Solutions Manual for Trigonometry 12th Edition by Larson

ISBN: 9798214407593

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Solution and Answer Guide

LARSON, TRIGONOMETRY, 12E, 2027, 9798214407593; CHAPTER P: PREREQUISITES

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P.1.1.

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Zero-Factor Property

P.1.3.

Yes. $|3-10|=|-7|=7$ and $|10-3|=|7|=7.$

P.1.4.

(a) iii
(b) iv
(c) ii
(d) v
(e) i

P.1.5.

$-9, -\frac{7}{2}, 5, \frac{2}{3}, \sqrt{3}, 0, 3, -4, 2, -11$

(a) Natural numbers: 5, 8, 2
(b) Whole numbers: 5, 0, 8, 2
(c) Integers: $-9, 5, 0, 8, -4, 2, -11$
(d) Rational numbers: $-9, -\frac{7}{2}, 5, \frac{2}{3}, 0, 8, -4, 2, -11$
(e) Irrational numbers: $\sqrt{3}$

P.1.6.

$\sqrt{5}, -7, -\frac{7}{3}, 0, 3.14, \frac{5}{4}, -3, 12, 5$

(a) Natural numbers: 12, 5
(b) Whole numbers: 0, 12, 5
(c) Integers: $-7, 0, -3, 12, 5$
(d) Rational numbers: $-7, -\frac{7}{3}, 0, 3.14, \frac{5}{4}, -3, 12, 5$
(e) Irrational numbers: $\sqrt{5}$

P.1.7.

$2.01, 0.\overline{6}, -13, 0.010110111\ldots, 1, -6$

(a) Natural numbers: 1
(b) Whole numbers: 1

(c) Integers: $-13,\ 1,\ -6$

(d) Rational numbers: $2.01, 0.\overline{6}, -13, 1, -6$

(e) Irrational numbers: $0.010110111\ldots$

P.1.8.

$25,\ -17,\ -\frac{12}{5},\ \sqrt{9},\ 3.12,\ \frac{1}{2}\pi,\ 18,\ -11.1,\ 13$

(a) Natural numbers: $25,\ \sqrt{9}=3,\ 18,\ 13$

(b) Whole numbers: $25,\ \sqrt{9}=3,\ 18,\ 13$

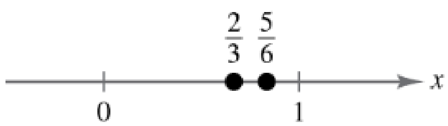
(c) Integers: $25,\ -17,\ \sqrt{9}=3,\ 18,\ 13$

(d) Rational numbers: $25,\ -17,\ -\frac{12}{5},\ \sqrt{9}=3,\ 3.12,\ 18,\ -11.1,\ 13$

(e) Irrational numbers: $\frac{1}{2}\pi$

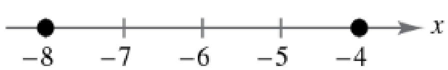
P.1.9.

$\frac{5}{6} > \frac{2}{3}$



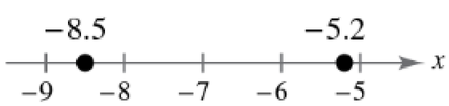
P.1.10.

$-4 > -8$



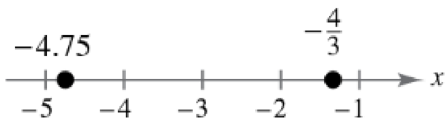
P.1.11.

$-5.2 > -8.5$



P.1.12.

$-\frac{4}{3} > -4.75$



P.1.13.

The inequality $x \le 5$ denotes the set of all real numbers less than or equal to 5.

P.1.14.

The inequality $x < 0$ denotes the set of all real numbers less than zero.

P.1.15.

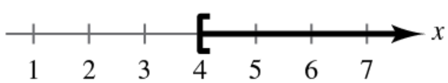
The inequality $-2 < x < 2$ denotes the set of all real numbers greater than -2 and less than 2.

P.1.16.

The inequality $0 < x \le 6$ denotes the set of all positive real numbers less than or equal to 6.

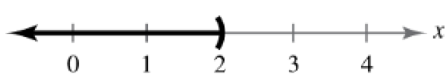
P.1.17.

The interval $\left[4, \infty\right)$ denotes the set of all real numbers greater than or equal to 4; $x \ge 4$.



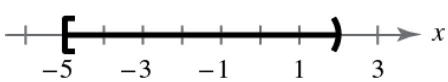
P.1.18.

The interval $\left(-\infty, 2\right)$ denotes the set of all real numbers less than 2; $x < 2$.



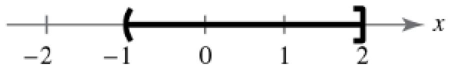
P.1.19.

The interval $\left[-5, 2\right)$ denotes the set of all real numbers greater than or equal to -5 and less than 2; $-5 \le x < 2$.



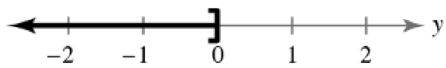
P.1.20.

The interval $(-1, 2]$ denotes the set of all real numbers greater than -1 and less than or equal to 2; $-1 < x \le 2$.



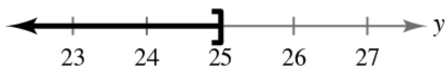
P.1.21.

$(-\infty, 0]$; $y \le 0$



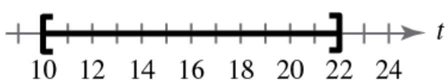
P.1.22.

$(-\infty, 25]$; $y \le 25$



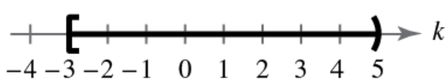
P.1.23.

$[10, 22]$; $10 \le t \le 22$



P.1.24.

$[-3, 5)$; $-3 \le k < 5$



P.1.25.

$|-10| = -(-10) = 10$

P.1.26.

$|0| = 0$

P.1.27.

$|-1| - |-2| = 1 - 2 = -1$

P.1.28.

$$-3-|-3|=-3-(3)=-6$$

P.1.29.

$$5\ |-5|=5(5)=25$$

P.1.30.

$$-4\ |-4|=-4(4)=-16$$

P.1.31.

If $x<-2$, then $x+2$ is negative.

So, $\dfrac{|x+2|}{x+2}=\dfrac{-(x+2)}{x+2}=-1.$

P.1.32.

If $x>1$, then $x-1$ is positive.

So, $\dfrac{|x-1|}{x-1}=\dfrac{x-1}{x-1}=1.$

P.1.33.

$|-4|=|4|$ because $|-4|=4$ and $|4|=4.$

P.1.34.

$-5=-\ |5|$ because $-5=-5.$

P.1.35.

$-|-6|<|-6|$ because $|-6|=6$ and $-|-6|=-(6)=-6.$

P.1.36.

$-|-2|=-\ |2|$ because $-2=-2.$

P.1.37.

$$d(126,\ 75)=|75-126|=51$$

P.1.38.

$$d(-20,\ 30)=|30-(-20)|=|50|=50$$

P.1.39.

$$d\left(-\tfrac{5}{2}, 0\right) = \left|0 - \left(-\tfrac{5}{2}\right)\right| = \tfrac{5}{2}$$

P.1.40.

$$d\left(-\tfrac{1}{4}, -\tfrac{11}{4}\right) = \left|-\tfrac{11}{4} - \left(-\tfrac{1}{4}\right)\right| = \left|-\tfrac{5}{2}\right| = \tfrac{5}{2}$$

P.1.41.

$7x + 4$
Terms: $7x,\ 4$
Coefficient: 7

P.1.42.

$6x^3 - 5x$
Terms: $6x^3,\ -5x$
Coefficients: $6,\ -5$

P.1.43.

$4x^3 + 0.5x - 5$
Terms: $4x^3,\ 0.5x,\ -5$
Coefficients: $4,\ 0.5$

P.1.44.

$3\sqrt{3}x^2 + 1$
Terms: $3\sqrt{3}x^2,\ 1$
Coefficient: $3\sqrt{3}$

P.1.45.

Receipts: R = \$3268.0 billion
Expenditures: E = \$3852.6 billion
$|R - E| = |3268.0 - 3852.6| = |-584.6| = \$584.6$ billion

P.1.46.

Receipts: R = \$3329.9 billion
Expenditures: E = \$4109.0 billion
$|R - E| = |3329.9 - 4109.0| = |-779.1| = \$779.1$ billion

P.1.47.

Receipts: R = \$3421.2 billion
Expenditures: E = \$6553.6 billion
$|R - E| = |3421.2 - 6553.6| = |-3132.4| = \$3132.4$ billion

P.1.48.

Receipts: R = \$4897.3 billion
Expenditures: E = \$6273.3 billion
$|R-E| = |4897.3-6273.3| = |-1376.0| = \$1376.0$ billion

P.1.49.

$x^2 - 3x + 2$

(a) $(0)^2 - 3(0) + 2 = 2$

(b) $(-1)^2 - 3(-1) + 2 = 1 + 3 + 2 = 6$

P.1.50.

$\dfrac{x-2}{x+2}$

(a) $\dfrac{2-2}{2+2} = \dfrac{0}{4} = 0$

(b) $\dfrac{-2-2}{-2+2} = \dfrac{-4}{0}$

Division by zero is undefined.

P.1.51.

$$\frac{2x}{3} - \frac{x}{4} = \frac{8x}{12} - \frac{3x}{12} = \frac{5x}{12}$$

P.1.52.

$$\frac{3x}{4} + \frac{x}{5} = \frac{15x}{20} + \frac{4x}{20} = \frac{19x}{20}$$

P.1.53.

$$\frac{3x}{10} \cdot \frac{5}{6} = \frac{x}{2} \cdot \frac{1}{2} = \frac{x}{4}$$

P.1.54.

$$\frac{2x}{3} \div \frac{6}{7} = \frac{2x}{3} \cdot \frac{7}{6} = \frac{x}{3} \cdot \frac{7}{3} = \frac{7x}{9}$$

P.1.55.

False. Because zero is nonnegative but not positive, not every nonnegative number is positive.

P.1.56.

True. The product of two negative numbers is positive.

P.1.57.

The Distributive Property was used incorrectly. The 5 was not distributed to the 3.

P.1.58.

(a) Because the price can only be a positive rational number with at most two decimal places, the description matches graph (ii).
A range of prices can only include zero and positive numbers with at most two decimal places. So, a range of prices can be represented by whole numbers and some noninteger positive fractions.

(b) Because the distance is a positive real number, the description matches graph (i).
A range of lengths can only include positive numbers. So, a range of lengths can be represented by positive real numbers.

P.1.59.

$5/n$ increases or decreases without bound. As n approaches 0, $5/n$ increases when n is positive and decreases when n is negative.

P.1.60.

$5/n$ approaches 0 as n increases without bound. As n increases, $5/n$ decreases but stays positive.

P.1.61.

The least common denominator is $3(4) = 12$.

P.1.62.

The least common denominator is 9.

P.1.63.

The least common denominator is $x - 4$.

P.1.64.

The least common denominator is $x^2 - 4 = (x+2)(x-2)$.

P.1.65.

$$3x^4 - 48x^2 = 3x^2\left(x^2 - 16\right) = 3x^2\left(x-4\right)\left(x+4\right)$$

P.1.66.

$$9x^4 - 12x^2 = 3x^2\left(3x^2 - 4\right)$$

P.1.67.

$$\begin{aligned} x^3 - 3x^2 + 3x - 9 &= x^2(x-3) + 3(x-3) \\ &= (x-3)(x^2+3) \end{aligned}$$

P.1.68.

$$\begin{aligned} x^3 - 5x^2 - 2x + 10 &= x^2(x-5) - 2(x-5) \\ &= (x-5)(x^2-2) \end{aligned}$$

P.1.69.

$$\begin{aligned} 6x^3 - 27x^2 - 54x &= 3x(2x^2 - 9x - 18) \\ &= 3x(2x+3)(x-6) \end{aligned}$$

P.1.70.

$$\begin{aligned} 12x^3 - 16x^2 - 60x &= 4x(3x^2 - 4x - 15) \\ &= 4x(3x+5)(x-3) \end{aligned}$$

P.1.71.

$$\begin{aligned} x^4 - 3x^2 + 2 &= (x^2-1)(x^2-2) \\ &= (x-1)(x+1)(x^2-2) \end{aligned}$$

P.1.72.

$$\begin{aligned} x^4 - 7x^2 + 12 &= (x^2-3)(x^2-4) \\ &= (x^2-3)(x-2)(x+2) \end{aligned}$$

P.1.73.

(a) When $x = 4$, $x + \sqrt{40-9x} = 4 + \sqrt{40-9(4)} = 4 + \sqrt{4} = 4 + 2 = 6.$

(b) When $x = -9$, $x + \sqrt{40-9x} = -9 + \sqrt{40-9(-9)} = -9 + \sqrt{121} = -9 + 11 = 2.$

P.1.74.

(a) When $x = -3$, $|x^2 - 3x| + 4x - 6 = |(-3)^2 - 3(-3)| + 4(-3) - 6 = |9+9| - 12 - 6 = 0.$

(b) When $x = 2$, $|x^2 - 3x| + 4x - 6 = |(2)^2 - 3(2)| + 4(2) - 6 = |4-6| + 8 - 6 = 4.$

SECTION P.2 SOLVING EQUATIONS

P.2.1.

$ax + b = 0$

P.2.2.

extraneous

P.2.3.

Square each side of the equation to produce the equation $x + 2 = x^2$.

P.2.4.

Four methods to solve a quadratic equation are: factoring, extracting square roots, completing the square, and using the Qualitative Formula.

P.2.5.

$$\begin{aligned} 7 - 2x &= 25 \\ 7 - 7 - 2x &= 25 - 7 \\ -2x &= 18 \\ \frac{-2x}{-2} &= \frac{18}{-2} \\ x &= -9 \end{aligned}$$

P.2.6.

$$\begin{aligned} 7x + 2 &= 23 \\ 7x + 2 - 2 &= 23 - 2 \\ 7x &= 21 \\ \frac{7x}{7} &= \frac{21}{7} \\ x &= 3 \end{aligned}$$

P.2.7.

$$\begin{aligned} 3x - 5 &= 2x + 7 \\ 3x - 2x - 5 &= 2x - 2x + 7 \\ x - 5 &= 7 \\ x - 5 + 5 &= 7 + 5 \\ x &= 12 \end{aligned}$$

P.2.8.

$$\begin{aligned}
4y+2-5y&=7-6y\\
4y-5y+2&=7-6y\\
-y+2&=7-6y\\
-y+6y+2&=7-6y+6y\\
5y+2&=7\\
5y+2-2&=7-2\\
5y&=5\\
\frac{5y}{5}&=\frac{5}{5}\\
y&=1
\end{aligned}$$

P.2.9.

$$\begin{aligned}
10-\frac{13}{x}&=4+\frac{5}{x}\\
\frac{10x-13}{x}&=\frac{4x+5}{x}\\
10x-13&=4x+5\\
6x&=18\\
x&=3
\end{aligned}$$

P.2.10.

$$\begin{aligned}
\frac{1}{x}+\frac{2}{x-5}&=0\\
x(x-5)\frac{1}{x}+x(x-5)\frac{2}{x-5}&=x(x-5)0\\
x-5+2x&=0\\
3x-5&=0\\
3x&=5\\
x&=\frac{5}{3}
\end{aligned}$$

P.2.11.

$$\frac{2}{(x-4)(x-2)}=\frac{1}{x-4}+\frac{2}{x-2}$$ Multiply each term by $(x-4)(x-2)$.

$$\begin{aligned}
2&=1(x-2)+2(x-4)\\
2&=x-2+2x-8\\
2&=3x-10\\
12&=3x\\
4&=x
\end{aligned}$$

A check reveals that $x=4$ yields a denominator of zero. So, $x=4$ is an extraneous solution, and the original equation has no real solution.

P.2.12.

$$\frac{12}{(x-1)(x+3)} = \frac{3}{x-1} + \frac{2}{x+3}$$

$$(x-1)(x+3)\frac{12}{(x-1)(x+3)} = (x-1)(x+3)\frac{3}{x-1} + (x-1)(x+3)\frac{2}{x+3} \quad \text{Multiply each term by } (x-1)(x+3).$$

$$\begin{aligned} 12 &= 3(x+3) + 2(x-1) \\ 12 &= 3x + 9 + 2x - 2 \\ 12 &= 5x + 7 \\ 5 &= 5x \\ x &= 1 \end{aligned}$$

A check reveals that $x = 1$ yields a denominator of zero. So, $x = 1$ is an extraneous solution, and the original equation has no real solution.

P.2.13.

$$\begin{aligned} x^2 + 10x + 25 &= 0 \\ (x+5)(x+5) &= 0 \\ x + 5 &= 0 \\ x &= -5 \end{aligned}$$

P.2.14.

$$\begin{aligned} x^2 - 2x - 8 &= 0 \\ (x-4)(x+2) &= 0 \end{aligned}$$

$$\begin{aligned} x - 4 &= 0 \quad \text{or} \quad x + 2 = 0 \\ x &= 4 \quad \text{or} \quad x = -2 \end{aligned}$$

P.2.15.

$$\begin{aligned} 16x^2 - 9 &= 0 \\ (4x+3)(4x+3) &= 0 \end{aligned}$$

$$4x + 3 = 0 \Rightarrow x = -\frac{3}{4}$$

$$4x - 3 = 0 \Rightarrow x = \frac{3}{4}$$

P.2.16.

$$\begin{aligned}
-x^2 + 8x &= 12 \\
-x^2 + 8x - 12 &= 0 \\
(-1)(-x^2 + 8x - 12) &= (-1)(0) \\
x^2 - 8x + 12 &= 0 \\
(x-6)(x-2) &= 0
\end{aligned}$$

$$x - 6 = 0 \quad \Rightarrow \quad x = 6$$
$$x - 2 = 0 \quad \Rightarrow \quad x = 2$$

P.2.17.

$$\begin{aligned}
3x^2 &= 81 \\
x^2 &= 27 \\
x &= \pm\, 3\sqrt{3} \\
&\approx \pm 5.20
\end{aligned}$$

P.2.18.

$$\begin{aligned}
9x^2 &= 36 \\
x^2 &= 4 \\
x &= \pm\sqrt{4} = \pm\, 2
\end{aligned}$$

P.2.19.

$$\begin{aligned}
(2x-1)^2 &= 18 \\
2x - 1 &= \pm\sqrt{18} \\
2x &= 1 \pm 3\sqrt{2} \\
x &= \frac{1 \pm 3\sqrt{2}}{2} \\
&\approx 2.62,\ -1.62
\end{aligned}$$

P.2.20.

$$\begin{aligned}
(x-7)^2 &= (x+3)^2 \\
x - 7 &= \pm(x+3)
\end{aligned}$$

$$\begin{array}{lcl}
x - 7 = x + 3 & \text{or} & x - 7 = -x - 3 \\
-7 \neq 3 & \text{or} & 2x = 4 \\
 & & x = 2
\end{array}$$

The only solution of the equation is $x = 2$.

