^2 + 2t, y = t + 1$

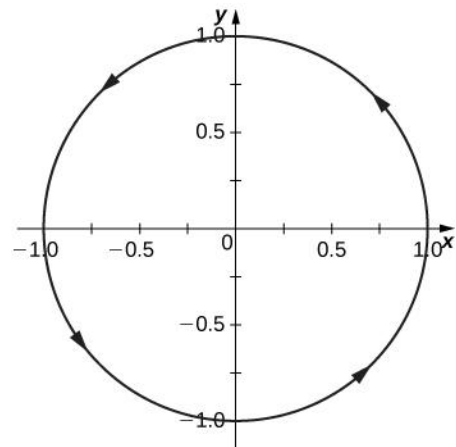
Answer:



orientation: bottom to top

2. $x = \cos(t), y = \sin(t), (0, 2\pi]$

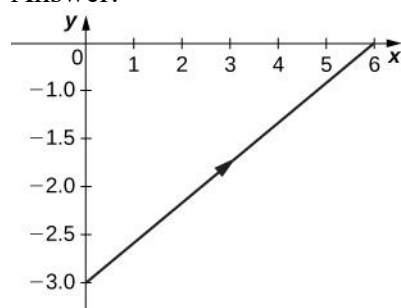
Answer:



orientation: counterclockwise

3. $x = 2t + 4$, $y = t - 1$

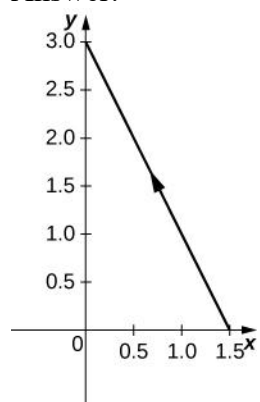
Answer:



orientation: left to right

4. $x = 3 - t$, $y = 2t - 3$, $1.5 \leq t \leq 3$

Answer:

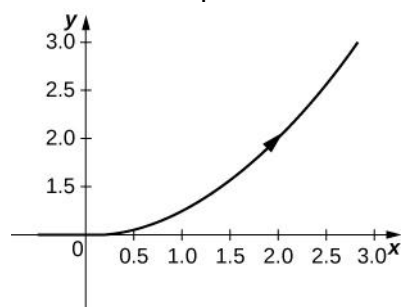


orientation: right to left

For the following exercises, eliminate the parameter and sketch the graphs.

5. $x = 2t^2$, $y = t^4 + 1$

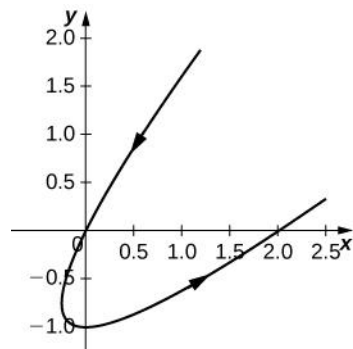
Answer: $y = \frac{x^2}{4} + 1$



For the following exercises, use technology (CAS or calculator) to sketch the parametric equations.

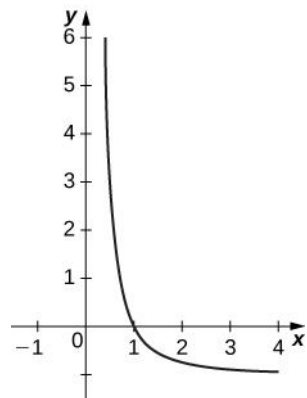
6. [T] $x = t^2 + t, \quad y = t^2 - 1$

Answer:



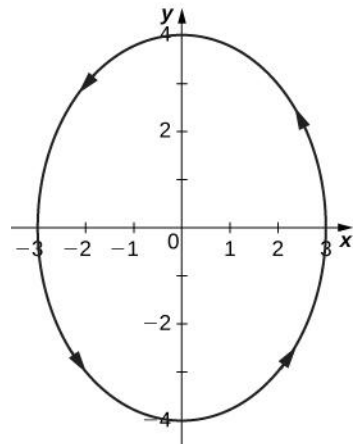
7. [T] $x = e^{-t}, \quad y = e^{2t} - 1$

Answer:



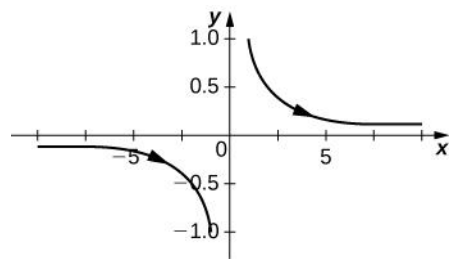
8. [T] $x = 3 \cos t, \quad y = 4 \sin t$

Answer:



9. [T] $x = \sec t$, $y = \cos t$

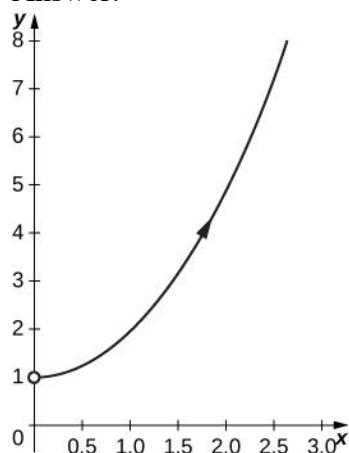
Answer:



For the following exercises, sketch the parametric equations by eliminating the parameter. Indicate any asymptotes of the graph.

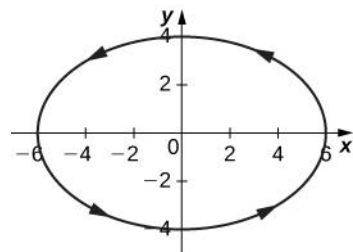
10. $x = e^t$, $y = e^{2t} + 1$

Answer:



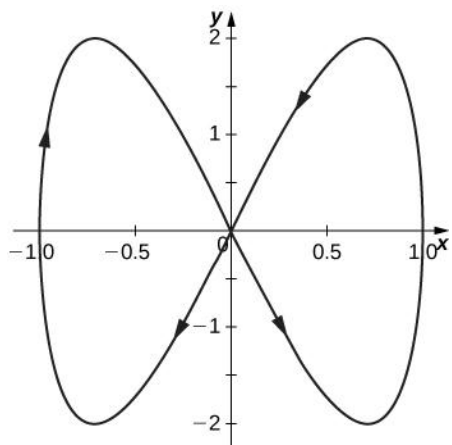
11. $x = 6 \sin(2\theta)$, $y = 4 \cos(2\theta)$

Answer:



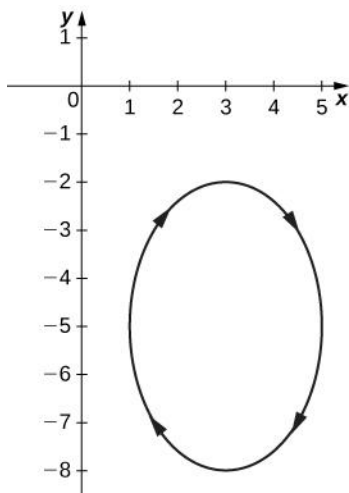
12. $x = \cos \theta$, $y = 2 \sin(2\theta)$

Answer:



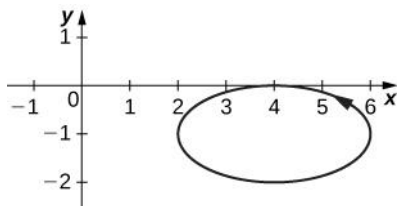
13. $x = 3 - 2 \cos \theta$, $y = -5 + 3 \sin \theta$

Answer:



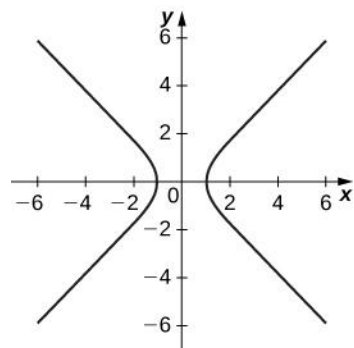
14. $x = 4 + 2 \cos \theta$, $y = -1 + \sin \theta$

Answer:



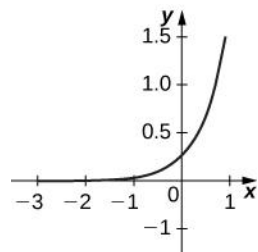
15. $x = \sec t, \quad y = \tan t$

Answer:

Asymptotes are $y = x$ and $y = -x$

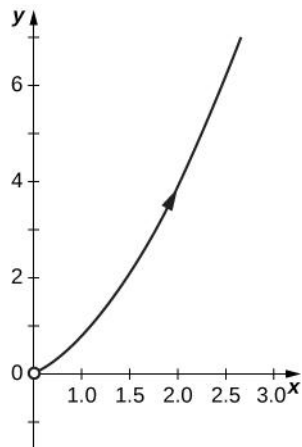
16. $x = \ln(2t), \quad y = t^2$

Answer:

 $y = 0$ is a horizontal asymptote.