P.2.21.

$$\begin{aligned}
x^2+4x-32&=0\\
x^2+4x&=32\\
x^2+4x+2^2&=32+2^2\\
(x+2)^2&=36\\
x+2&=\pm 6\\
x&=-2\pm\ \ 6\\
x&=4 \quad\text{or}\quad x=-8
\end{aligned}$$

P.2.22.

$$\begin{aligned}
x^2-2x-3&=0\\
x^2-2x&=3\\
x^2-2x+(-1)^2&=3+(1)^2\\
(x-1)^2&=4\\
x-1&=\pm\sqrt{4}\\
x&=1\pm\ \ 2\\
x&=3 \quad\text{or}\quad x=-1
\end{aligned}$$

P.2.23.

$$\begin{aligned}
6x^2-12x&=-3\\
x^2-2x&=-\frac{1}{2}\\
x^2-2x+1^2&=-\frac{1}{2}+1^2\\
(x-1)^2&=\frac{1}{2}\\
x-1&=\pm\sqrt{\frac{1}{2}}\\
x&=1\pm\sqrt{\frac{1}{2}}\\
x&=1\pm\frac{\sqrt{2}}{2}
\end{aligned}$$

P.2.24.

$$
\begin{aligned}
4x^2 - 4x &= 1 \\
x^2 - x &= \frac{1}{4} \\
x^2 - x + \left(-\frac{1}{2}\right)^2 &= \frac{1}{4} + \left(-\frac{1}{2}\right)^2 \\
\left(x - \frac{1}{2}\right)^2 &= \frac{1}{2} \\
x - \frac{1}{2} &= \pm\frac{\sqrt{2}}{2} \\
x &= \frac{1}{2} \pm \frac{\sqrt{2}}{2} \\
x &= \frac{1 \pm \sqrt{2}}{2}
\end{aligned}
$$

P.2.25.

$$
\begin{aligned}
&2x^2 + x - 1 = 0 \\
&x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&\quad = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)} \\
&\quad = \frac{-1 \pm 3}{4} = \frac{1}{2},\quad -1
\end{aligned}
$$

P.2.26.

$$
\begin{aligned}
&2x^2 - x - 1 = 0 \\
&x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&\quad = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)} \\
&\quad = \frac{1 \pm \sqrt{1+8}}{4} \\
&\quad = \frac{1 \pm 3}{4} = 1, -\frac{1}{2}
\end{aligned}
$$

P.2.27.

$$\begin{aligned}
2x^2 - 7x + 1 &= 0 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(1)}}{2(2)} \\
&= \frac{7 \pm \sqrt{49 - 8}}{2(2)} \\
&= \frac{7 \pm \sqrt{41}}{4} \\
&= \frac{7}{4} \pm \frac{\sqrt{41}}{4}
\end{aligned}$$

P.2.28.

$$\begin{aligned}
3x + x^2 - 1 &= 0 \\
x^2 + 3x - 1 &= 0 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)} \\
&= \frac{-3 \pm \sqrt{13}}{2} = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}
\end{aligned}$$

P.2.29.

$$\begin{aligned}
2 + 2x - x^2 &= 0 \\
-x^2 + 2x + 2 &= 0 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-2 \pm \sqrt{2^2 - 4(-1)(2)}}{2(-1)} \\
&= \frac{-2 \pm 2\sqrt{3}}{-2} \\
&= 1 \pm \sqrt{3}
\end{aligned}$$

P.2.30.

$$\begin{aligned}
x^2+10+8x&=0\\
x^2+8x+10&=0\\
x&=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\\
&=\frac{-(8)\pm\sqrt{(8)^2-4(1)(10)}}{2(1)}\\
&=\frac{-8\pm\sqrt{64-40}}{2(1)}\\
&=\frac{-8\pm\sqrt{24}}{2}\\
&=\frac{-8\pm2\sqrt{6}}{2}\\
&=\frac{2\left(-4\pm\sqrt{6}\right)}{2}\\
&=-4\pm\sqrt{6}
\end{aligned}$$

P.2.31.

$$\begin{aligned}
(y-5)^2&=2y\\
y^2-12y+25&=0\\
y&=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\\
&=\frac{-(-12)\pm\sqrt{(-12)^2-4(1)(25)}}{2(1)}\\
&=\frac{12\pm2\sqrt{11}}{2}=6\pm\sqrt{11}
\end{aligned}$$

P.2.32.

$$\begin{aligned}
(z+6)^2&=-2z\\
z^2+12z+36&=-2z\\
z^2+14z+36&=0
\end{aligned}$$

$$\begin{aligned} z &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-14 \pm \sqrt{14^2 - 4(1)(36)}}{2(1)} \\ &= \frac{-14 \pm \sqrt{52}}{2} = -7 \pm \sqrt{13} \end{aligned}$$

P.2.33.

$$\begin{aligned} 6x^4 - 54x^2 &= 0 \\ 6x^2\left(x^2 - 9\right) &= 0 \\ 6x^2 = 0 &\Rightarrow x = 0 \\ x^2 - 9 = 0 &\Rightarrow x = \pm 3 \end{aligned}$$

P.2.34.

$$\begin{aligned} 5x^3 + 3 - x^2 + 45x &= 0 \\ 5x\left(x^2 + 6x + 9\right) &= 0 \\ 5x\left(x + 3\right)^2 &= 0 \\ 5x = 0 &\Rightarrow x = 0 \\ x + 3 = 0 &\Rightarrow x = -3 \end{aligned}$$

P.2.35.

$$\begin{aligned} x^3 + 2x^2 - 8x &= 16 \\ x^3 + 2x^2 - 8x - 16 &= 0 \\ x^2\left(x + 2\right) - 8\left(x + 2\right) &= 0 \\ \left(x + 2\right)\left(x^2 - 8\right) &= 0 \\ x + 2 = 0 &\Rightarrow x = -2 \\ x^2 - 8 = 0 &\Rightarrow \pm\sqrt{8} = \pm 2\sqrt{2} \end{aligned}$$

P.2.36.

$$\begin{aligned}
x^3 - 3x^2 - x &= -3\\
x^3 - 3x^2 - x + 3 &= 0\\
x^2(x-3) - (x-3) &= 0\\
(x-3)(x^2-1) &= 0\\
(x-3)(x+1)(x-1) &= 0\\
x - 3 = 0 &\Rightarrow x = 3\\
x + 1 = 0 &\Rightarrow x = -1\\
x - 1 = 0 &\Rightarrow x = 1
\end{aligned}$$

P.2.37.

$$\begin{aligned}
4 + \sqrt[3]{2x-9} &= 0\\
\sqrt[3]{2x-9} &= -4\\
\left(\sqrt[3]{2x-9}\right)^3 &= (-4)^3\\
2x - 9 &= -64\\
2x &= -55\\
x &= -\frac{55}{2}
\end{aligned}$$

P.2.38.

$$\begin{aligned}
\sqrt[3]{12-x} - 3 &= 0\\
\sqrt[3]{12-x} &= 3\\
\left(\sqrt[3]{12-x}\right)^3 &= (3)^3\\
12 - x &= 27\\
-x &= 15\\
x &= -15
\end{aligned}$$

P.2.39.

$$\begin{aligned}
\sqrt{x-3}+1 &= \sqrt{x}\\
\sqrt{x-3} &= \sqrt{x}-1\\
\left(\sqrt{x-3}\right)^2 &= \left(\sqrt{x}-1\right)^2\\
x-3 &= x-2\sqrt{x}+1\\
-4 &= -2\sqrt{x}\\
2 &= \sqrt{x}\\
(2)^2 &= \left(\sqrt{x}\right)^2\\
4 &= x
\end{aligned}$$

P.2.40.

$$\begin{aligned}
2\sqrt{x+1}-\sqrt{2x+3} &= 1\\
2\sqrt{x+1} &= 1+\sqrt{2x+3}\\
\left(2\sqrt{x+1}\right)^2 &= \left(1+\sqrt{2x+3}\right)^2\\
4(x+1) &= 1+2\sqrt{2x+3}+2x+3\\
2x &= 2\sqrt{2x+3}\\
x &= \sqrt{2x+3}\\
x^2 &= 2x+3\\
x^2-2x-3 &= 0\\
(x-3)(x+1) &= 0\\
x-3=0 &\Rightarrow x=3\\
x+1=0 &\Rightarrow x=-1, \text{ extraneous}
\end{aligned}$$

P.2.41.

$$\begin{aligned}
(x-5)^{3/2} &= 8\\
(x-5)^3 &= 8^2\\
x-5 &= \sqrt[3]{64}\\
x &= 5+4=9
\end{aligned}$$

P.2.42.

$$\begin{aligned}\left(x^2-x-22\right)^{3/2}&=27\\ x^2-x-22&=27^{2/3}\\ x^2-x-22&=9\\ x^2-x-31&=0\\ x&=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-31)}}{2(1)}=\frac{1\pm\sqrt{125}}{2}=\frac{1\pm5\sqrt{5}}{2}\end{aligned}$$

P.2.43.

$$\begin{aligned}3x(x-1)^{1/2}+2(x-1)^{3/2}&=0\\ (x-1)^{1/2}\left[3x+2(x-1)\right]&=0\\ (x-1)^{1/2}(5x-2)&=0\\ (x-1)^{1/2}&=0\Rightarrow x-1=0\Rightarrow x=1\\ 5x-2&=0\Rightarrow x=\tfrac{2}{5},\ \text{extraneous}\end{aligned}$$

P.2.44.

$$\begin{aligned}4x^2(x-1)^{1/3}+6x(x-1)^{4/3}&=0\\ 2x\left[2x(x-1)^{1/3}+3(x-1)^{4/3}\right]&=0\\ 2x(x-1)^{1/3}\left[2x+3(x-1)\right]&=0\\ 2x(x-1)^{1/3}(5x-3)&=0\\ 2x&=0\Rightarrow x=0\\ x-1&=0\Rightarrow x=1\\ 5x-3&=0\Rightarrow x=\tfrac{3}{5}\end{aligned}$$

P.2.45.

$$\begin{aligned}|2x-5|&=11\\ 2x-5&=11\Rightarrow x=8\\ -(2x-5)&=11\Rightarrow x=-3\end{aligned}$$

P.2.46.

$$\left|3x+2\right|=7$$
$$3x+2=7 \Rightarrow x=\tfrac{5}{3}$$
$$-\left(3x+2\right)=7$$
$$-3x-2=7 \Rightarrow x=-3$$

P.2.47.

$$\left|x+1\right|=x^2-5$$

First equation:

$$x+1=x^2-5$$
$$x^2-x-6=0$$
$$\left(x-3\right)\left(x+2\right)=0$$
$$x-3=0 \Rightarrow x=3$$
$$x+2=0 \Rightarrow x=-2$$

Second equation:

$$-\left(x+1\right)=x^2-5$$
$$-x-1=x^2-5$$
$$x^2+x-4=0$$
$$x=\frac{-1\pm\sqrt{17}}{2}$$

Only $x=3$ and $x=\dfrac{-1-\sqrt{17}}{2}$ are solutions of the original equation. $x=-2$ and $x=\dfrac{-1+\sqrt{17}}{2}$ are extraneous.

P.2.48.

$$\left|x^2+6x\right|=3x+18$$

First equation:

$$x^2+6x=3x+18$$
$$x^2+3x-18=0$$
$$\left(x-3\right)\left(x+6\right)=0$$
$$x-3=0 \Rightarrow x=3$$
$$x+6=0 \Rightarrow x=-6$$

Second equation:

$$-\left(x^2+6x\right)=3x+18$$
$$0=x^2+9x+18$$
$$0=\left(x+3\right)\left(x+6\right)$$
$$0=x+3 \Rightarrow x=-3$$
$$0=x+6 \Rightarrow x=-6$$

The solutions of the original equation are $x=\pm 3$ and $x=-6$.

P.2.49.

Let $y=18$.

$$y=0.514x-14.75$$
$$18=0.514x-14.75$$
$$32.75=0.514x$$
$$\frac{32.75}{0.514}=x$$
$$63.7\approx x$$

So, the height of the female is about 63.7 inches.

P.2.50.

Let $y = 23.$

$$\begin{aligned} y &= 0.532x - 17.03 \\ 23 &= 0.532x - 17.03 \\ 40.03 &= 0.532x \\ \frac{40.03}{0.532} &= x \\ 75.2 &\approx x \end{aligned}$$

Because 75.2 in. $\left(\dfrac{1\text{ ft}}{12\text{ in.}}\right) \approx 6.27\text{ ft}$ is about 6 feet 3 inches, it is possible the femur belongs to the missing man.

P.2.51.

False.

$$\begin{aligned} \sqrt{2x+1} &= -2 + \sqrt{x+1} \\ 2x+1 &= 4 - 4\sqrt{x+1} + (x+1) \\ x - 4 &= -4\sqrt{x+1} \\ x^2 - 8x + 16 &= 16(x+1) \\ x^2 - 24x &= 0 \\ x(x-24) &= 0 \end{aligned}$$

$x = 0 \qquad 1 \neq -2+1, \quad x = 24 \qquad 5 \neq -2+5$

P.2.52.

$$\begin{aligned} 2(x-3)+1 &= 2x-5 \\ 2x-6+1 &= 2x-5 \\ 2x-5 &= 2x-5 \end{aligned}$$

False. The equation is an identity, so every real number is a solution.

P.2.53.

$$\begin{aligned} \sqrt{x-10} - \sqrt{x-10} &= 0 \\ \sqrt{x-10} &= \sqrt{x-10} \end{aligned}$$

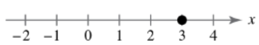
False. The equation is an identity, so every real number is a solution.

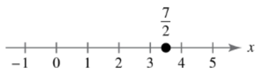
P.2.54.

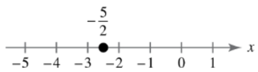
(a) The formula for volume of the glass cube is V = Length × Width × Height.
The volume of water in the cube is the length × width × height of the water.
So, the volume is $x \cdot x \cdot (x-3) = x^2(x-3)$.

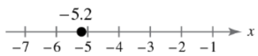
(b) Given the equation $x^2(x-3) = 320$. The dimensions of the glass cube can be found by solving for x. Then, substitute that value into the expression x^3 to find the volume of the cube.

P.2.55.

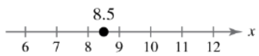
(a) 

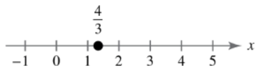
(b) 

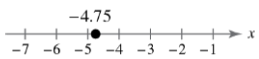
(c) 

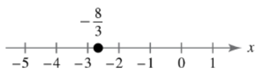
(d) 

P.2.56.

(a) 

(b) 

(c) 

(d) 

P.2.57.

$$\sqrt{25+20} = \sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$$

P.2.58.

$$\sqrt{284+321} = \sqrt{605} = \sqrt{5 \cdot 11^2} = 11\sqrt{5}$$

P.2.59.

$4x - 6$

(a) $4(-1) - 6 = -4 - 6 = -10$

(b) $4(0) - 6 = 0 - 6 = -6$

P.2.60.

$9 - 7x$

(a) $9 - 7(-3) = 9 + 21 = 30$

(b) $9 - 7(3) = 9 - 21 = -12$

P.2.61.

(a) When $x = -3$, $2x^3 = 2(-3)^3 = 2(-27) = -54$.

(b) When $x = 0$, $2x^3 = 2(0)^3 = 0$.

P.2.62.

(a) When $x = 0$, $-3x^{-4} = \dfrac{-3}{x^4} = -\dfrac{3}{0}$. Division by 0 is undefined.

(b) When $x = -2$, $-3x^{-4} = \dfrac{-3}{x^4} = \dfrac{-3}{(-2)^4} = -\dfrac{3}{16}$.

P.2.63.

$$-x^2 + (-x)^2 = -x^2 + x^2 = 0$$

P.2.64.

$$\begin{aligned} -x^3 + (-x)^3 + (-x)^4 &= -x^3 - x^3 + x^4 \\ &= x^4 - 2x^3 \end{aligned}$$

Section P.3 The Cartesian Plane and Graphs of Equations

P.3.1.

Cartesian

P.3.2.

Midpoint Formula

P.3.3.

solution or solution point

P.3.4.

graph

P.3.5.

(a) The x-axis is the horizontal real number line in the Cartesian plane.

(b) The y-axis is the vertical real number line in the Cartesian plane.

(c) The origin $(0, 0)$ is the point of intersection of the x- and y-axes.

(d) The x- and y-axes divide the Cartesian plane into four quadrants.

(e) The x-coordinate of the point (x, y) is the directed distance from the y-axis to the point.

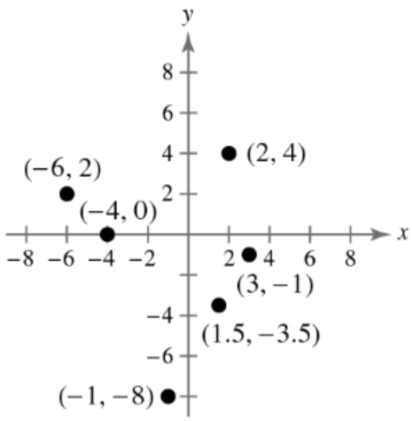
(f) The y-coordinate of the point (x, y) is the directed distance from the x-axis to the point.

P.3.6.

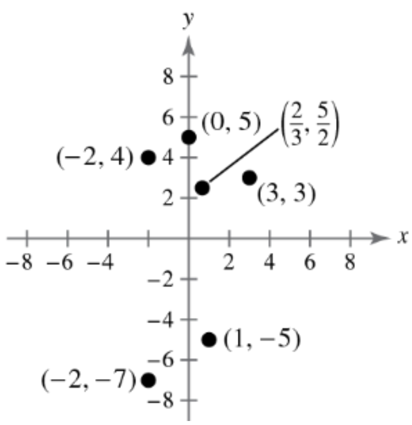
Let (x, y) be any point on the circle. The distance between the center (h, k)

and $(x,\ y)$ is the radius r. So, $\sqrt{(x-\ h)^2+(y-k)^2}=r$.

P.3.7.



P.3.8.



P.3.9.

$(-3, 4)$

P.3.10.

$(-12, 0)$

P.3.11.

$x>0$ and $y<0$ in Quadrant IV.

P.3.12.

$x<0$ and $y<0$ in Quadrant III.

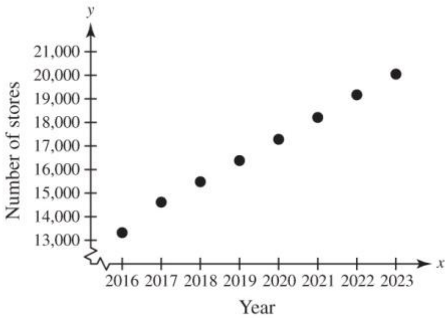
P.3.13.

$x+y=0, x\neq 0, y\neq 0$ means $x=-y$ or $y=-x$. This occurs in Quadrant II or IV.

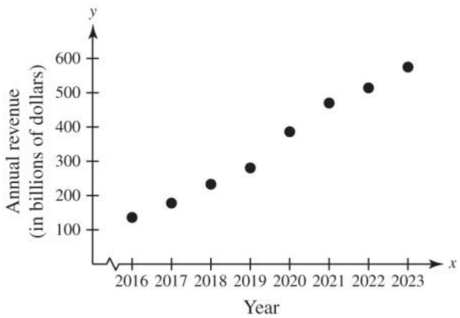
P.3.14.

(x, y), $xy > 0$ means x and y have the same signs. This occurs in Quadrant I or III.

P.3.15.



P.3.16.



P.3.17.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - (-2))^2 + (-6 - 6)^2} \\ &= \sqrt{(5)^2 + (-12)^2} \\ &= \sqrt{25 + 144} \\ &= 13 \text{ units} \end{aligned}$$

P.3.18.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0-8)^2 + (20-5)^2} \\ &= \sqrt{(-8)^2 + (15)^2} \\ &= \sqrt{64 + 225} \\ &= \sqrt{289} \\ &= 17 \text{ units} \end{aligned}$$

P.3.19.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(2 - \frac{1}{2}\right)^2 + \left(-1 - \frac{4}{3}\right)^2} \\ &= \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{7}{3}\right)^2} \\ &= \sqrt{\frac{9}{4} + \frac{49}{9}} \\ &= \sqrt{\frac{277}{36}} \\ &= \frac{\sqrt{277}}{6} \text{ units} \end{aligned}$$

P.3.20.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3.9 - 9.5)^2 + (8.2 - (-2.6))^2} \\ &= \sqrt{(-13.4)^2 + (10.8)^2} \\ &= \sqrt{179.56 + 116.64} \\ &= \sqrt{296.2} \end{aligned}$$

P.3.21.

(a) $(1, 0), (13, 5)$

$$\begin{aligned} \text{Distance} &= \sqrt{(13-1)^2 + (5-0)^2} \\ &= \sqrt{12^2 + 5^2} = \sqrt{169} = 13 \end{aligned}$$

$(13, 5), (13, 0)$

$\text{Distance} = |5 - 0| = |5| = 5$

$(1, 0), (13, 0)$

$\text{Distance} = |1 - 13| = |-12| = 12$

(b) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

P.3.22.

(a) The distance between $(-1, 1)$ and $(9, 1)$ is 10.

The distance between $(9, 1)$ and $(9, 4)$ is 3.

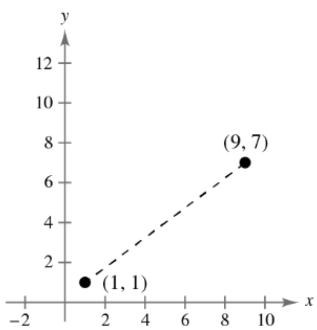
The distance between $(-1, 1)$ and $(9, 4)$ is

$$\sqrt{(9-(-1))^2 + (4-1)^2} = \sqrt{100+9} = \sqrt{109}.$$

(b) $10^2 + 3^2 = 109 = \left(\sqrt{109}\right)^2$

P.3.23.

(a)

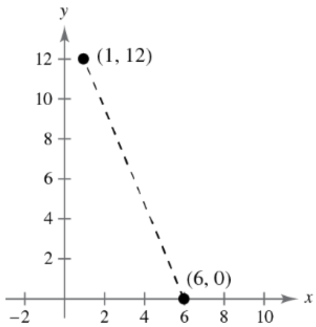


(b) $d = \sqrt{(9-1)^2 + (7-1)^2} = \sqrt{64+36} = 10$

(c) $\left(\dfrac{9+1}{2}, \dfrac{7+1}{2}\right) = (5, 4)$

P.3.24.

(a)

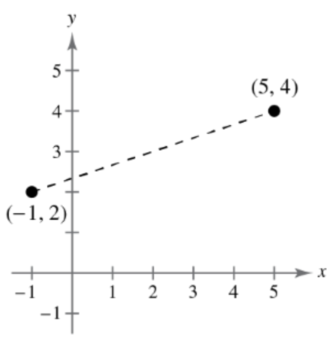


(b) $d = \sqrt{(1-6)^2 + (12-0)^2} = \sqrt{25+144} = 13$

(c) $\left(\dfrac{1+6}{2}, \dfrac{12+0}{2}\right) = \left(\dfrac{7}{2}, 6\right)$

P.3.25.

(a)

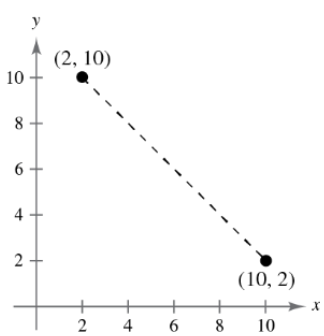


(b) $d = \sqrt{(5+1)^2 + (4-2)^2} + = \sqrt{36+4} = 2\sqrt{10}$

(c) $\left(\dfrac{-1+5}{2}, \dfrac{2+4}{2}\right) = (2, 3)$

P.3.26.

(a)



(b) $d = \sqrt{(2-10)^2 + (10-2)^2}$
$= \sqrt{64+64} = 8\sqrt{2}$

(c) $\left(\dfrac{2+10}{2}, \dfrac{10+2}{2}\right) = (6, 6)$

P.3.27.

$$\begin{aligned} d &= \sqrt{(42-18)^2 + (50-12)^2} \\ &= \sqrt{24^2 + 38^2} \\ &= \sqrt{2020} \\ &= 2\sqrt{505} \\ &\approx 45 \end{aligned}$$

The pass is about 45 yards.

P.3.28.

$$\begin{aligned} d &= \sqrt{120^2 + 150^2} \\ &= \sqrt{36,900} \\ &= 30\sqrt{41} \\ &\approx 192.09 \end{aligned}$$

The plane flies about 192 kilometers.

P.3.29.

$$\begin{aligned} \text{midpoint} &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{2021+2023}{2}, \frac{572.8+648.1}{2}\right) \\ &= (2022, 610.45) \end{aligned}$$

In 2022, the sales were about $610.45 billion.

P.3.30.

$$\text{midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

In this problem, you have $\left(\dfrac{2022+2024}{2}, \dfrac{8.59+y_2}{2}\right) = (2023, 14.87)$. So,

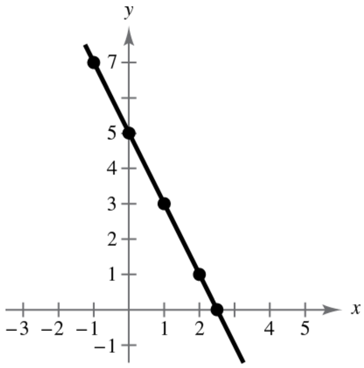
$$\frac{8.59 + y_2}{2} = 14.87$$
$$8.59 + y_2 = 29.74$$
$$y_2 = 21.15.$$

In 2024, the earnings per share were about \$21.15.

P.3.31.

$y = -2x + 5$

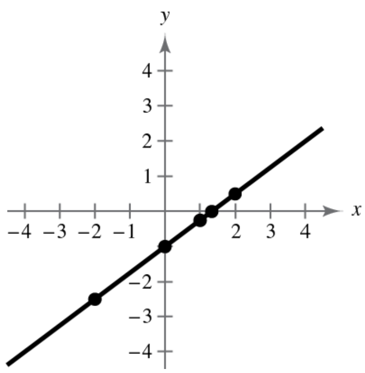
x	-1	0	1	2	$\frac{5}{2}$
y	7	5	3	1	0
(x, y)	$(-1, 7)$	$(0, 5)$	$(1, 3)$	$(2, 1)$	$\left(\frac{5}{2}, 0\right)$



P.3.32.

$y = \frac{3}{4}x - 1$

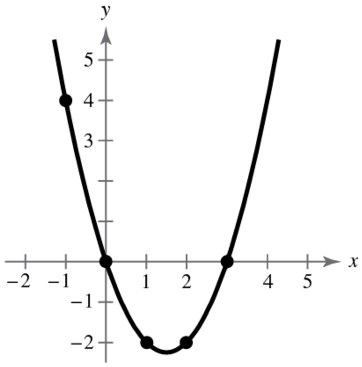
x	-2	0	1	$\frac{4}{3}$	2
y	$-\frac{5}{2}$	-1	$-\frac{1}{4}$	0	$\frac{1}{2}$
(x, y)	$\left(-2, -\frac{5}{2}\right)$	$(0, -1)$	$\left(1, -\frac{1}{4}\right)$	$\left(\frac{4}{3}, 0\right)$	$\left(2, \frac{1}{2}\right)$



P.3.33.

$y = x^2 - 3x$

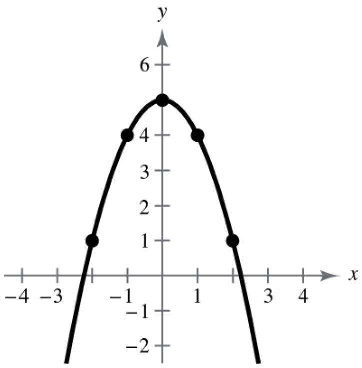
x	-1	0	1	2	3
y	4	0	-2	-2	0
(x, y)	$(-1, 4)$	$(0, 0)$	$(1, -2)$	$(2, -2)$	$(3, 0)$



P.3.34.

$y = 5 - x^2$

x	-2	-1	0	1	2
y	1	4	5	4	1
(x, y)	$(-2, 1)$	$(-1, 4)$	$(0, 5)$	$(1, 4)$	$(2, 1)$



P.3.35.

$y = 5x - 6$

x-intercept: $0 = 5x - 6$

$6 = 5x$

$\frac{6}{5} = x$

$\left(\frac{6}{5},\ 0\right)$

y-intercept: $y = 5(0) - 6 = -6$

$(0,\ -6)$

P.3.36.

$$y = 8 - 3x$$

x-intercept: $$\begin{aligned} 0 &= 8 - 3x \\ 3x &= 8 \\ x &= \tfrac{8}{3} \end{aligned}$$

$\left(\frac{8}{3},\ 0\right)$

y-intercept: $y = 8 - 3(0) = 8$

$(0,\ 8)$

P.3.37.

$$y = \sqrt{x+4}$$

x-intercept: $$\begin{aligned} 0 &= \sqrt{x+4} \\ 0 &= x + 4 \\ -4 &= x \end{aligned}$$

$(-4,\ 0)$

y-intercept: $y = \sqrt{0+4} = 2$

$(0,\ 2)$

P.3.38.

$$y = \sqrt{2x-1}$$

x-intercept: $$\begin{aligned} 0 &= \sqrt{2x-1} \\ 2x - 1 &= 0 \\ x &= \tfrac{1}{2} \end{aligned}$$

$\left(\frac{1}{2},\ 0\right)$

y-intercept: $$\begin{aligned} y &= \sqrt{2(0)-1} \\ &= \sqrt{-1} \end{aligned}$$

There is no real solution. There is no y-intercept.

P.3.39.

$y = |3x - 7|$

x-intercept: $0 = |3x - 7|$

$0 = 3x - 7$

$\frac{7}{3} = 0$

$\left(\frac{7}{3},\ 0\right)$

y-intercept: $y = |3(0) - 7| = 7$

$(0,\ 7)$

P.3.40.

$y = -\ |x + 10|$

x-intercept: $0 = -\ |x + 10|$

$x + 10 = 0$

$x = -10$

$(-10,\ 0)$

y-intercept: $y = -\ |0 + 10|$

$= -\ |10| = -10$

$(0,\ -10)$

P.3.41.

$y = 2x^3 - 4x^2$

x-intercept: $0 = 2x^3 - 4x^2$

$0 = 2x^2(x - 2)$

$x = 0 \quad \text{or} \quad x = 2$

$(0,\ 0),\ (2,\ 0)$

y-intercept: $y = 2(0)^3 - 4(0)^2$

$y = 0$

$(0,\ 0)$

P.3.42.

$y = x^4 - 25$

x-intercepts: $0 = x^4 - 25$

$x^4 = 25$

$x = \pm\sqrt[4]{5^2} = \pm\sqrt{5}$

$\left(\pm\sqrt{5},\ 0\right)$

y-intercept: $y = (0)^4 - 25 = -25$

$(0,\ -25)$

P.3.43.

$x^2 - y = 0$

$(-x)^2 - y = 0 \Rightarrow x^2 - y = 0 \Rightarrow$ y-axis symmetry

$x^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No x-axis symmetry

$(-x)^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No origin symmetry

P.3.44.

$x - y^2 = 0$

$(-x) - y^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No y-axis symmetry

$x - (-y)^2 = 0 \Rightarrow x - y^2 = 0 \Rightarrow$ x-axis symmetry

$(-x) - (-y)^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No origin symmetry

P.3.45.

$y = x^3$

$y = (-x)^3 \Rightarrow y = -x^3 \Rightarrow$ No y-axis symmetry

$-y = x^3 \Rightarrow y = -x^3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^3 \Rightarrow -y = -x^3 \Rightarrow y = x^3 \Rightarrow$ Origin symmetry

P.3.46.

$y = x^4 - x^2 + 3$

$y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = x^4 - x^2 + 3 \Rightarrow$ y-axis symmetry

$-y = x^4 - x^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No origin symmetry

P.3.47.

$$y = \frac{x}{x^2+1}$$
$$y = \frac{-x}{(-x)^2+1} \Rightarrow y = \frac{-x}{x^2+1} \Rightarrow \text{No } y\text{-axis symmetry}$$
$$-y = \frac{x}{x^2+1} \Rightarrow y = \frac{-x}{x^2+1} \Rightarrow \text{No } x\text{-axis symmetry}$$
$$-y = \frac{-x}{(-x)^2+1} \Rightarrow -y = \frac{-x}{x^2+1} \Rightarrow y = \frac{x}{x^2+1} \Rightarrow \text{Origin symmetry}$$

P.3.48.

$$y = \frac{1}{1+x^2}$$
$$y = \frac{1}{1+(-x)^2} \Rightarrow y = \frac{1}{1+x^2} \Rightarrow y\text{-axis symmetry}$$
$$-y = \frac{1}{1+x^2} \Rightarrow y = \frac{-1}{1+x^2} \Rightarrow \text{No } x\text{-axis symmetry}$$
$$-y = \frac{1}{1+(-x)^2} \Rightarrow y = \frac{-1}{1+x^2} \Rightarrow \text{No origin symmetry}$$

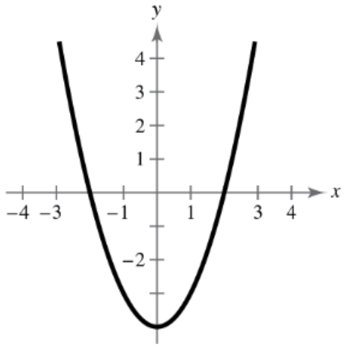
P.3.49.

$$xy^2 + 10 = 0$$
$$(-x)y^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow \text{No } y\text{-axis symmetry}$$
$$x(-y)^2 + 10 = 0 \Rightarrow xy^2 + 10 = 0 \Rightarrow x\text{-axis symmetry}$$
$$(-x)(-y)^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow \text{No origin symmetry}$$

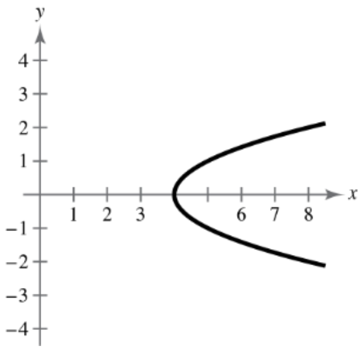
P.3.50.

$$xy = 4$$
$$(-x)y = 4 \Rightarrow xy = -4 \Rightarrow \text{No } y\text{-axis symmetry}$$
$$x(-y) = 4 \Rightarrow xy = -4 \Rightarrow \text{No } x\text{-axis symmetry}$$
$$(-x)(-y) = 4 \Rightarrow xy = 4 \Rightarrow \text{Origin symmetry}$$

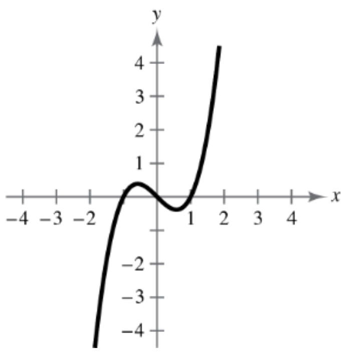
P.3.51.



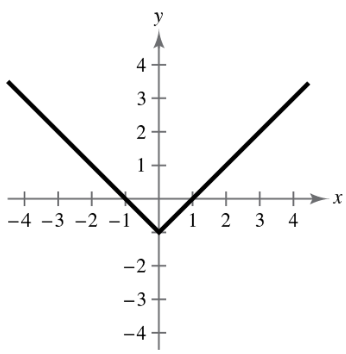
P.3.52.



P.3.53.



P.3.54.

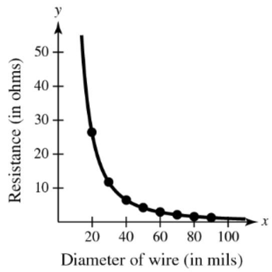


P.3.55.

(a)

x	5	10	20	30	40	50	60	70	80	90	100
y	414.8	103.7	25.93	11.52	6.48	4.15	2.88	2.12	1.62	1.28	1.04

(b)



When $x = 85.5$, the resistance is about 1.4 ohms.

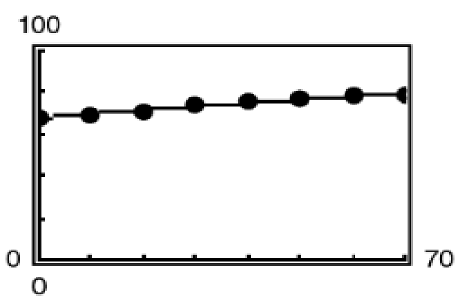
(c) When $x = 85.5$,

$$y = \frac{10,370}{(85.5)^2} = 1.42 \text{ ohms.}$$

(d) As the diameter of the copper wire increases, the resistance decreases.

P.3.56.

(a)



The model fits the data well.

Each data value is close to the graph of the model.

(b) Using the graph or the model $(\text{with } t = 40)$, the life expectancy is approximately 75.2 years.

(c) The model and the line $y = 70.1$ intersect at $t = 11.1$, corresponding to the year 1961.

Algebraically, $\dfrac{68.0 + 0.33(11.1)}{1 + 0.002(11.1)} \approx 70.1.$

(d) The y-intercept is $(0, 68.0)$. In 1950 $(t = 0)$, the life expectancy of a child (at birth) was 68.0 years.

(e) Answers will vary.

P.3.57.

Center: $(0, 0)$; Radius: 3

$$(x-0)^2 + (y-0)^2 = 3^2$$
$$x^2 + y^2 = 9$$

P.3.58.

Center: $(0, 0)$; Radius: 7

$$(x-0)^2 + (y-0)^2 = 7^2$$
$$x^2 + y^2 = 49$$

P.3.59.

Center: $(-4, 5)$; Radius: 2

$$(x-h)^2 + (y-k)^2 = r^2$$
$$[x-(-4)]^2 + [y-5]^2 = 2^2$$
$$(x+4)^2 + (y-5)^2 = 4$$

P.3.60.

Center: $(1, -3)$; Radius: $\sqrt{11}$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-1)^2 + [y-(-3)]^2 = \sqrt{11}^2$$
$$(x-1)^2 + (y+3)^2 = 11$$

P.3.61.

Center: $(3, 8)$; Solution point: $(-9, 13)$

$$\begin{aligned} r &= \sqrt{(x-h)^2 + (y-k)^2} \\ &= \sqrt{(-9-3)^2 + (13-8)^2} \\ &= \sqrt{(-12)^2 + (5)^2} \\ &= \sqrt{144+25} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-3)^2+(y-8)^2=13^2$$
$$(x-3)^2+(y-8)^2=169$$

P.3.62.

Center: $(-2,-6)$; Solution point: $(1,-10)$

$$\begin{aligned} r &= \sqrt{(x-h)^2+(y-k)^2} \\ &= \sqrt{[1-(-2)]^2+[-10-(-6)]^2} \\ &= \sqrt{(3)^2+(-4)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$(x-h)^2+(y-k)^2=r^2$$
$$[x-(-2)]^2+[y-(-6)]^2=5^2$$
$$(x+2)^2+(y+6)^2=25$$

P.3.63.

Endpoints of a diameter: $(3, 2), (-9, -8)$

$$\begin{aligned} r &= \frac{1}{2}\sqrt{(-9-3)^2+(-8-2)^2} \\ &= \frac{1}{2}\sqrt{(-12)^2+(-10)^2} \\ &= \frac{1}{2}\sqrt{144+100} \\ &= \frac{1}{2}\sqrt{244}=\frac{1}{2}(2)\sqrt{61}=\sqrt{61} \end{aligned}$$

$$(h,k): \left(\frac{3+(-9)}{2}\cdot\frac{2+(-8)}{2}\right)=\left(\frac{-6}{2}\cdot\frac{-6}{2}\right)$$
$$=(-3,-3)$$

$$(x-h)^2+(y-k)^2=r^2$$
$$[x-(-3)]^2+[y-(-3)]^2=\left(\sqrt{61}\right)^2$$
$$(x+3)^2+(y+3)^2=61$$

P.3.64.

Endpoints of a diameter: $(11, -5), (3, 15)$

$$
\begin{aligned}
r &= \frac{1}{2}\sqrt{(3-11)^2 + [15-(-5)]^2} \\
&= \frac{1}{2}\sqrt{(-8)^2 + (20)^2} \\
&= \frac{1}{2}\sqrt{64+400} \\
&= \frac{1}{2}\sqrt{464} = \frac{1}{2}(4)\sqrt{29} = 2\sqrt{29}
\end{aligned}
$$

$$(h, k): \left(\frac{11+3}{2} \cdot \frac{-5+15}{2}\right) = \left(\frac{14}{2}, \frac{10}{2}\right) = (7, 5)$$

$$
\begin{aligned}
(x-h)^2 + (y-k)^2 &= r^2 \\
(x-7)^2 + (y-5)^2 &= \left(2\sqrt{29}\right)^2 \\
(x-7)^2 + (y-5)^2 &= 116
\end{aligned}
$$

P.3.65.

False. The line $y = x$ is symmetric with respect to the origin.

P.3.66.

True. Because $x < 0$ and $y > 0$, $2x < 0$ and $-3y < 0$, which is located in Quadrant III.

P.3.67.

The test is for symmetry with respect to the x-axis. The statement should read: The graph of $x = 3y^2$ is symmetric with respect to the x-axis because replacing y with $-y$ yields an equivalent equation.

P.3.68.

x-axis symmetry:

$$
\begin{aligned}
x^2 + y^2 &= 1 \\
x^2 + (-y)^2 &= 1 \\
x^2 + y^2 &= 1
\end{aligned}
$$

y-axis symmetry:

$$
\begin{aligned}
x^2 + y^2 &= 1 \\
(-x)^2 + y^2 &= 1 \\
x^2 + y^2 &= 1
\end{aligned}
$$

Origin symmetry:

$$\begin{aligned} x^2 + y^2 &= 1 \\ (-x)^2 + (-y)^2 &= 1 \\ x^2 + y^2 &= 1 \end{aligned}$$

So, the graph of the equation is symmetric with respect to x-axis, y-axis, and origin.

P.3.69.

$$\frac{5-7}{12-18} = \frac{-2}{-6} = \frac{1}{3}$$

P.3.70.

$$\frac{16-6}{6-11} = \frac{10}{-5} = -2$$

P.3.71.

$$\frac{3-3}{4-0} = \frac{0}{4} = 0$$

P.3.72.

$$\frac{1-(-1)}{(9-9)} = \frac{2}{0} \rightarrow \text{Division by 0 is undefined.}$$

P.3.73.

$$\begin{aligned} 3y + 5 &= 0 \\ 3y &= -5 \\ y &= -\tfrac{5}{3} \end{aligned}$$

P.3.74.

$$\begin{aligned} 2x - 3y &= 5 \\ -3y &= 5 - 2x \\ y &= \frac{2x-5}{3} \end{aligned}$$

P.3.75.

$$\begin{aligned} 5x + 2y &= 8 \\ 2y &= 8 - 5x \\ y &= \frac{8-5x}{2} \end{aligned}$$

P.3.76.

$$x + 3y + 8 = 0 \Rightarrow 3y = -x - 8 \Rightarrow y = -\frac{x+8}{3}$$

P.3.77.

(a) $P = R - C$

$= 135x - (93x + 35{,}000)$

$= 42x - 35{,}000$

(b) $P = 42(5000) - 35{,}000 = \$175{,}000$

P.3.78.

(a) Time to copy one page: $\frac{1}{50}$ min

(b) Time to copy x pages: $\frac{x}{50}$ min

(c) Time to copy 120 pages: $\frac{120}{50} = \frac{12}{5} = 2.4$ min

(d) Time to copy 10,000 pages: $\frac{10{,}000}{50} = 200$ min

SECTION P.4 LINEAR EQUATIONS IN TWO VARIABLES

P.4.1.

linear

P.4.2.

slope

P.4.3.

point-slope

P.4.4.

parallel

P.4.5.

rate or rate of change

P.4.6.

linear extrapolation

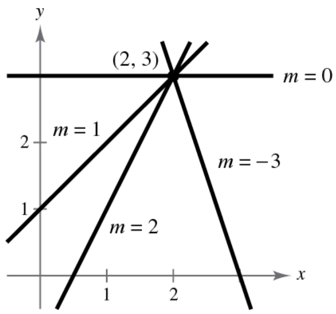
P.4.7.

They are perpendicular to each other.