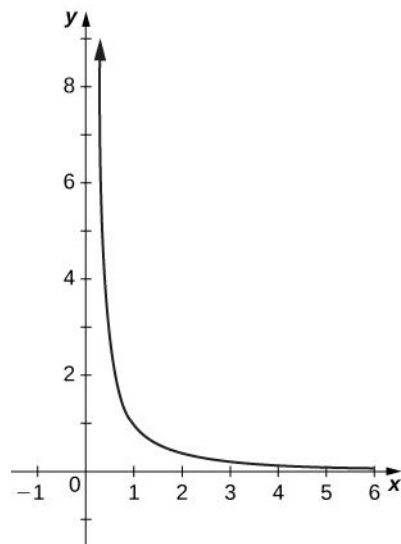
17. $x = e^t, \quad y = e^{2t}$

Answer:



18. $x = e^{-2t}$, $y = e^{3t}$

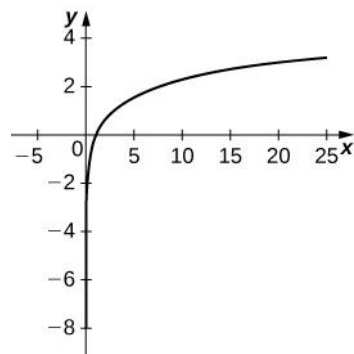
Answer:



Horizontal asymptote: $y = 0$. Vertical asymptote: $x = 0$.

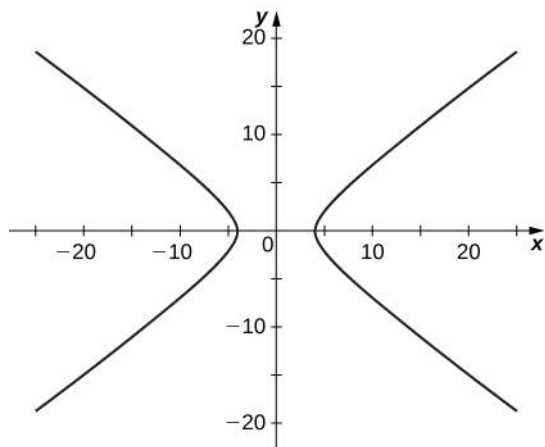
19. $x = t^3$, $y = 3\ln t$

Answer:



20. $x = 4 \sec \theta$, $y = 3 \tan \theta$

Answer:



For the following exercises, convert the parametric equations of a curve into rectangular form. No sketch is necessary. State the domain of the rectangular form.

21. $x = t^2 - 1$, $y = \frac{t}{2}$

Answer: $y = \frac{\sqrt{x+1}}{2}$; domain: $x \in [-1, \infty)$

22. $x = \frac{1}{\sqrt{t+1}}$, $y = \frac{t}{1+t}$, $t > -1$

Answer: $y = 1 - x^2$; domain: $x \in (0, \infty)$.

23. $x = 4 \cos \theta$, $y = 3 \sin \theta$, $\theta \in (0, 2\pi]$

Answer: $\frac{x^2}{16} + \frac{y^2}{9} = 1$; domain $x \in [-4, 4]$.

24. $x = \cosh t$, $y = \sinh t$

Answer: $x^2 - y^2 = 1$; domain $x \in [1, \infty) \cup (-\infty, -1]$.

25. $x = 2t - 3$, $y = 6t - 7$

Answer: $y = 3x + 2$; domain: all real numbers.

26. $x = t^2$, $y = t^3$

Answer: $y = \pm x^{3/2}$; domain: $x \in [0, \infty)$.

27. $x = 1 + \cos t$, $y = 3 - \sin t$

Answer: $(x-1)^2 + (y-3)^2 = 1$; domain: $x \in [0, 2]$.

28. $x = \sqrt{t}$, $y = 2t + 4$

Answer: $y = 2x^2 + 4$, $x \geq 0$; domain: $x \in [0, \infty)$.