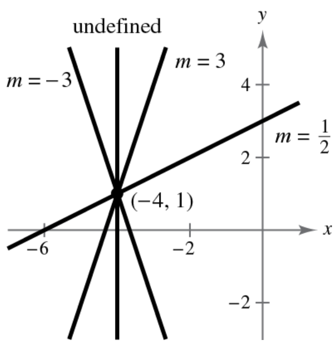
P.4.8.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - y_1 &= mx - mx_1 \\ mx_1 - y_1 &= mx - y \\ 0 &= mx - y - mx_1 + y_1 \\ mx - y - (mx_1 - y_1) &= 0 \end{aligned}$$

P.4.9.



P.4.10.



P.4.11.

Two points on the line: $(0, 0)$ and $(4, 6)$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6}{4} = \frac{3}{2}$$

P.4.12.

The line appears to go through $(0, 7)$ and $(7, 0)$.

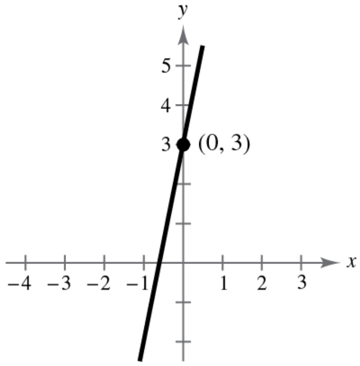
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 7}{7 - 0} = -1$$

P.4.13.

$y = 5x + 3$

Slope: $m = 5$

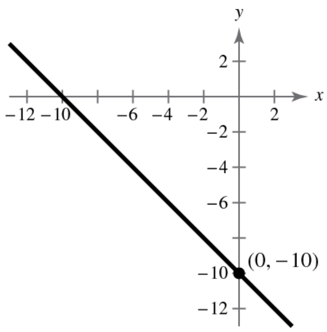
y-intercept: $(0, 3)$



P.4.14.

Slope: $m = -1$

y-intercept: $(0, -10)$

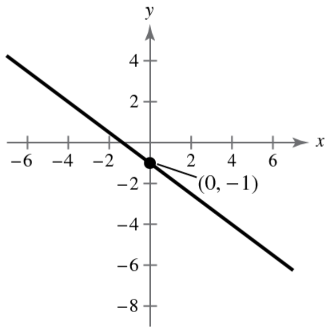


P.4.15.

$y = -\frac{3}{4}x - 1$

Slope: $m = -\frac{3}{4}$

y-intercept: $(0, -1)$

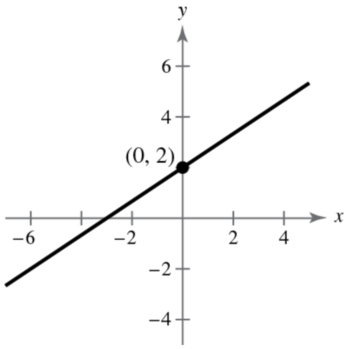


P.4.16.

$$y = \frac{2}{3}x + 2$$

Slope: $m = \frac{2}{3}$

y-intercept: $(0, 2)$

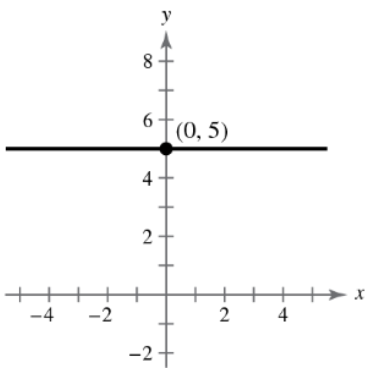


P.4.17.

$$y - 5 = 0$$
$$y = 5$$

Slope: $m = 0$

y-intercept: $(0, 5)$

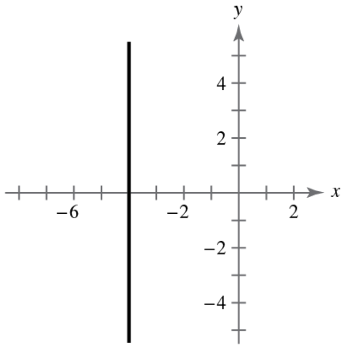


P.4.18.

$$x + 4 = 0$$
$$x = -4$$

Slope: undefined (vertical line)

y-intercept: none

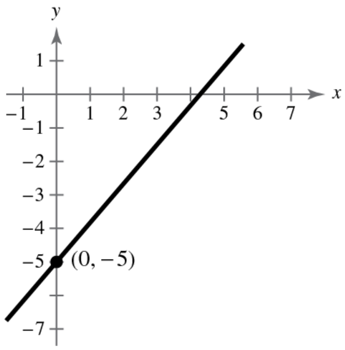


P.4.19.

$$7x - 6y = 30$$
$$-6y = -7x + 30$$
$$y = \frac{7}{6}x - 5$$

Slope: $m = \frac{7}{6}$

y-intercept: $(0, -5)$

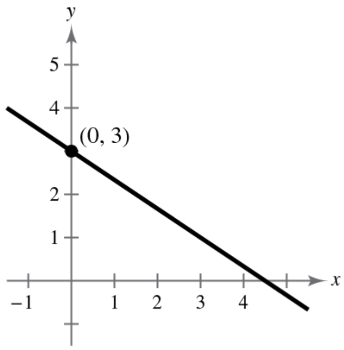


P.4.20.

$$2x + 3y = 9$$
$$3y = -2x + 9$$
$$y = -\tfrac{2}{3}x + 3$$

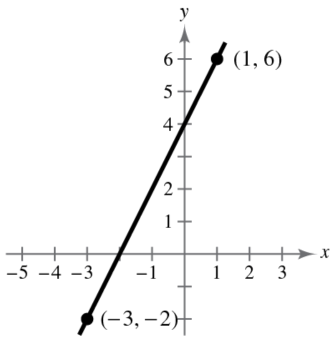
Slope: $m = -\frac{2}{3}$

y-intercept: $(0, 3)$



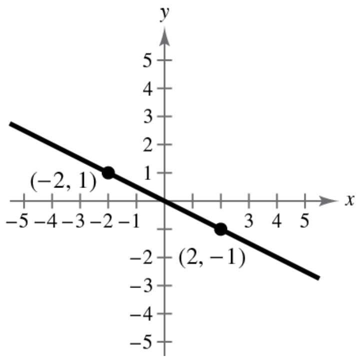
P.4.21.

$$m = \frac{6-(-2)}{1-(-3)} = \frac{8}{4} = 2$$



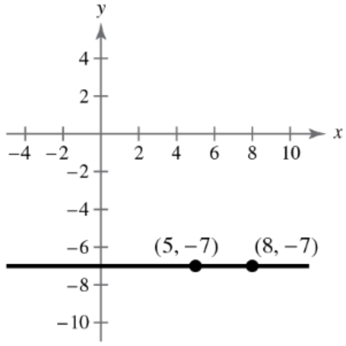
P.4.22.

$$m = \frac{1-(-1)}{-2-2} = \frac{2}{-4} = -\frac{1}{2}$$



P.4.23.

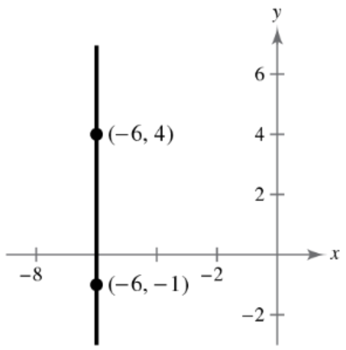
$$m = \frac{-7-(-7)}{8-5} = \frac{0}{3} = 0$$



P.4.24.

$$m = \frac{4-(-1)}{-6-(-6)} = \frac{5}{0}$$

m is undefined.



P.4.25.

Point: $(-5, 4)$, Slope: $m = 2$

Because $m = 2 = \frac{2}{1}$, y increases by 2 for every one unit increase in x. Three additional points are $(-4, 6)$, $(-3, 8)$, and $(-2, 10)$.

P.4.26.

Point: $(0, -9)$, Slope: $m = -2$

Because $m = -2$, y decreases by 2 for every one unit increase in x. Three other points are $(-2, -5)$, $(1, -11)$, and $(3, -15)$.

P.4.27.

Point: $(-4, 3)$, Slope is undefined.

Because m is undefined, x does not change. Three points are $(-4, 0)$, $(-4, 5)$, and $(-4, 2)$.

P.4.28.

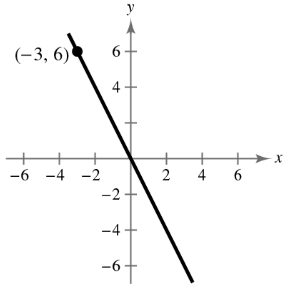
Point: $(2, 14)$, Slope is undefined.

Because m is undefined, x does not change. Three other points are $(2, -3)$, $(2, 0)$, and $(2, 4)$.

P.4.29.

Point: $(-3, 6)$; $m = -2$

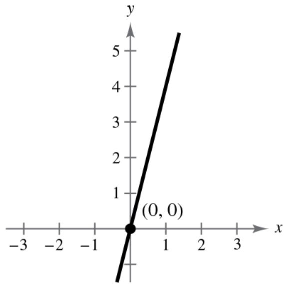
$$y - 6 = -2(x + 3)$$
$$y = -2x$$



P.4.30.

Point: $(0, 0)$; $m = 4$

$$y - 0 = 4(x - 0)$$
$$y = 4x$$

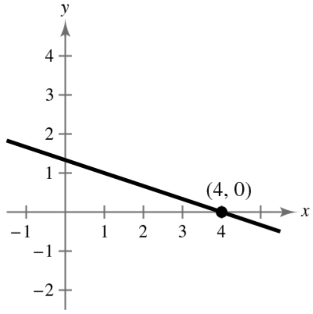


P.4.31.

Point: $(4, 0)$; $m = -\dfrac{1}{3}$

$$y - 0 = -\frac{1}{3}(x - 4)$$

$$y = -\frac{1}{3}x + \frac{4}{3}$$



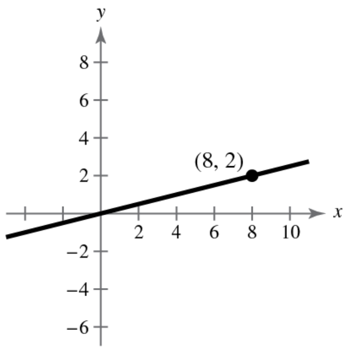
P.4.32.

Point: $(8, 2)$; $m = \dfrac{1}{4}$

$$y - 2 = \frac{1}{4}(x - 8)$$

$$y - 2 = \frac{1}{4}x - 2$$

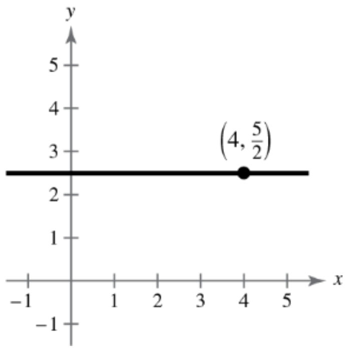
$$y = \frac{1}{4}x$$



P.4.33.

Point: $\left(4, \frac{5}{2}\right)$; $m = 0$

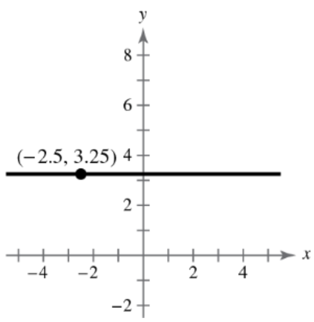
$$y - \frac{5}{2} = 0(x - 4)$$
$$y - \frac{5}{2} = 0$$
$$y = \frac{5}{2}$$



P.4.34.

Point: $(-2.5, 3.25)$; $m = 0$

$$y - 3.25 = 0(x - (-2.5))$$
$$y - 3.25 = 0$$
$$y = 3.25$$



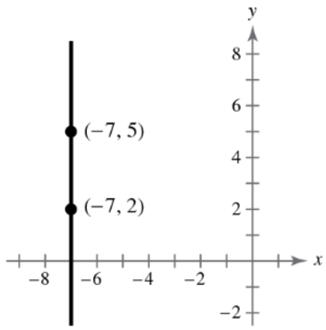
P.4.35.

$(-7, 2), (-7, 5)$

$$m = \frac{5 - 2}{-7 - (-7)} = \frac{3}{0}$$

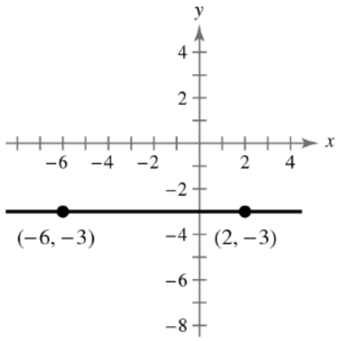
m is undefined.

$x = -7$



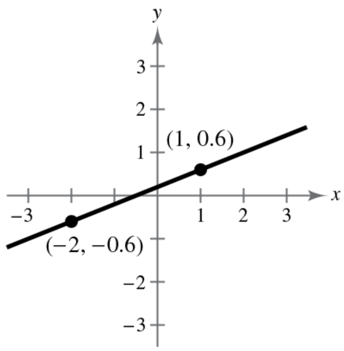
P.4.36.

$$(-6,-3),(2,-3)$$
$$y-4=\frac{-3-(-3)}{2-(-6)}(x+6)$$
$$y+3=0(x+6)$$
$$y+3=0$$
$$y=-3$$



P.4.37.

$$(1,0.6),(-2,-0.6)$$
$$y-0.6=\frac{-0.6-0.6}{-2-1}(x-1)$$
$$y=0.4(x-1)+0.6$$
$$y=0.4x+0.2$$



P.4.38.

$(-8, 0.6), (2, -2.4)$

$$y - 0.6 = \frac{-2.4 - 0.6}{2 - (-8)}(x + 8)$$

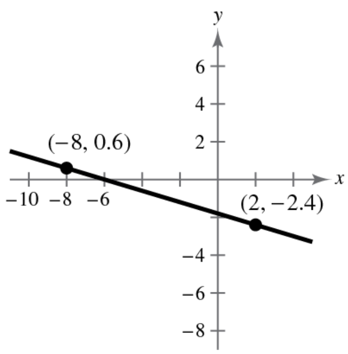
$$y - 0.6 = -\frac{3}{10}(x + 8)$$

$$10y - 6 = -3(x + 8)$$

$$10y - 6 = -3x - 24$$

$$10y = -3x - 18$$

$$y = -\frac{3}{10}x - \frac{9}{5} \text{ or } y = -0.3x - 1.8$$



P.4.39.

L_1: $(0, -1), (5, 9)$

$$m_1 = \frac{9 + 1}{5 - 0} = 2$$

L_2: $(0, 3), (4, 1)$

$$m_2 = \frac{1 - 3}{4 - 0} = -\frac{1}{2}$$

The slopes are negative reciprocals, so the lines are perpendicular.

P.4.40.

L_1: $(-2, -1), (1, 5)$

$$m_1 = \frac{5-(-1)}{1-(-2)} = \frac{6}{3} = 2$$

L_2: $(1, 3), (5, -5)$

$$m_2 = \frac{-5-3}{5-1} = \frac{-8}{4} = -2$$

The lines are neither parallel nor perpendicular.

P.4.41.

L_1: $(-6, -3), (2, -3)$

$$m_1 = \frac{-3-(-3)}{2-(-6)} = \frac{0}{8} = 0$$

L_2: $\left(3, -\frac{1}{2}\right), \left(6, -\frac{1}{2}\right)$

$$m_2 = \frac{-\frac{1}{2}-\left(-\frac{1}{2}\right)}{6-3} = \frac{0}{3} = 0$$

L_1 and L_2 are both horizontal lines, so they are parallel.

P.4.42.

L_1: $(4, 8), (-4, 2)$

$$m_1 = \frac{2-8}{-4-4} = \frac{-6}{-8} = \frac{3}{4}$$

L_2: $(3, -5), \left(-1, \frac{1}{3}\right)$

$$m_2 = \frac{\frac{1}{3}-(-5)}{-1-3} = \frac{\frac{16}{3}}{-4} = -\frac{4}{3}$$

The slopes are negative reciprocals, so the lines are perpendicular.

P.4.43.

$$4x - 2y = 3$$
$$y = 2x - \tfrac{3}{2}$$

Slope: $m = 2$

(a) $(2, 1), m = 2$

$$y - 1 = 2(x-2)$$
$$y = 2x - 3$$

(b) $(2, 1)$, $m = -\frac{1}{2}$

$$y - 1 = -\tfrac{1}{2}(x - 2)$$
$$y = -\tfrac{1}{2}x + 2$$

P.4.44.

$$x + y = 7$$
$$y = -x + 7$$

Slope: $m = -1$

(a) $m = -1$, $(-3, 2)$

$$y - 2 = -1(x + 3)$$
$$y - 2 = -x - 3$$
$$y = -x - 1$$

(b) $m = 1$, $(-3, 2)$

$$y - 2 = 1(x + 3)$$
$$y = x + 5$$

P.4.45.

$$3x + 4y = 7$$
$$y = -\frac{3}{4}x + \frac{7}{4}$$

Slope: $m = -\frac{3}{4}$

(a) $\left(-\frac{2}{3}, \frac{7}{8}\right)$, $m = -\frac{3}{4}$

$$y - \frac{7}{8} = -\frac{3}{4}\left(x - \left(-\frac{2}{3}\right)\right)$$
$$y = -\frac{3}{4}x + \frac{3}{8}$$

(b) $\left(-\frac{2}{3}, \frac{7}{8}\right)$, $m = \frac{4}{3}$

$$y - \frac{7}{8} = \frac{4}{3}\left(x - \left(-\frac{2}{3}\right)\right)$$
$$y = \frac{4}{3}x + \frac{127}{72}$$

P.4.46.

$$\begin{aligned} 5x + 3y &= 0 \\ 3y &= -5x \\ y &= -\frac{5}{3}x \end{aligned}$$

Slope: $m = -\frac{5}{3}$

(a) $m = -\frac{5}{3}, \left(\frac{7}{8}, \frac{3}{4}\right)$

$$\begin{aligned} y - \frac{3}{4} &= -\frac{5}{3}\left(x - \frac{7}{8}\right) \\ 24y - 18 &= -40\left(x - \frac{7}{8}\right) \\ 24y - 18 &= -40x + 35 \\ 24y &= -40x + 53 \\ y &= -\frac{5}{3}x + \frac{53}{24} \end{aligned}$$

(b) $m = \frac{3}{5}, \left(\frac{7}{8}, \frac{3}{4}\right)$

$$\begin{aligned} y - \frac{3}{4} &= \frac{3}{5}\left(x - \frac{7}{8}\right) \\ 40y - 30 &= 24\left(x - \tfrac{7}{8}\right) \\ 40y - 30 &= 24x - 21 \\ 40y &= 24x + 9 \\ y &= \frac{3}{5}x + \frac{9}{40} \end{aligned}$$

P.4.47.

$$\begin{aligned} y + 5 &= 0 \\ y &= -5 \end{aligned}$$

Slope: $m = 0$

(a) $(-2, 4)$, $m = 0$

$y = 4$

(b) $(-2, 4)$, m is undefined.

$x = -2$

P.4.48.

$$x - 4 = 0$$
$$x = 4$$

Slope: m is undefined.

(a) $(3, -2)$, m is undefined.

$$x = 3$$

(b) $(3, -2)$, $m = 0$

$$y = -2$$

P.4.49.

$$x - y = 4$$
$$y = x - 4$$

Slope: $m = 1$

(a) $(2.5, 6.8)$, $m = 1$

$$y - 6.8 = 1(x - 2.5)$$
$$y = x + 4.3$$

(b) $(2.5, 6.8)$, $m = -1$

$$y - 6.8 = (-1)(x - 2.5)$$
$$y = -x + 9.3$$

P.4.50.

$$6x + 2y = 9$$
$$2y = -6x + 9$$
$$y = -3x + \tfrac{9}{2}$$

Slope: $m = -3$

(a) $(-3.9, -1.4)$, $m = -3$

$$y - (-1.4) = -3(x - (-3.9))$$
$$y + 1.4 = -3x - 11.7$$
$$y = -3x - 13.1$$

(b) $(-3.9, -1.4)$, $m = \tfrac{1}{3}$

$$y - (-1.4) = \tfrac{1}{3}(x - (-3.9))$$
$$y + 1.4 = \tfrac{1}{3}x + 1.3$$
$$y = \tfrac{1}{3}x - 0.1$$

P.4.51.

$(3, 0), (0, 5)$

$$\frac{x}{3}+\frac{y}{5}=1$$

$$(15)\left(\frac{x}{3}+\frac{y}{5}\right)=1(15)$$

$$5x+3y-15=0$$

P.4.52.

$(-3, 0), (0, 4)$

$$\frac{x}{-3}+\frac{y}{4}=1$$

$$(-12)\frac{x}{-3}+(-12)\frac{y}{4}=(-12)\cdot 1$$

$$4x-3y+12=0$$

P.4.53.

$\left(-\frac{1}{6}, 0\right), \left(0, -\frac{2}{3}\right)$

$$\frac{x}{-1/6}+\frac{y}{-2/3}=1$$

$$6x+\frac{3}{2}y=-1$$

$$12x+3y+2=0$$

P.4.54.

$\left(\frac{2}{3}, 0\right), (0, -2)$

$$\frac{x}{2/3}+\frac{y}{-2}=1$$

$$\frac{3x}{2}-\frac{y}{2}=1$$

$$3x-y-2=0$$

P.4.55.

(a) $m=135.$ The sales are increasing 135 units per year.

(b) $m=0.$ There is no change in sales during the year.

(c) $m=-40.$ The sales are decreasing 40 units per year.

P.4.56.

(a) The slopes of each line segment are:

$$\frac{265.60-229.23}{18-17}=36.37 \qquad \frac{260.17-265.60}{19-18}=-5.43$$

$$\frac{274.52-260.17}{20-19}=14.35 \qquad \frac{365.82-274.52}{21-20}=91.3$$

$$\frac{394.33-365.82}{22-21}=28.51 \qquad \frac{383.29-394.33}{23-22}=-11.04$$

The sales showed the greatest increase from 2020 to 2021. The sales showed the greatest decrease from 2022 to 2023.

(b) $m=\dfrac{383.29-229.23}{23-17}=\dfrac{154.06}{6}\approx 25.68$

The slope of the line is about 25.68.

(c) Sales increased at an average rate of about \$25.68 billion per year from 2017 to 2023.

P.4.57.

Using the points $(0, 32)$ and $(100, 212)$, where the first coordinate represents a temperature in degrees Celsius and the second coordinate represents a temperature in degrees Fahrenheit, you have

$$m=\frac{212-32}{100-0}=\frac{180}{100}=\frac{9}{5}.$$

Since the point $(0, 32)$ is the F-intercept, $b=32$, the equation is

$$F=1.8C+32 \text{ or } C=\frac{5}{9}F-\frac{160}{9}.$$

P.4.58.

(a) Using the points $(1, 970)$ and $(3, 1270)$, you have $m=\dfrac{1270-970}{3-1}=\dfrac{300}{2}=150.$

Using the point-slope form with $m=150$ and the point $(1, 970)$, you have

$$\begin{aligned} y-y_1&=m(t-t_1)\\ y-970&=150(t-1)\\ y-970&=150t-150\\ y&=150t+820. \end{aligned}$$

(b) The slope is $m=150.$ The slope tells you the amount of increase in the weight of an average male child's brain each year.

(c) Let $t = 2$:

$$y = 150(2) + 820$$
$$y = 300 + 820$$
$$y = 1120$$

The average brain weight at age 2 is 1120 grams.

(d) Answers will vary.

(e) Answers will vary. *Sample answer:* No. The brain stops growing after reaching a certain age.

P.4.59.

Using the points $(0, 1100)$ and $(5, 0)$, where the first coordinate represents the year t and the second coordinate represents the value V, you have

$$V = \frac{0 - 1100}{5 - 0}t + 1100 = -220t + 1100,\ 0 \le t \le 5.$$

P.4.60.

Using the points $(0, 30{,}000)$ and $(10, 2500)$, where the first coordinate represents the year t and the second coordinate represents the value V, you have

$$m = \frac{2500 - 30{,}000}{10 - 0} = \frac{-27{,}500}{10} = -2750.$$

The point $(0, 30{,}000)$ is the V-intercept, so $b = 30{,}000$. The equation is

$$V = -2750t + 30{,}000,\ 0 \le t \le 10.$$

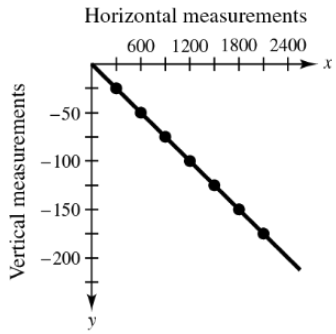
P.4.61.

$$y = \frac{6}{100}x$$
$$y = \frac{6}{100}(200) = 12 \text{ feet}$$

P.4.62.

(a) and (b)

x	300	600	900	1200	1500	1800	2100
y	–25	–50	–75	–100	–125	–150	–175



(c) $m = \dfrac{-50-(-25)}{600-300} = \dfrac{-25}{300} = -\dfrac{1}{12}$

$$y-(-50) = -\frac{1}{12}(x-600)$$
$$y+50 = -\frac{1}{12}x+50$$
$$y = -\frac{1}{12}x$$

(d) Because $m = -\dfrac{1}{12}$, for every change in the horizontal measurement of 12 feet, the vertical measurement decreases by 1 foot.

(e) $\dfrac{1}{12} \approx 0.083 = 8.3\%$ grade

P.4.63.

(a) Total Cost = cost for fuel and maintainance + cost for operator + purchase cost

$$C = 9.5t + 11.5t + 42{,}000$$
$$C = 21t + 42{,}000$$

(b) Revenue = Rate per hour · Hours

$$R = 45t$$

(c) $P = R - C$

$$P = 45t - (21t + 42{,}000)$$
$$P = 24t - 42{,}000$$

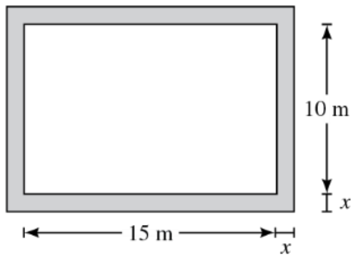
(d) Let $P = 0$, and solve for t.

$$0 = 24t - 42{,}000$$
$$42{,}000 = 24t$$
$$1750 = t$$

The equipment must be used 1750 hours to yield a profit of 0 dollars.

P.4.64.

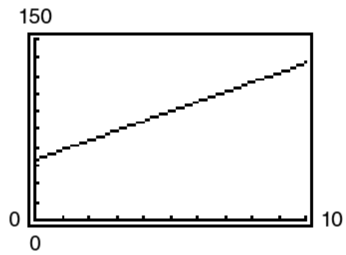
(a)



(b) $y = 2(15 + 2x) + 2(10 + 2x) = 8x + 50$

Because $m = 8$, each 1-meter increase in x will increase y by 8 meters.

(c)



P.4.65.

False. The slope with the greatest magnitude corresponds to the steepest line.

P.4.66.

$(-8, 2)$ and $(-1, 4)$: $m_1 = \dfrac{4-2}{-1-(-8)} = \dfrac{2}{7}$

$(0, -4)$ and $(-7, 7)$: $m_2 = \dfrac{7-(-4)}{-7-0} = \dfrac{11}{-7}$

False. The lines are not parallel.

P.4.67.

$$\begin{aligned} d_1 &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-0)^2 + (m_1 - 0)^2} \\ &= \sqrt{1 + (m_1)^2} \end{aligned}$$

$$\begin{aligned} d_2 &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-0)^2 + (m_2 - 0)^2} \\ &= \sqrt{1 + (m_2)^2} \end{aligned}$$

Using the Pythagorean Theorem:

$$(d_1)^2 + (d_2)^2 = \left(\text{distance between } (1, m_1), \text{ and } (1, m_2)\right)^2$$

$$\left(\sqrt{1+(m_1)^2}\right)^2 + \left(\sqrt{1+(m_2)^2}\right)^2 = \left(\sqrt{(1-1)^2+(m_2-m_1)^2}\right)^2$$

$$1+(m_1)^2+1+(m_2)^2 = (m_2-m_1)^2$$

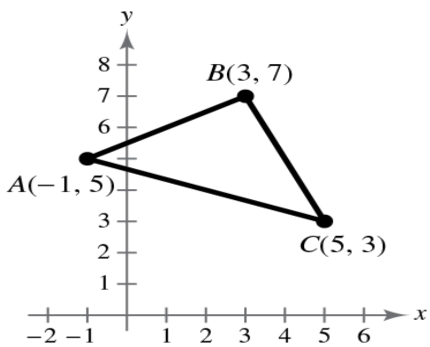
$$(m_1)^2+(m_2)^2+2 = (m_2)^2 - 2m_1m_2 + (m_1)^2$$

$$2 = -2m_1m_2$$

$$-\frac{1}{m_2} = m_1$$

P.4.68.

Find the slope of the line segments between the points *A* and *B*, and *B* and *C*.



$$m_{AB} = \frac{7-5}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$$

$$m_{BC} = \frac{3-7}{5-3} = \frac{-4}{2} = -2$$

Since the slopes are negative reciprocals, the line segments are perpendicular and therefore intersect to form a right angle. So, the triangle is a right triangle.

P.4.69.

(a) The slope is $\frac{0-(-1)}{2-0} = \frac{1}{2}$, not 2.

(b) The y-intercept is $(0, -3)$, not $(0, 4)$.

P.4.70.

(a) Matches graph (ii).

The slope is −20, which represents the decrease in the amount of the loan each week. The y-intercept is $(0, 200)$, which represents the original amount of the loan.

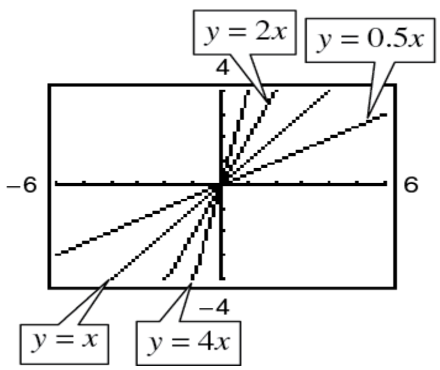
(b) Matches graph (iii).
The slope is 2, which represents the increase in the hourly wage for each unit produced. The y-intercept is $(0, 12.5)$, which represents the hourly rate if the employee produces no units.

(c) Matches graph (i).
The slope is 0.32, which represents the increase in travel cost for each mile driven. The y-intercept is $(0, 32)$, which represents the fixed cost of \$30 per day for meals. This amount does not depend on the number of miles driven.

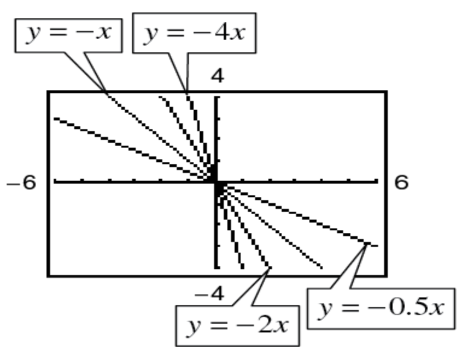
(d) Matches graph (iv).
The slope is –100, which represents the amount by which the computer depreciates each year. The y-intercept is $(0, 750)$, which represents the original purchase price.

P.4.71.

The line $y = 4x$ rises most quickly.



The line $y = -4x$ falls most quickly.



The greater the magnitude of the slope (the absolute value of the slope), the faster the line rises or falls.

P.4.72.

Because $\left|-4\right| > \left|\frac{5}{2}\right|$, the steeper line is the one with a slope of −4. The slope with the greatest magnitude corresponds to the steepest line.

P.4.73.

$$-\frac{x}{4}$$

When $x = -2$: $-\dfrac{x}{4} = -\dfrac{(-2)}{4} = \dfrac{1}{2}$

When $x = 0$: $-\dfrac{x}{4} = -\dfrac{0}{4} = 0$

When $x = 3$: $-\dfrac{x}{4} = -\dfrac{3}{4}$

When $x = 6$: $-\dfrac{x}{4} = -\dfrac{6}{4} = -\dfrac{3}{2}$

P.4.74.

$$x^2 - x$$

When $x = -2$: $(-2)^2 - (-2) = 4 + 2 = 6$

When $x = 0$: $0^2 - 0 = 0$

When $x = 3$: $3^2 - 3 = 9 - 3 = 6$

When $x = 6$: $6^2 - 6 = 36 - 6 = 30$

P.4.75.

$$\frac{7 - x^3}{10}$$

When $x = -2$: $\dfrac{7 - (-2)^3}{10} = \dfrac{7 + 8}{10} = \dfrac{15}{10} = \dfrac{3}{2}$

When $x = 0$: $\dfrac{7 - 0^3}{10} = \dfrac{7}{10}$

When $x = 3$: $\dfrac{7 - 3^3}{10} = \dfrac{7 - 27}{10} = -\dfrac{20}{10} = -2$

When $x = 6$: $\dfrac{7 - 6^3}{10} = \dfrac{7 - 216}{10} = -\dfrac{209}{10}$

P.4.76.

$$\frac{4\sqrt{2+x}}{3}$$

When $x = -2$: $\dfrac{4\sqrt{2+(-2)}}{3} = \dfrac{4\sqrt{0}}{3} = 0$

When $x = 0$: $\dfrac{4\sqrt{2+0}}{3} = \dfrac{4\sqrt{2}}{3}$

When $x = 3$: $\dfrac{4\sqrt{2+3}}{3} = \dfrac{4\sqrt{5}}{3}$

When $x = 6$: $\dfrac{4\sqrt{2+6}}{3} = \dfrac{4\ \ 2\sqrt{2}}{3} = \dfrac{8\sqrt{2}}{3}$

P.4.77.

$$\begin{aligned} 2x^3 - 5x^2 - 2x + 5 &= 0 \\ x^2(2x-5) - (2x-5) &= 0 \\ (x^2-1)(2x-5) &= 0 \\ x &= \pm 1, \frac{5}{2} \end{aligned}$$

P.4.78.

$$\begin{aligned} x^3 - 3x^2 - 11x + 28 &= 1 - 2x \\ x^3 - 3x^2 - 9x + 27 &= 0 \\ x^2(x-3) - 9(x-3) &= 0 \\ (x^2-9)(x-3) &= 0 \end{aligned}$$

$x^2 - 9 = 0 \Rightarrow x = \pm 3$

$x - 3 = 0 \Rightarrow x = 3$

The solutions are $x = \pm 3$.

P.4.79.

$$\begin{aligned} x - 6\sqrt{x} + 5 &= 0 \\ (\sqrt{x}-1)(\sqrt{x}-5) &= 0 \end{aligned}$$

$\sqrt{x} - 1 = 0 \Rightarrow x = 1$

$\sqrt{x} - 5 = 0 \Rightarrow x = 25$

The solutions are $x = 1,\ 25$.

P.4.80.

$$\begin{aligned} x^4 - 5x^2 + 4 &= 0 \\ (x^2 - 4)(x^2 - 1) &= 0 \\ (x+2)(x-2)(x+1)(x-1) &= 0 \\ x &= \pm 2, \pm 1 \end{aligned}$$

P.4.81.

$$\begin{aligned} 3\sqrt{x} - 5\sqrt{18} &= 0 \\ 3\sqrt{x} &= 5(3)\sqrt{2} \\ \sqrt{x} &= 5\sqrt{2} \\ x &= 25(2) = 50 \end{aligned}$$

P.4.82.

$$\begin{aligned} (x+1)^{3/5} - 8 &= 0 \\ (x+1)^{3/5} &= 8 \\ (x+1) &= 8^{5/3} = 2^5 = 32 \\ x &= 31 \end{aligned}$$

P.4.83.

$$\begin{aligned} x &= \frac{2}{x} + 1 \\ x^2 &= 2 + x \\ x^2 - x - 2 &= 0 \\ (x-2)(x+1) &= 0 \\ x &= 2, -1 \end{aligned}$$

P.4.84.

$$\begin{aligned} \frac{3}{x} - \frac{1}{2} &= \frac{x}{16} \\ \frac{6-x}{2x} &= \frac{x}{16} \\ 96 - 16x &= 2x^2 \\ x^2 + 8x - 48 &= 0 \\ (x+12)(x-4) &= 0 \end{aligned}$$

$x + 12 = 0 \Rightarrow x = -12$

$x - 4 = 0 \Rightarrow x = 4$

The solutions are $x = -12,\ 4$.

P.4.85.

$$x + \left| x - 9 \right| = 7$$
$$\left| x - 9 \right| = 7 - x$$

If $x - 9 \geq 0,$ then $x - 9 = 7 - x \Rightarrow 2x = 16 \Rightarrow x = 8,$ which is extraneous.

If $x - 9 < 0,$ then $-\left(x - 9\right) = 7 - x,$ which is extraneous.

No real solution.

P.4.86.

$$\left| 3x - 6 \right| = x^2 - 4$$

If $3x - 6 \geq 0$, then $3x - 6 = x^2 - 4 \Rightarrow x^2 - 3x - 2 = 0$

$\Rightarrow \left(x - 2\right)\left(x - 1\right) = 0 \Rightarrow x = 1,\ 2$

$\left(x = 1 \text{ is extraneous.}\right)$

s

If $3x - 6 < 0$, then $6 - 3x = x^2 - 4 \Rightarrow x^2 + 3x - 10 = 0$

$\Rightarrow \left(x + 5\right)\left(x - 2\right) = 0 \Rightarrow x = 2, -5$

Solutions: $x = 2, -5$

P.4.87.

$$\frac{\left[\left(x - 1\right)^2 + 1\right] - \left(x^2 + 1\right)}{x} = \frac{\left(x^2 - 2x + 1 + 1\right) - \left(x^2 + 1\right)}{x}$$
$$= \frac{-2x + 1}{x}$$

P.4.88.

$$\frac{\left[\left(t + 1\right)^2 - \left(t + 1\right) - 2\right] - \left(t^2 - t - 2\right)}{t} = \frac{\left(t^2 + 2t + 1 - t - 1 - 2\right) - \left(t^2 - t - 2\right)}{t}$$
$$= \frac{t^2 + t - 2 - t^2 + t + 2}{t}$$
$$= \frac{2t}{t} = 2,\ t \neq 0$$

P.4.89.

$$\begin{aligned}\frac{4-x^2-3}{x-1}+x^2+x+4&=\frac{4-x^2-3+(x-1)(x^2+x+4)}{x-1}\\&=\frac{4-x^2-3+(x^3+x^2+4x-x^2-x-4)}{x-1}\\&=\frac{4-x^2-3+x^3+3x-4}{x-1}\\&=\frac{x^3-x^2+3x-3}{x-1}\\&=\frac{x^2(x-1)+3(x-1)}{x-1}\\&=\frac{(x-1)(x^2+3)}{x-1}\\&=x^2+3,\quad x\neq 1\end{aligned}$$

Alternate solution:

$$\begin{aligned}\frac{4-x^2-3}{x-1}+x^2+x+4&=\frac{(1-x)(1+x)}{x-1}+x^2+x+4\\&=-1-x+x^2+x+4,\quad x\neq 1\\&=x^2+3,\ x\neq 1\end{aligned}$$

P.4.90.

$$\begin{aligned}\frac{x^3+x}{x^2-4}-\frac{5}{2(x+2)}+\frac{5}{2(x-2)}+2&=\frac{2(x^3+x)-5(x-2)+5(x+2)+4(x^2-4)}{2(x^2-4)}\\&=\frac{2x^3+2x-5x+10+5x+10+4x^2-16}{2(x^2-4)}\\&=\frac{2x^3+4x^2+2x+4}{2(x^2-4)}\\&=\frac{x^3+2x^2+x+2}{x^2-4}\\&=\frac{x^2(x+2)+(x+2)}{x^2-4}\\&=\frac{(x+2)(x^2+1)}{(x+2)(x-2)}\\&=\frac{x^2+1}{x-2},\ x\neq -2\end{aligned}$$

Section P.5 Functions

P.5.1.

independent; dependent

P.5.2.

difference quotient

P.5.3.

A relation is any set of ordered pairs. A function is a relation in which no two ordered pairs have the same first component and different second components.

P.5.4.

For $g(x) = 4x - 5$, to find $g(x+2)$, substitute $x+2$ for x, and simplify:

$$\begin{aligned} g(x+2) &= 4(x+2) - 5 \\ &= 4x + 3. \end{aligned}$$

P.5.5.

The domain of a piece-wise function must be explicitly described, so that it can determine which equation is used to evaluate the function.

P.5.6.

The domain of $y = \sqrt{4 - x^2}$ is determined by solving the inequality $4 - x^2 \geq 0$.

P.5.7.

Yes, the relationship is a function. Each domain value is matched with exactly one range value.

P.5.8.

No, the relationship is not a function. The domain value of –1 is matched with two output values.

P.5.9.

No, it does not represent a function. The input values of 10 and 7 are each matched with two output values.

P.5.10.

Yes, the table does represent a function. Each input value is matched with exactly one output value.

Input, x	–2	0	2	4	6
Output, y	1	1	1	1	1

P.5.11.

$$x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$$

No, y *is not* a function of x.

P.5.12.

$$x^2 - y = 9 \Rightarrow y = x^2 - 9$$

Yes, y *is* a function of x.

P.5.13.

$$y = \sqrt{16 - x^2}$$

Yes, y *is* a function of x.

P.5.14.

$$y = \sqrt{x + 5}$$

Yes, y *is* a function of x.

P.5.15.

$$y = 4 - |x|$$

Yes, y *is* a function of x.

P.5.16.

$$|y| = 4 - x \Rightarrow y = 4 - x \text{ or } y = -(4 - x)$$

No, y *is not* a function of x.

P.5.17.

$$y = -75 \text{ or } y = -75 + 0x$$

Yes, y *is* a function of x.

P.5.18.

$$x - 1 = 0$$
$$x = 1$$

No, this *is not* a function of x.

P.5.19.

$$g(t) = 4t^2 - 3t + 5$$

(a) $g(2) = 4(2)^2 - 3(2) + 5 = 16 - 6 + 5 = 15$

(b) $g(-1) = 4(-1)^2 - 3(-1) + 5 = 4 + 3 + 5 = 12$

(c) $g(t+2) = 4(t+2)^2 - 3(t+2) + 5$
$= 4(t^2 + 4t + 4) - 3t - 6 + 5$
$= 4t^2 + 13t + 15$

P.5.20.