29. $x = \sec t$, $y = \tan t$, $\pi \leq t < \frac{3\pi}{2}$

Answer: $y = \sqrt{x^2 - 1}$; domain: $x \in (-\infty, 1]$

30. $x = 2 \cosh t$, $y = 4 \sinh t$

Answer: $\frac{x^2}{4} - \frac{y^2}{16} = 1$

31. $x = \cos(2t)$, $y = \sin t$

Answer: $y^2 = \frac{1-x}{2}$; domain: $x \in [2, \infty) \cup (-\infty, -2]$.

32. $x = 4t + 3$, $y = 16t^2 - 9$

Answer: $y = (x-3)^2 - 9$; domain: all real numbers.

33. $x = t^2$, $y = 2 \ln t$, $t \geq 1$

Answer: $y = \ln x$; domain: $x \in [1, \infty)$.

34. $x = t^3$, $y = 3 \ln t$, $t \geq 1$

Answer: $y = \ln x$; domain: $x \in (0, \infty)$.

35. $x = t^n$, $y = n \ln t$, $t \geq 1$, where n is a natural number

Answer: $y = \ln x$; domain: $x \in (0, \infty)$.

36. $x = \ln(5t)$
 $y = \ln(t^2)$ where $1 \leq t \leq e$

Answer: $y = 2x - \ln 25$, a linear function; domain: $x \in [\ln(5), \ln(5) + 1]$.

37. $x = 2 \sin(8t)$
 $y = 2 \cos(8t)$

Answer: $x^2 + y^2 = 4$; domain: $x \in [-2, 2]$.

38. $x = \tan t$
 $y = \sec^2 t - 1$

Answer: $y = x^2$; domain: all real numbers.

For the following exercises, the pairs of parametric equations represent lines, parabolas, circles, ellipses, or hyperbolas. Name the type of basic curve that each pair of equations represents.

39.
$$\begin{aligned}x &= 3t + 4 \\ y &= 5t - 2\end{aligned}$$

Answer: line

40.
$$\begin{aligned}x - 4 &= 5t \\ y + 2 &= t\end{aligned}$$

Answer: line

41.
$$\begin{aligned}x &= 2t + 1 \\ y &= t^2 - 3\end{aligned}$$

Answer: parabola

42.
$$\begin{aligned}x &= 3 \cos t \\ y &= 3 \sin t\end{aligned}$$

Answer: circle

43.
$$\begin{aligned}x &= 2 \cos(3t) \\ y &= 2 \sin(3t)\end{aligned}$$

Answer: circle

44.
$$\begin{aligned}x &= \cosh t \\ y &= \sinh t\end{aligned}$$

Answer: hyperbola

45.
$$\begin{aligned}x &= 3 \cos t \\ y &= 4 \sin t\end{aligned}$$

Answer: ellipse

46.
$$\begin{aligned}x &= 2 \cos(3t) \\ y &= 5 \sin(3t)\end{aligned}$$

Answer: ellipse

47.
$$\begin{aligned}x &= 3 \cosh(4t) \\ y &= 4 \sinh(4t)\end{aligned}$$

Answer: hyperbola

48. $x = 2 \cosh t$
 $y = 2 \sinh t$

Answer: hyperbola

49. Show that $\begin{matrix} x = h + r \cos \theta \\ y = k + r \sin \theta \end{matrix}$ represents the equation of a circle.

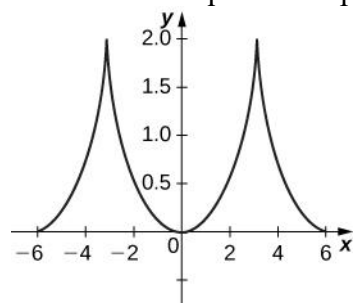
50. Use the equations in the preceding problem to find a set of parametric equations for a circle whose radius is 5 and whose center is $(-2, 3)$.

Answer: $x = -2 + 5 \cos \theta$
 $y = 3 + 5 \sin \theta$

For the following exercises, use a graphing utility to graph the curve represented by the parametric equations and identify the curve from its equation.

51. [T] $x = \theta + \sin \theta$
 $y = 1 - \cos \theta$

Answer: The equations represent a cycloid.



52. [T] $x = 2t - 2 \sin t$
 $y = 2 - 2 \cos t$

Answer: The equations represent the mirror image of a cycloid, the graph of which is symmetric with the y-axis.