$V(r) = \frac{4}{3}\pi r^3$

(a) $V(3) = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi$

(b) $V\left(\frac{3}{2}\right) = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{27}{8}\right) = \frac{9}{2}\pi$

(c) $V(2r) = \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi(8r^3) = \frac{32}{3}\pi r^3$

P.5.21.

$f(y) = 3 - \sqrt{y}$

(a) $f(4) = 3 - \sqrt{4} = 1$

(b) $f(0.25) = 3 - \sqrt{0.25} = 2.5$

(c) $f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$

P.5.22.

$f(x) = \sqrt{x+8} + 2$

(a) $f(-8) = \sqrt{(-8)+8} + 2 = 2$

(b) $f(1) = \sqrt{(1)+8} + 2 = 5$

(c) $f(x-8) = \sqrt{(x-8)+8} + 2 = \sqrt{x} + 2$

P.5.23.

$q(x) = \dfrac{1}{x^2 - 9}$

(a) $q(0) = \dfrac{1}{0^2 - 9} = -\dfrac{1}{9}$

(b) $q(3) = \dfrac{1}{3^2 - 9}$ is undefined.

(c) $q(y+3) = \dfrac{1}{(y+3)^2 - 9} = \dfrac{1}{y^2 + 6y}$

P.5.24.

$$q(t)=\frac{2t^2+3}{t^2}$$

(a) $q(2)=\dfrac{2(2)^2+3}{(2)^2}=\dfrac{8+3}{4}=\dfrac{11}{4}$

(b) $q(0)=\dfrac{2(0)^2+3}{(0)^2}$

Division by zero is undefined.

(c) $q(-x)=\dfrac{2(-x)^2+3}{(-x)^2}=\dfrac{2x^2+3}{x^2}$

P.5.25.

$$f(x)=\frac{|x|}{x}$$

(a) $f(2)=\dfrac{|2|}{2}=1$

(b) $f(-2)=\dfrac{|-2|}{-2}=-1$

(c) $f(x-1)=\dfrac{|x-1|}{x-1}=\begin{cases}-1, & \text{if } x<1\\ 1, & \text{if } x>1\end{cases}$

P.5.26.

$$f(x)=|x|+4$$

(a) $f(2)=|2|+4=6$

(b) $f(-2)=|-2|+4=6$

(c) $f(x^2)=|x^2|+4=x^2+4$

P.5.27.

$$\begin{aligned} f(x)&=15-3x\\ 15-3x&=0\\ 3x&=15\\ x&=5\end{aligned}$$

P.5.28.

$$\begin{aligned} f(x) &= 4x + 6 \\ 4x + 6 &= 0 \\ 4x &= -6 \\ x &= -\tfrac{3}{2} \end{aligned}$$

P.5.29.

$$\begin{aligned} f(x) &= \frac{3x-4}{5} \\ \frac{3x-4}{5} &= 0 \\ 3x - 4 &= 0 \\ x &= \frac{4}{3} \end{aligned}$$

P.5.30.

$$\begin{aligned} f(x) &= \frac{12-x^2}{8} \\ \frac{12-x^2}{8} &= 0 \\ x^2 &= 12 \\ x &= \pm\sqrt{12} = \pm 2\sqrt{3} \end{aligned}$$

P.5.31.

$$\begin{aligned} f(x) &= x^2 - 81 \\ x^2 - 81 &= 0 \\ x^2 &= 81 \\ x &= \pm 9 \end{aligned}$$

P.5.32.

$$\begin{aligned} f(x) &= x^2 - 6x - 16 \\ x^2 - 6x - 16 &= 0 \\ (x-8)(x+2) &= 0 \\ x - 8 = 0 &\Rightarrow x = 8 \\ x + 2 = 0 &\Rightarrow x = -2 \end{aligned}$$

P.5.33.

$$f(x) = x^3 - x$$
$$x^3 - x = 0$$
$$x(x^2 - 1) = 0$$
$$x(x+1)(x-1) = 0$$
$$x = 0,\ x = -1, \text{ or } x = 1$$

P.5.34.

$$f(x) = x^3 - x^2 - 3x + 3$$
$$x^3 - x^2 - 3x + 3 = 0$$
$$x^2(x-1) - 3(x-1) = 0$$
$$(x-1)(x^2-3) = 0$$
$$x - 1 = 0 \Rightarrow x = 1$$
$$x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3}$$

P.5.35.

$$f(x) = g(x)$$
$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x - 2 = 0 \quad x + 1 = 0$$
$$x = 2 \quad\quad x = -1$$

P.5.36.

$$f(x) = g(x)$$
$$x^2 + 2x + 1 = 5x + 19$$
$$x^2 - 3x - 18 = 0$$
$$(x-6)(x+3) = 0$$
$$x - 6 = 0 \quad x + 3 = 0$$
$$x = 6 \quad\quad x = -3$$

P.5.37.

$$f(x) = g(x)$$
$$x^4 - 2x^2 = 2x^2$$
$$x^4 - 4x^2 = 0$$
$$x^2(x^2 - 4) = 0$$
$$x^2(x+2)(x-2) = 0$$

$$x^2 = 0 \Rightarrow x = 0$$
$$x + 2 = 0 \Rightarrow x = -2$$
$$x - 2 = 0 \Rightarrow x = 2$$

P.5.38.

$$f(x) = g(x)$$
$$\sqrt{x} - 4 = 2 - x$$
$$x + \sqrt{x} - 6 = 0$$
$$(\sqrt{x} + 3)(\sqrt{x} - 2) = 0$$

$\sqrt{x} + 3 = 0 \Rightarrow \sqrt{x} = -3,$ which is a contradiction, since $\sqrt{x}$ represents the principal square root.

$\sqrt{x} - 2 = 0 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$

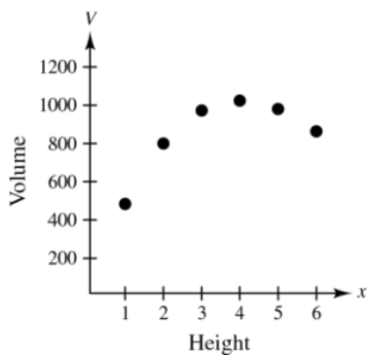
P.5.39.

(a)

Height, x	Volume, V
1	484
2	800
3	972
4	1024
5	980
6	864

The volume is maximum when $x = 4$ and $V = 1024$ cubic centimeters.

(b)



V is a function of x.

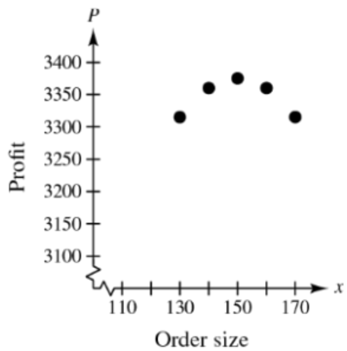
$V = x(24 - 2x)^2$

Domain: $0 < x < 12$

P.5.40.

(a) The maximum profit is \$3375.

(b)



Yes, P is a function of x.

$$\begin{aligned}\text{Profit} &= \text{Revenue} - \text{Cost}\\ &= \begin{pmatrix}\text{price}\\ \text{per unit}\end{pmatrix}\begin{pmatrix}\text{number}\\ \text{of units}\end{pmatrix} - (\text{cost})\begin{pmatrix}\text{number}\\ \text{of units}\end{pmatrix}\\ &= \left[90 - (x - 100)(0.15)\right]x - 60x, \quad x > 100\\ &= (90 - 0.15x + 15)x - 60x\\ &= (105 - 0.15x)x - 60x\\ &= 105x - 0.15x^2 - 60x\\ &= 45x - 0.15x^2, \quad x > 100\end{aligned}$$

P.5.41.

$$y = -\frac{1}{10}x^2 + 3x + 6$$

$$y(25) = -\frac{1}{10}(25)^2 + 3(25) + 6 = 18.5 \text{ feet}$$

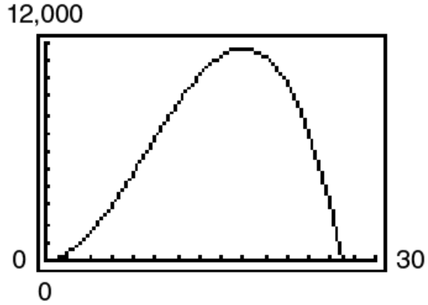
If the child holds a glove at a height of 5 feet, then the ball *will* be over the child's head because it will be at a height of 18.5 feet.

P.5.42.

(a) $V = l \cdot w \cdot h = x \cdot y \cdot x = x^2 y$ where $4x + y = 108$. So, $y = 108 - 4x$ and $V = x^2(108 - 4x) = 108x^2 - 4x^3$.

Domain: $0 < x < 27$

(b)

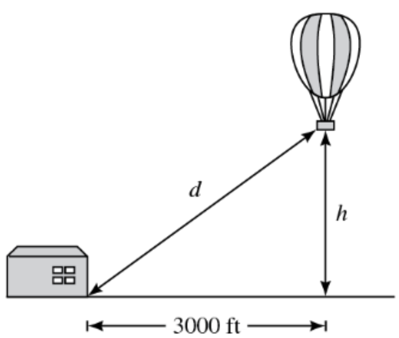


(c) The dimensions that will maximize the volume of the package are $18 \times 18 \times 36$. From the graph, the maximum volume occurs when $x = 18$. To find the dimension for y, use the equation $y = 108 - 4x$.

$$y = 108 - 4x = 108 - 4(18) - 72 = 36$$

P.5.43.

(a)



(b) $(3000)^2 + h^2 = d^2$

$$h = \sqrt{d^2 - (3000)^2}$$

Domain: $d \geq 3000$ (because both $d \geq 0$ and $d^2 - (3000)^2 \geq 0$)

P.5.44.

$$p(t)=\begin{cases}0.38t+81.2, & 17\le t\le 21\\ 1.35t+57.5, & 21\le t\le 23\end{cases}$$

For 2017 to 2020, use the first equation.

2017: $p(17) = 0.38(17) + 81.2 = 87.66\%$

2018: $p(18) = 0.38(18) + 81.2 = 88.04\%$

2019: $p(19) = 0.38(19) + 81.2 = 88.42\%$

2020: $p(20) = 0.38(20) + 81.2 = 88.80\%$

For 2021 through 2023, use the second equation.

2021: $p(21) = 1.35(21) + 57.5 = 85.85\%$

2022: $p(22) = 1.35(22) + 57.5 = 87.20\%$

2023: $p(23) = 1.35(23) + 57.5 = 88.55\%$

P.5.45.

$$p(t)=\begin{cases}4.011t^2-76.89t+586.7, & 7\le t<12\\ 14.94t+70.0, & 12\le t<17\\ 8.352t^2-304.97t-3098.5, & 17\le t<22\\ 0.234t^2-16.93t-692.5, & 22\le t\le 24\end{cases}$$

For 2007 through 2011, use the first equation.

$$2007: p(7)=4.011(7)^2-76.89(7)+586.7=245.009\Rightarrow \$245,009$$

$$2008: p(8)=4.011(8)^2-76.89(8)+586.7=228.284\Rightarrow \$228,284$$

$$2009: p(9)=4.011(9)^2-76.89(9)+586.7=219.581\Rightarrow \$219,581$$

$$2010: p(10)=4.011(10)^2-76.89(10)+586.7=218.900\Rightarrow \$218,900$$

$$2011: p(11)=4.011(11)^2-76.89(11)+586.7=226.241\Rightarrow \$226,241$$

For 2012 through 2016, use the second equation.

$$2012: p(12)=14.94(12)+70.0=249.28\Rightarrow \$249,280$$

$$2013: p(13)=14.94(13)+70.0=264.22\Rightarrow \$264,220$$

$$2014: p(14)=14.94(14)+70.0=279.16\Rightarrow \$279,160$$

$$2015: p(15)=14.94(15)+70.0=294.1\Rightarrow \$294,100$$

$$2016: p(16)=14.94(16)+70.0=309.04\Rightarrow \$309,040$$

For 2017 through 2021, use the third equation.

$$2017: p(17) = 8.352(17)^2 - 304.97(17) + 3098.5 = 327.738 \Rightarrow \$327,738$$
$$2018: p(18) = 8.352(18)^2 - 304.97(18) + 3098.5 = 315.088 \Rightarrow \$315,088$$
$$2019: p(19) = 8.352(19)^2 - 304.97(19) + 3098.5 = 319.142 \Rightarrow \$319,142$$
$$2020: p(20) = 8.352(20)^2 - 304.97(20) + 3098.5 = 339.9 \Rightarrow \$339,900$$
$$2021: p(21) = 8.352(21)^2 - 304.97(21) + 3098.5 = 377.362 \Rightarrow \$377,362$$

For 2022 through 2024, use the fourth equation.

$$2022: p(22) = 0.234(22)^2 + 16.93(22) + 292.5 = 433.296 \Rightarrow \$433,296$$
$$2023: p(23) = 0.234(23)^2 + 16.93(23) + 692.5 = 426.896 \Rightarrow \$426,896$$
$$2024: p(24) = 0.234(23)^2 + 16.93(24) + 692.5 = 420.964 \Rightarrow \$420,964$$

P.5.46.

$$F(y) = 149.76\sqrt{10}y^{5/2}$$

(a)

y	5	10	20	30	40
$F(y)$	26,474.08	149,760.00	847,170.49	2,334,527.36	4,792,320

The force, in tons, of the water against the dam increases with the depth of the water.

(b) It appears that approximately 21 feet of water would produce 1,000,000 tons of force.

(c) $$1,000,000 = 149.76\sqrt{10}y^{5/2}$$
$$\frac{1,000,000}{149.76\sqrt{10}} = y^{5/2}$$
$$2111.56 \approx y^{5/2}$$
$$21.37 \text{ feet} \approx y$$

P.5.47.

(a) $$R = n(\text{rate}) = n\left[8.00 - 0.05(n - 80)\right], n \geq 80$$
$$R = 12.00n - 0.05n^2 = 12n - \frac{n^2}{20} = \frac{240n - n^2}{20}, n \geq 80$$

(b)

n	90	100	110	120	130	140	150
$R(n)$	\$675	\$700	\$715	\$720	\$715	\$700	\$675

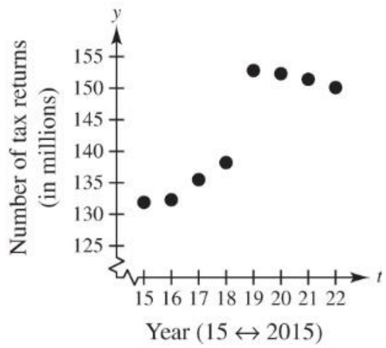
The revenue is maximum when 120 people take the trip.

P.5.48.

(a) $\dfrac{f(2022)-f(2015)}{2022-2015}=\dfrac{150.1-131.9}{7}=\dfrac{18.2}{7}=2.6$

The number of e-filed tax returns increased at an average rate of 2,600,000 per year from 2015 to 2022.

(b)



(c) *Sample answer:* $N=2.6t+92.9,\ 15\le t\le 22$

(d) *Sample answer:*

t	15	16	17	18	19	20	21	22
N	131.9	134.5	137.1	139.7	142.3	144.9	147.5	150.1

(e) The model does not appear to be a good fit for the actual data.

(f) Using technology, $N=3.43t+79.7$. The models are similar.

P.5.49.

$$\begin{aligned}
f(x)&=x^2-2x+4\\
f(2+h)&=(2+h)^2-2(2+h)+4\\
&=4+4h+h^2-4-2h+4\\
&=h^2+2h+4\\
f(2)&=(2)^2-2(2)+4=4\\
f(2+h)-f(2)&=h^2+2h\\
\frac{f(2+h)-f(2)}{h}&=\frac{h^2+2h}{h}=h+2,\ h\neq 0
\end{aligned}$$

P.5.50.

$$f(x) = 5x - x^2$$

$$\begin{aligned} f(5+h) &= 5(5+h) - (5+h)^2 \\ &= 25 + 5h - (25 + 10h + h^2) \\ &= 25 + 5h - 25 - 10h - h^2 \\ &= -h^2 - 5h \end{aligned}$$

$$\begin{aligned} f(5) &= 5(5) - (5)^2 \\ &= 25 - 25 = 0 \end{aligned}$$

$$\begin{aligned} \frac{f(5+h) - f(5)}{h} &= \frac{-h^2 - 5h}{h} \\ &= \frac{-h(h+5)}{h} = -(h+5),\ h \neq 0 \end{aligned}$$

P.5.51.

$$f(x) = x^3 + 3x$$

$$\begin{aligned} f(x+h) &= (x+h)^3 + 3(x+h) \\ &= x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h \end{aligned}$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{(x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h) - (x^3 + 3x)}{h} \\ &= \frac{h(3x^2 + 3xh + h^2 + 3)}{h} \\ &= 3x^2 + 3xh + h^2 + 3,\ h \neq 0 \end{aligned}$$

P.5.52.

$$f(x) = 4x^3 - 2x$$

$$\begin{aligned} f(x+h) &= 4(x+h)^3 - 2(x+h) \\ &= 4(x^3 + 3x^2h + 3xh^2 + h^3) - 2x - 2h \\ &= 4x^3 + 12x^2h + 12xh^2 + 4h^3 - 2x - 2h \end{aligned}$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{(4x^3 + 12x^2h + 12xh^2 + 4h^3 - 2x - 2h) - (4x^3 - 2x)}{h} \\ &= \frac{12x^2h + 12xh^2 + 4h^3 - 2h}{h} \\ &= \frac{h(12x^2 + 12xh + 4h^2 - 2)}{h} \\ &= 12x^2 + 12xh + 4h^2 - 2,\ h \neq 0 \end{aligned}$$

P.5.53.

$$g(x) = \frac{1}{x^2}$$

$$\frac{g(x) - g(3)}{x - 3} = \frac{\frac{1}{x^2} - \frac{1}{9}}{x - 3}$$

$$= \frac{9 - x^2}{9x^2(x - 3)}$$

$$= \frac{-(x + 3)(x - 3)}{9x^2(x - 3)}$$

$$= -\frac{x + 3}{9x^2},\ x \neq 3$$

P.5.54.

$$f(t) = \frac{1}{t - 2}$$

$$f(1) = \frac{1}{1 - 2} = -1$$

$$\frac{f(t) - f(1)}{t - 1} = \frac{\frac{1}{t - 2} - (-1)}{t - 1}$$

$$= \frac{1 + (t - 2)}{(t - 2)(t - 1)}$$

$$= \frac{(t - 1)}{(t - 2)(t - 1)}$$

$$= \frac{1}{t - 2},\ t \neq 1$$

P.5.55.

$$f(x) = \sqrt{5x}$$

$$\frac{f(x) - f(5)}{x - 5} = \frac{\sqrt{5x} - 5}{x - 5},\ x \neq 5$$

P.5.56.

$$f(x) = x^{2/3} + 1$$

$$f(8) = 8^{2/3} + 1 = 5$$

$$\frac{f(x) - f(8)}{x - 8} = \frac{x^{2/3} + 1 - 5}{x - 8} = \frac{x^{2/3} - 4}{x - 8},\ x \neq 8$$

P.5.57.

False. The equation $y^2 = x^2 + 4$ is a relation between *x* and *y*. However, $y = \pm\sqrt{x^2 + 4}$ does not represent a function.

P.5.58.

True. A function is a relation by definition.

P.5.59.

False. The range is $[-1, \infty)$.

P.5.60.

True. The set represents a function. Each *x*-value is mapped to exactly one *y*-value.

P.5.61.

By plotting the points, you have a parabola, so $g(x) = cx^2$. Because $(-4, -32)$ is on the graph, you have $-32 = c(-4)^2 \Rightarrow c = -2$. So, $g(x) = -2x^2$.

P.5.62.

By plotting the data, you can see that they represent a line, or $f(x) = cx$. Because $(0, 0)$ and $\left(1, \frac{1}{4}\right)$ are on the line, the slope is $\frac{1}{4}$. So, $f(x) = \frac{1}{4}x$.

P.5.63.

Because the function is undefined at 0, you have $r(x) = c/x$. Because $(-4, -8)$ is on the graph, you have $-8 = c/-4 \Rightarrow c = 32$. So, $r(x) = 32/x$.

P.5.64.

By plotting the data, you can see that they represent $h(x) = c\sqrt{|x|}$. Because $\sqrt{|-4|} = 2$ and $\sqrt{|-1|} = 1$, and the corresponding *y*-values are 6 and 3, $c = 3$ and $h(x) = 3\sqrt{|x|}$.

P.5.65.

The domain of $f(x) = \sqrt{x - 1}$ includes $x = 1$, $x \geq 1$ and the domain of $g(x) = \dfrac{1}{\sqrt{x - 1}}$ does not include $x = 1$ because you cannot divide by 0. The domain of $g(x) = \dfrac{1}{\sqrt{x-1}}$ is $x > 1$. So, the functions do not have the same domain.

P.5.66.

(a) The height h is a function of t because for each value of t there is a corresponding value of h for $0 \le t \le 2.6$.

(b) Using the graph when $t = 0.5, h \approx 20$ feet and when $t = 1.25, h \approx 28$ feet.

(c) The domain of h is approximately $0 \le t \le 2.6$.

(d) No, the time t is not a function of the height h because some values of h correspond to more than one value of t.

P.5.67.

No; x is the independent variable, f is the name of the function.

P.5.68.

$$f(x) = x^{2/3} + 1$$
$$f(8) = 8^{2/3} + 1 = 5$$
$$\frac{f(x) - f(8)}{x-8} = \frac{x^{2/3} + 1 - 5}{x-8} = \frac{x^{2/3} - 4}{x-8}, \ x \ne 8$$

P.5.69.

x-intercepts: $(-2, 0), (1, 0)$

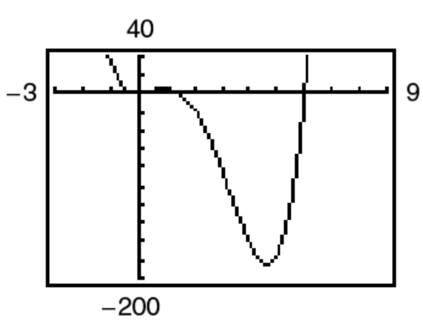
y-intercept: $(0, -2)$

P.5.70.

x-intercepts: $(-1, 0), (3, 0)$

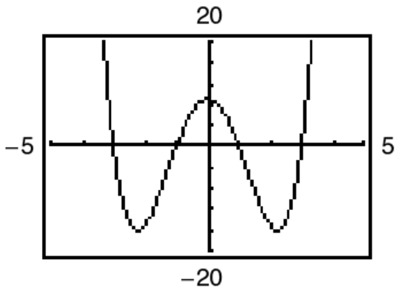
y-intercept: $(0, 3)$

P.5.71.



x-intercepts: $(0, 0), \left(\frac{3}{2}, 0\right), (6, 0)$

P.5.72.



x-intercepts: $(\pm 1, 0), (\pm 3, 0)$

P.5.73.

$y = x^2 - 3.61x + 2.86$

Using technology, the x-intercepts are $(1.17, 0)$ and $(2.44, 0)$.

P.5.74.

$y = x^3 + 1.27x^2 + 5.49$

Using technology, the x-intercept is $(-2.30, 0)$.

P.5.75.

$$\frac{(3^2 + 4) - (1^2 + 4)}{3 - 1} = \frac{13 - 5}{2} = \frac{8}{2} = 4$$

P.5.76.

$$\frac{9^{3/2} - 4^{3/2}}{9 - 4} = \frac{27 - 8}{5} = \frac{19}{5}$$

P.5.77.

$$\frac{\frac{1}{3} - \frac{1}{2}}{6 - 4} = \frac{-\frac{1}{6}}{2} = \frac{-1}{12}$$

P.5.78.

$$\frac{-\sqrt{\frac{1}{9}} + \sqrt{\frac{1}{4}}}{9 - 4} = \frac{-\frac{1}{3} + \frac{1}{2}}{5} = \frac{\frac{1}{6}}{5} = \frac{1}{30}$$

Section P.6 Analyzing Graphs of Functions

P.6.1.

zeros

P.6.2.

decreasing

P.6.3.

average rate of change; secant

P.6.4.

odd

P.6.5.

No. If a vertical line intersects the graph more than once, then it does not represent *y* as a function of *x*.

P.6.6.

If $f(2) \geq f(2)$ for all *x* in $(0, 3)$, the $(2, f(2))$ is a relative maximum of *f*.

P.6.7.

Domain: $(-2, 2]$; Range: $[-1, 8]$

(a) $f(-1) = -1$

(b) $f(0) = 0$

(c) $f(1) = -1$

(d) $f(2) = 8$

P.6.8.

Domain: $[-1, \infty)$; Range: $(-\infty, 7]$

(a) $f(-1) = 4$

(b) $f(0) = 3$

(c) $f(1) = 6$

(d) $f(3) = 0$

P.6.9.

Domain: $(-\infty, \infty)$; Range: $(-2, \infty)$

(a) $f(2) = 0$

(b) $f(1) = 1$

(c) $f(3) = 2$

(d) $f(-1) = 3$

P.6.10.

Domain: $(-\infty, \infty)$; Range: $(-\infty, 1]$

(a) $f(-2) = -3$

(b) $f(1) = 0$

(c) $f(0) = 1$

(d) $f(2) = -3$

P.6.11.

A vertical line intersects the graph at most once, so *y is* a function of *x*.

P.6.12.

y is not a function of *x*. Some vertical lines intersect the graph twice.

P.6.13.

A vertical line intersects the graph more than once, so *y is not* a function of *x*.

P.6.14.

A vertical line intersects the graph at most once, so *y is* a function of *x*.

P.6.15.

$$f(x) = 2x^2 - 7x - 30$$

$$2x^2 - 7x - 30 = 0$$

$$(2x+5)(x-6) = 0$$

$$2x + 5 = 0 \quad \text{or} \quad x - 6 = 0$$

$$x = -\tfrac{5}{2} \qquad\qquad x = 6$$

P.6.16.

$$f(x) = 3x^2 + 22x - 16$$

$$3x^2 + 22x - 16 = 0$$
$$(3x - 2)(x + 8) = 0$$
$$3x - 2 = 0 \Rightarrow x = \tfrac{2}{3}$$
$$x + 8 = 0 \Rightarrow x = -8$$

P.6.17.

$$f(x) = \tfrac{1}{3}x^3 - 2x$$

$$\tfrac{1}{3}x^3 - 2x = 0$$
$$(3)\left(\tfrac{1}{3}x^3 - 2x\right) = 0(3)$$
$$x^3 - 6x = 0$$
$$x(x^2 - 6) = 0$$
$$x = 0 \quad \text{or} \quad x^2 - 6 = 0$$
$$x^2 = 6$$
$$x = \pm\sqrt{6}$$

P.6.18.

$$f(x) = -25x^4 + 9x^2$$

$$-x^2(25x^2 - 9) = 0$$
$$-x^2 = 0 \quad \text{or} \quad 25x^2 - 9 = 0$$
$$x = 0 \qquad 25x^2 = 9$$
$$x^2 = \tfrac{9}{25}$$
$$x = \pm\tfrac{3}{5}$$

P.6.19.

$$f(x) = \sqrt{2x} - 1$$

$$\sqrt{2x} - 1 = 0$$
$$\sqrt{2x} = 1$$
$$2x = 1$$
$$x = \tfrac{1}{2}$$

P.6.20.

$$\begin{aligned} f(x) &= \sqrt{3x+2} \\ \sqrt{3x+2} &= 0 \\ 3x+2 &= 0 \\ -\tfrac{2}{3} &= x \end{aligned}$$

P.6.21.

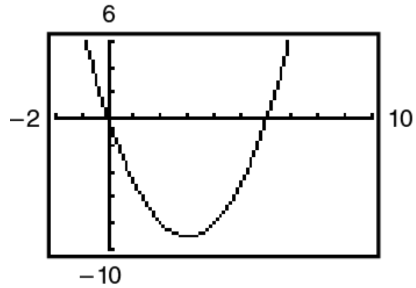
$$\begin{aligned} f(x) &= \frac{x+3}{2x^2-6} \\ \frac{x+3}{2x^2-6} &= 0 \\ x+3 &= 0 \\ x &= -3 \end{aligned}$$

P.6.22.

$$\begin{aligned} f(x) &= \frac{x^2-9x+14}{4x} \\ \frac{x^2-9x+14}{4x} &= 0 \\ (x-7)(x-2) &= 0 \\ x-7 = 0 &\Rightarrow x = 7 \\ x-2 = 0 &\Rightarrow x = 2 \end{aligned}$$

P.6.23.

(a)



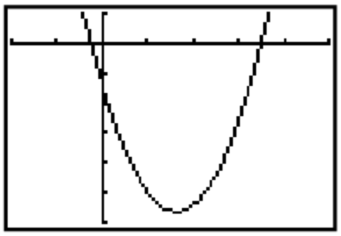
Zeros: $x = 0, 6$

(b)

$$\begin{aligned} f(x) &= x^2 - 6x \\ x^2 - 6x &= 0 \\ x(x-6) &= 0 \\ x = 0 &\Rightarrow x = 0 \\ x-6 = 0 &\Rightarrow x = 6 \end{aligned}$$

P.6.24.

(a)



Zeros: $x = -0.5, 7$

(b) $$f(x) = 2x^2 - 13x - 7$$

$$2x^2 - 13x - 7 = 0$$

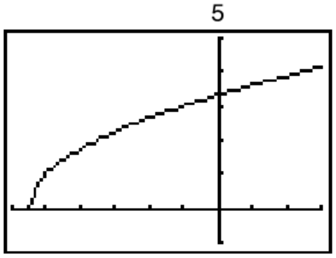
$$(2x+1)(x-7) = 0$$

$$2x + 1 = 0 \Rightarrow x = -\tfrac{1}{2}$$

$$x - 7 = 0 \Rightarrow x = 7$$

P.6.25.

(a)



Zero: $x = -5.5$

(b) $$f(x) = \sqrt{2x+11}$$

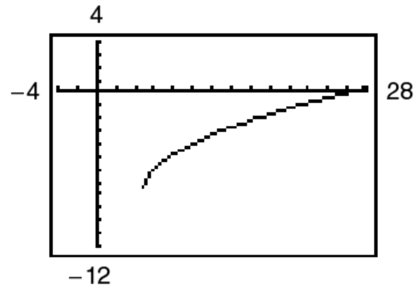
$$\sqrt{2x+11} = 0$$

$$2x + 11 = 0$$

$$x = -\tfrac{11}{2}$$

P.6.26.

(a)



Zero: $x = 26$

(b) $f(x) = \sqrt{3x-14} - 8$

$$\sqrt{3x-14} - 8 = 0$$
$$\sqrt{3x-14} = 8$$
$$3x - 14 = 64$$
$$x = 26$$

P.6.27.

$f(x) = -\frac{1}{2}x^3$

The function is decreasing on $(-\infty, \infty)$.

P.6.28.

$f(x) = x^2 - 4x$

The function is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

P.6.29.

$f(x) = \sqrt{x^2 - 1}$

The function is decreasing on $(-\infty, -1)$ and increasing on $(1, \infty)$.

P.6.30.

$f(x) = x^3 - 3x^2 + 2$

The function is increasing on $(-\infty, 0)$ and $(2, \infty)$ and decreasing on $(0, 2)$.

P.6.31.

$$f(x) = \begin{cases} 2x+1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$$

The function is decreasing on $(-1, 0)$ and increasing on $(-\infty, -1)$ and $(0, \infty)$.

P.6.32.

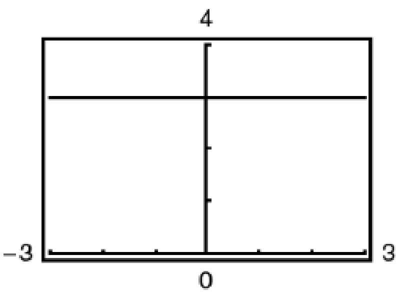
$$f(x) = \begin{cases} x+3, & x \le 0 \\ 3, & 0 < x \le 2 \\ 2x+1, & x > 2 \end{cases}$$

The function is increasing on $(-\infty, 0)$ and $(2, \infty)$.

The function is constant on $(0, 2)$.

P.6.33.

$f(x) = 3$

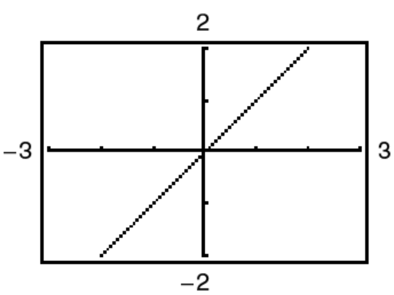


Constant on $(-\infty, \infty)$

x	-2	-1	0	1	2
$f(x)$	3	3	3	3	3

P.6.34.

$g(x) = x$

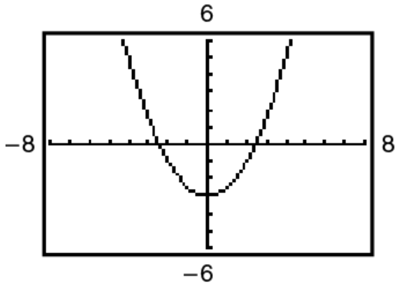


Increasing on $(-\infty, \infty)$

x	-2	-1	0	1	2
$g(x)$	-2	-1	0	1	2

P.6.35.

$g(x) = \frac{1}{2}x^2 - 3$



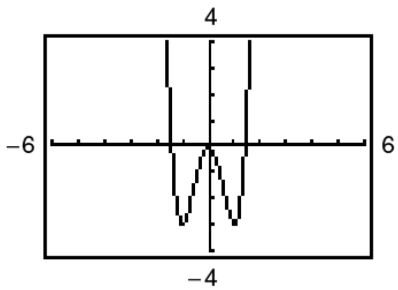
Decreasing on $(-\infty, 0)$.

Increasing on $(0, \infty)$.

x	-2	-1	0	1	2
$g(x)$	-1	$-\frac{5}{2}$	-3	$-\frac{5}{2}$	-1

P.6.36.

$f(x) = 3x^4 - 6x^2$

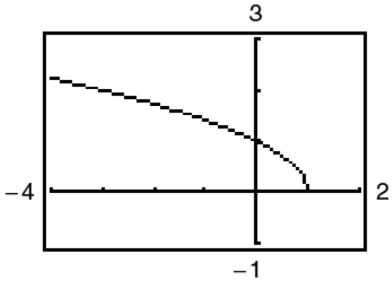


Increasing on $(-1, 0), (1, \infty)$; Decreasing on $(-\infty, -1), (0, 1)$

x	-2	-1	0	1	2
$f(x)$	24	-3	0	-3	24

P.6.37.

$f(x) = \sqrt{1-x}$

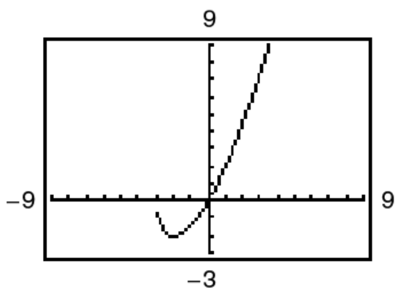


Decreasing on $(-\infty, 1)$

x	−3	−2	−1	0	1
$f(x)$	2	$\sqrt{3}$	$\sqrt{2}$	1	0

P.6.38.

$f(x) = x\sqrt{x+3}$

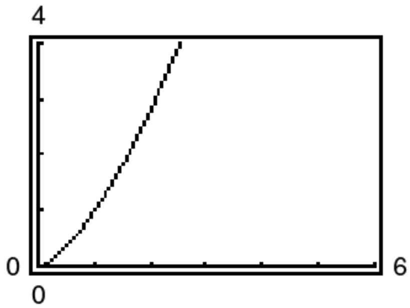


Increasing on $(-2, \infty)$; Decreasing on $(-3, -2)$

x	−3	−2	−1	0	1
$f(x)$	0	−2	−1.414	0	2

P.6.39.

$f(x) = x^{3/2}$

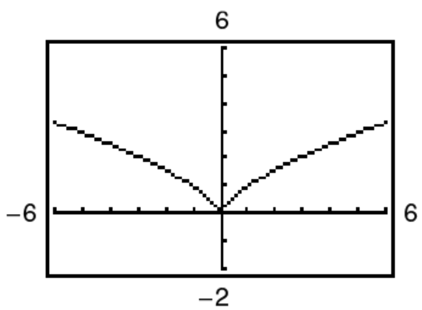


Increasing on $(0, \infty)$

x	0	1	2	3	4
$f(x)$	0	1	2.8	5.2	8

P.6.40.

$f(x) = x^{2/3}$

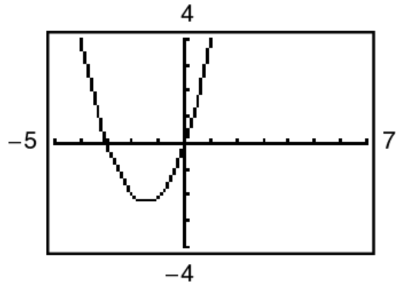


Decreasing on $(-\infty, 0)$; Increasing on $(0, \infty)$

x	−2	−1	0	1	2
$f(x)$	1.59	1	0	1	1.59

P.6.41.

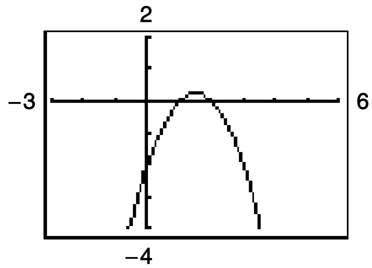
$$f(x) = x(x+3)$$



Relative minimum: $(-1.5, -2.25)$

P.6.42.

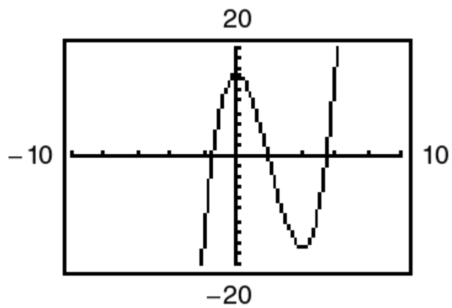
$$f(x) = -x^2 + 3x - 2$$



Relative maximum: $(1.5, 0.25)$

P.6.43.

$$h(x) = x^3 - 6x^2 + 15$$

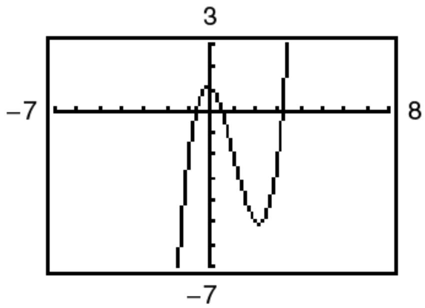


Relative minimum: $(4, -17)$

Relative maximum: $(0, 15)$

P.6.44.

$f(x) = x^3 - 3x^2 - x + 1$

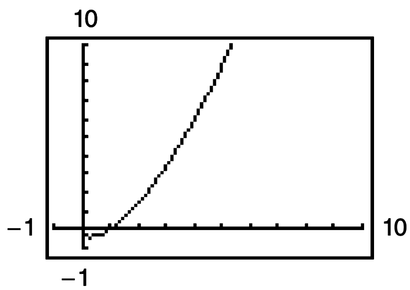


Relative maximum: $(-0.15, 1.08)$

Relative minimum: $(2.15, -5.08)$

P.6.45.

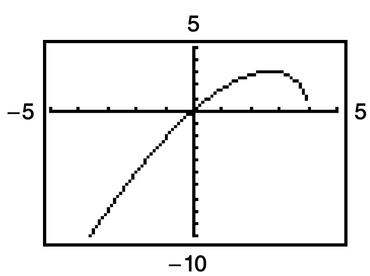
$h(x) = (x-1)\sqrt{x}$



Relative minimum: $(0.33, -0.38)$

P.6.46.

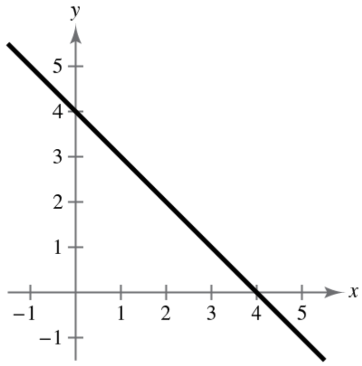
$g(x) = x\sqrt{4-x}$



Relative maximum: $(2.67, 3.08)$

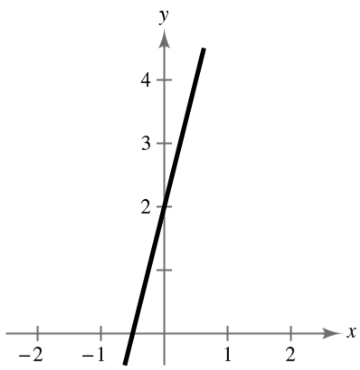
P.6.47.

$$f(x) = 4 - x$$
$$f(x) \geq 0 \text{ on } (-\infty, 4]$$



P.6.48.

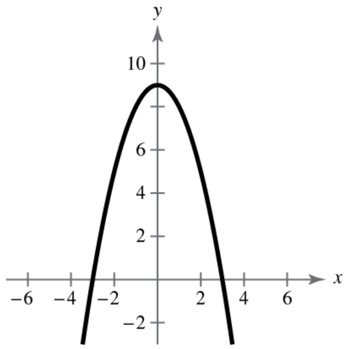
$$f(x) = 4x + 2$$
$$f(x) \geq 0 \text{ on } \left[-\tfrac{1}{2}, \infty\right)$$
$$4x + 2 \geq 0$$
$$4x \geq -2$$
$$x \geq -\tfrac{1}{2}$$
$$\left[-\tfrac{1}{2}, \infty\right)$$



P.6.49.

$$f(x) = 9 - x^2$$

$$f(x) \geq 0 \text{ on } [-3, 3]$$



P.6.50.

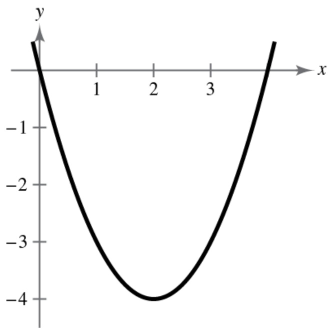
$$f(x) = x^2 - 4x$$

$$f(x) \geq 0 \text{ on } (-\infty, 0] \text{ and } [4, \infty)$$

$$x^2 - 4x \geq 0$$

$$x(x-4) \geq 0$$

$$(-\infty, 0], [4, \infty)$$



P.6.51.

$$f(x) = \sqrt{x-1}$$

$$f(x) \geq 0 \text{ on } [1, \infty)$$

$$\sqrt{x-1} \geq 0$$

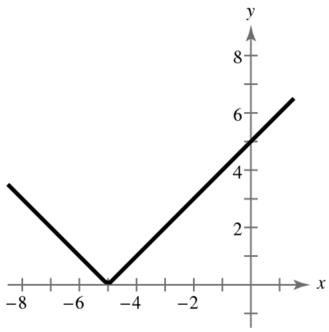
$$x - 1 \geq 0$$

$$x \geq 1$$

$$[1, \infty)$$

P.6.52.

$$f(x) = |\ x + 5\ |$$



$f(x)$ is always greater than or equal to 0. $f(x) \geq 0$ for all x.

$(-\infty, \infty)$

P.6.53.

$$f(x) = -2x + 15$$

$$\frac{f(3) - f(0)}{3 - 0} = \frac{9 - 15}{3} = -2$$

The average rate of change from $x_1 = 0$ to $x_2 = 3$ is -2.

P.6.54.

$$f(x) = x^2 - 2x + 8$$

$$\frac{f(5) - f(1)}{5 - 1} = \frac{23 - 7}{4} = \frac{16}{4} = 4$$

The average rate of change from $x_1 = 1$ to $x_2 = 5$ is 4.

P.6.55.

$$f(x) = x^3 - 3x^2 - x$$

$$\frac{f(2) - f(-1)}{2 - (-1)} = \frac{-6 - (-3)}{3} = \frac{-3}{3} = -1$$

The average rate of change from $x_1 = -1$ to $x_2 = 2$ is -1.

P.6.56.

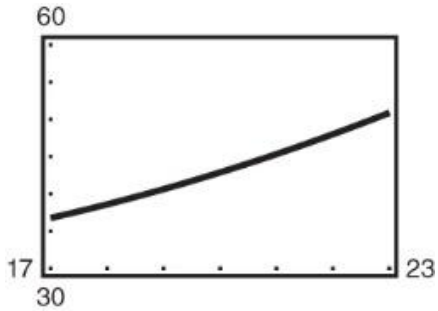
$f(x) = -x^3 + 6x^2 + x$

$\dfrac{f(6) - f(1)}{6 - 1} = \dfrac{6 - 6}{5} = 0$

The average rate of change from $x_1 = 1$ to $x_2 = 6$ is 0.

P.6.57.

(a) $y = 0.1053t^2 - 1.861t + 37.96,\ 17 \le t \le 23$



(b) $\dfrac{y(23) - y(17)}{23 - 17} \approx \dfrac{50.8607 - 36.7547}{6} \approx 2.35$

The amount the U.S. federal government spent on applied research increased by an average rate of about \$2.35 billion per year from 2017 to 2023.

P.6.58.

$$\begin{aligned}\text{Average rate of change} &= \frac{s(t_2) - s(t_1)}{t_2 - t_1}\\ &= \frac{s(9) - s(0)}{9 - 0}\\ &= \frac{540 - 0}{9 - 0}\\ &= 60 \text{ feet per second.}\end{aligned}$$

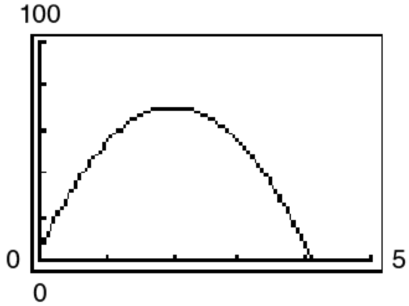
As the time traveled increases, the distance increases rapidly, causing the average speed to increase with each time increment. From $t = 0$ to $t = 4$, the average speed is less than from $t = 4$ to $t = 9$. Therefore, the overall average from $t = 0$ to $t = 9$ falls below the average found in part (b).

P.6.59.

$s_0 = 6,\ v_0 = 64$

(a) $s = -16t^2 + 64t + 6$

(b)



(c) $\dfrac{s(3) - s(0)}{3 - 0} = \dfrac{54 - 6}{3} = 16$

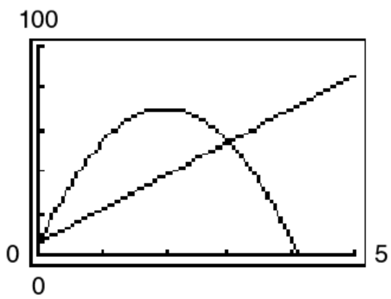
(d) The slope of the secant line is positive.

(e) $s(0) = 6, m = 16$

Secant line: $y - 6 = 16(t - 0)$

$y = 16t + 6$

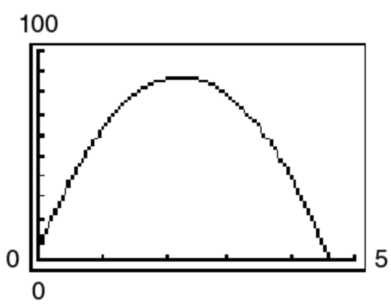
(f)



P.6.60.

(a) $s = -16t^2 + 72t + 6.5$

(b)



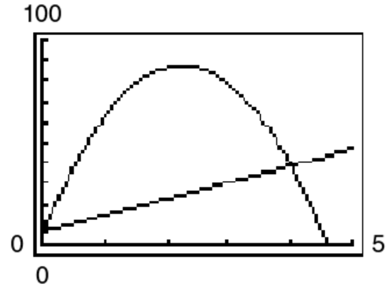
(c) The average rate of change from $t = 0$ to $t = 4$:

$\dfrac{s(4) - s(0)}{4 - 0} = \dfrac{38.5 - 6.5}{4} = \dfrac{32}{4} = 8$ feet per second

(d) The slope of the secant line through $(0, s(0))$ and $(4, s(4))$ is positive.

(e) The equation of the secant line: $m = 8$, $y = 8t + 6.5$

(f)

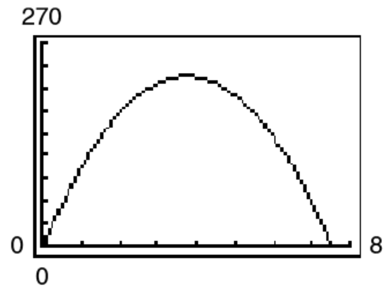


P.6.61.

$v_0 = 120$, $s_0 = 0$

(a) $s = -16t^2 + 120t$

(b)



(c) The average rate of change from $t = 3$ to $t = 5$:

$$\frac{s(5) - s(3)}{5 - 3} = \frac{200 - 216}{2} = -\frac{16}{2} = -8 \text{ feet per second}$$

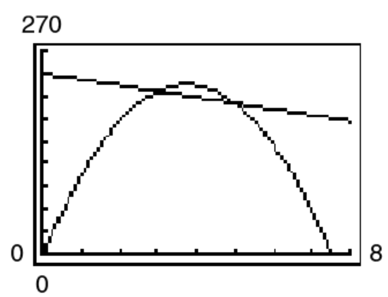
(d) The slope of the secant line through $(3, s(3))$ and $(5, s(5))$ is negative.

(e) The equation of the secant line: $m = -8$

Using $(5, s(5)) = (5, 200)$ we have

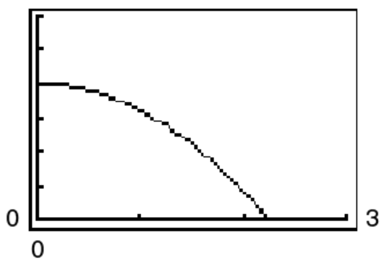
$$y - 200 = -8(t - 5)$$
$$y = -8t + 240.$$

(f)



P.6.62.

(a) $s = -16t^2 + 80$

(b)


(c) The average rate of change from $t = 1$ to $t = 2$:

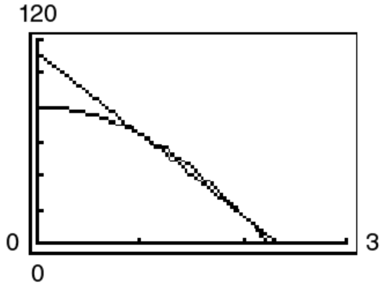
$$\frac{s(2) - s(1)}{2 - 1} = \frac{16 - 64}{1} = -\frac{48}{1} = -48 \text{ feet per second}$$

(d) The slope of the secant line through $(1, s(1))$ and $(2, s(2))$ is negative.

(e) The equation of the secant line: $m = -48$

Using $(1, s(1)) = (1, 64)$ we have

$$y - 64 = -48(t - 1)$$
$$y = -48t + 112.$$

(f)


P.6.63.

$$\begin{aligned} f(x) &= x^6 - 2x^2 + 3 \\ f(-x) &= (-x)^6 - 2(-x)^2 + 3 \\ &= x^6 - 2x^2 + 3 \\ &= f(x) \end{aligned}$$

The function is even. y-axis symmetry.

P.6.64.

$$\begin{aligned} g(x) &= x^3 - 5x \\ g(-x) &= (-x)^3 - 5(-x) \\ &= -x^3 + 5x \\ &= -g(x) \end{aligned}$$

The function is odd. Origin symmetry.

P.6.65.

$$\begin{aligned} h(x) &= x\sqrt{x+5} \\ h(-x) &= (-x)\sqrt{-x+5} \\ &= -x\sqrt{5-x} \\ &\neq h(x) \\ &\neq -h(x) \end{aligned}$$

The function is neither odd nor even. No symmetry.

P.6.66.

$$\begin{aligned} f(x) &= x\sqrt{1-x^2} \\ f(-x) &= -x\sqrt{1-(-x)^2} \\ &= -x\sqrt{1-x^2} \\ &= -f(x) \end{aligned}$$

The function is odd. Origin symmetry.

P.6.67.

$$\begin{aligned} f(s) &= 4s^{3/2} \\ &= 4(-s)^{3/2} \\ &\neq f(s) \\ &\neq -f(s) \end{aligned}$$

The function is neither odd nor even. No symmetry.

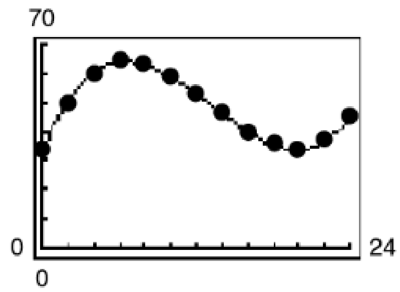
P.6.68.

$$\begin{aligned} g(s) &= 4s^{2/3} \\ g(-s) &= 4(-s)^{2/3} \\ &= 4s^{2/3} \\ &= g(s) \end{aligned}$$

The function is even. y-axis symmetry.

P.6.69.

(a)



(b) The model is an excellent fit.

(c) The temperature was increasing from 6 A.M. until noon $(x = 0 \text{ to } x = 6)$. Then it decreases until 2 A.M. $(x = 6 \text{ to } x = 20)$. Then the temperature increases until 6 A.M. $(x = 20 \text{ to } x = 24)$.

(d) The maximum temperature according to the model is about 63.93°F. According to the data, it is 64°F. The minimum temperature according to the model is about 33.98°F. According to the data, it is 34°F.

(e) Answers may vary. Temperatures will depend upon the weather patterns, which usually change from day to day.

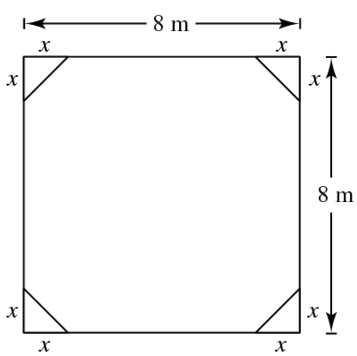
P.6.70.

$$\begin{aligned} L &= \text{right} - \text{left} \\ &= 2 - \sqrt[3]{2y} \end{aligned}$$

P.6.71.

$$\begin{aligned} L &= \text{right} - \text{left} \\ &= \frac{2}{y} - 0 \\ &= \frac{2}{y} \end{aligned}$$

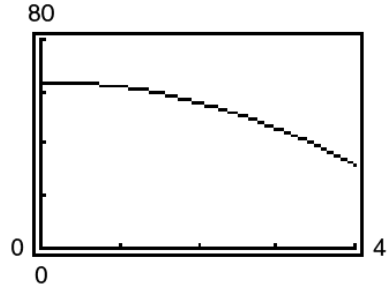
P.6.72.



(a) $A = (8)(8) - 4\left(\frac{1}{2}\right)(x)(x) = 64 - 2x^2$

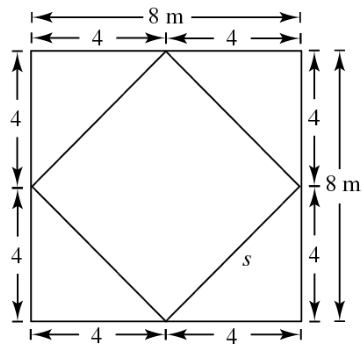
Domain: $0 \le x \le 4$

(b)



Range: $32 \le A \le 64$

(c) When $x = 4$, the resulting figure is a square.



By the Pythagorean Theorem, $4^2 + 4^2 = s^2 \Rightarrow s = \sqrt{32} = 4\sqrt{2}$ meters.

P.6.73.

False. The function $f(x) = \sqrt{x^2 + 1}$ has a domain of all real numbers.

P.6.74.

False. An odd function is symmetric with respect to the origin, so its domain must include negative values.

P.6.75.

The error is that $-2x^3 - 5 \ne -(2x^3 - 5)$. The correct process is as follows.

$$\begin{aligned} f(x) &= 2x^3 - 5 \\ f(-x) &= 2(-x)^3 - 5 \\ &= -2x^3 - 5 \\ &= -(2x^3 + 5) \end{aligned}$$

$f(-x) \ne -f(x)$ and $f(-x) \ne f(x)$, so the function $f(x) = 2x^3 - 5$ is neither odd nor even.

P.6.76.

(a) Domain: $[-4, 5)$; Range: $[0, 9]$

(b) $(3, 0)$

(c) Increasing: $(-4, 0)\cup(3, 5)$; Decreasing: $(0, 3)$

(d) Relative minimum: $(3, 0)$

Relative maximum: $(0, 9)$

(e) Neither

P.6.77.

$\left(-\frac{5}{3}, -7\right)$

(a) If f is even, another point is $\left(\frac{5}{3}, -7\right)$.

(b) If f is odd, another point is $\left(\frac{5}{3}, 7\right)$.

P.6.78.

$(2a, 2c)$

(a) $(-2a, 2c)$

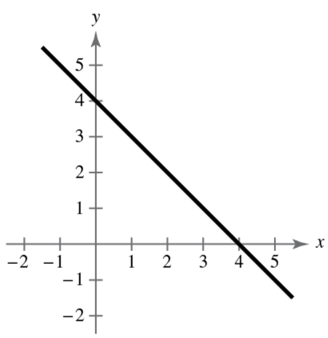
(b) $(-2a, -2c)$

P.6.79.

$(1, 3), (4, 0)$

Slope: $m = \dfrac{3-0}{1-4} = -1$

$$y - 0 = -1(x-4)$$
$$y = -x + 4$$

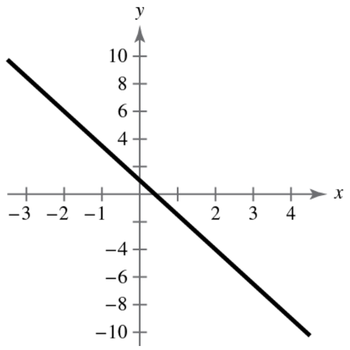


P.6.80.

$(-2, 6), (4, -9)$

Slope: $m = \dfrac{6-(-9)}{-2-4} = \dfrac{15}{-6} = \dfrac{-5}{2}$

$$y - 6 = -\tfrac{5}{2}(x+2)$$
$$y = -\tfrac{5}{2}x - 5 + 6$$
$$y = -\tfrac{5}{2}x + 1$$

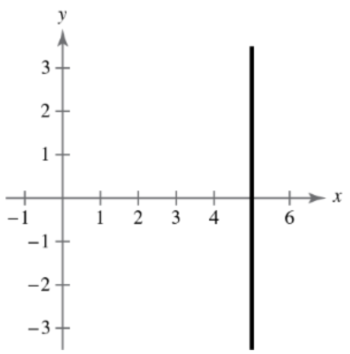


P.6.81.

$(5, 0), (5, 1)$

Slope: $\dfrac{0-1}{5-5} = \dfrac{-1}{0} \Rightarrow m$ is undefined.

$$x = 5$$

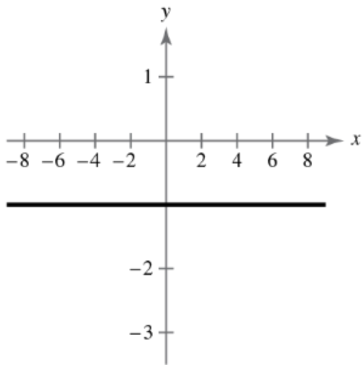


P.6.82.

$(6, -1), (-6, -1)$

Slope: $m = \dfrac{-1-(-1)}{6-(-6)} = 0$

$y = -1$



P.6.83.

$$f(x) = \begin{cases} 2x+3, & x \le 1 \\ -x+4, & x > 1 \end{cases}$$

(a) $f(0) = 2(0) + 3 = 3$

(b) $f(1) = 2(1) + 3 = 5$

(c) $f(2) = -2 + 4 = 2$

P.6.84.

$$f(x) = \begin{cases} -\frac{1}{2}x - 6, & x \le -4 \\ x+5, & x > -4 \end{cases}$$

(a) $f(-8) = -\frac{1}{2}(-8) - 6 = 4 - 6 = -2$

(b) $f(-4) = -\frac{1}{2}(-4) - 6 = 2 - 6 = -4$

(c) $f(0) = 0 + 5 = 5$

P.6.85.

Verbal Model: $(\text{Sum}) = (\text{first number}) + (\text{second number})$

Labels: $\text{Sum} = S$, first number $= n$, second number $= n + 1$

Equation: $S = n + (n+1) = 2n + 1$

P.6.86.

Verbal Model: $\text{Product} = (\text{first number}) \cdot (\text{second number})$

Labels: Product = P, first number = n, second number = $n + 1$

Equation: $P = n(n+1) = n^2 + n$

Section P.7 A Library of Parent Functions

P.7.1.

Greatest integer function

P.7.2.

Identity function

P.7.3.

Reciprocal function

P.7.4.

Squaring function

P.7.5.

Square root function

P.7.6.

Constant function

P.7.7.

Absolute value function

P.7.8.

Cubic function

P.7.9.

Linear function

P.7.10.

linear

P.7.11.

(a) $f(1) = 4, f(0) = 6$

$(1, 4), (0, 6)$

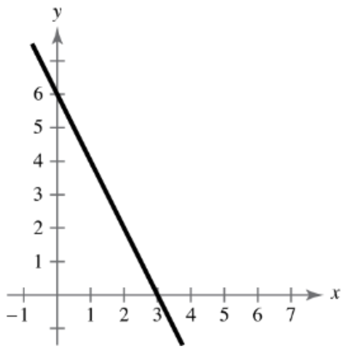
$$m = \frac{6-4}{0-1} = -2$$

$$y - 6 = -2(x - 0)$$

$$y = -2x + 6$$

$$f(x) = -2x + 6$$

(b)



P.7.12.

(a) $f(-3) = -8, f(1) = 2$

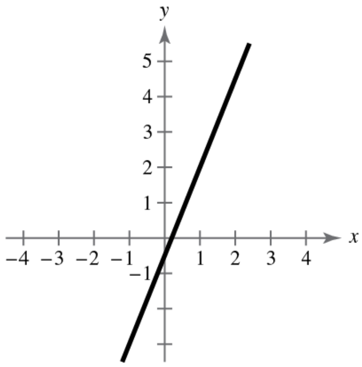
$(-3, -8), (1, 2)$

$$m = \frac{2-(-8)}{1-(-3)} = \frac{10}{4} = \frac{5}{2}$$

$$f(x) - 2 = \frac{5}{2}(x-1)$$

$$f(x) = \frac{5}{2}x - \frac{1}{2}$$

(b)



P.7.13.

(a) $f\left(\frac{1}{2}\right) = -\frac{5}{3},\ f(6) = 2$

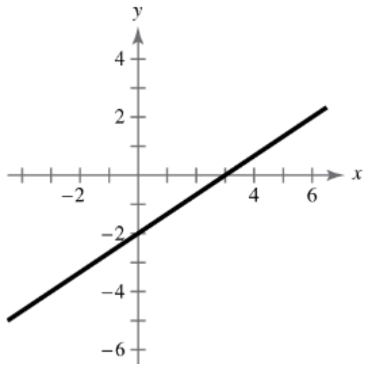
$\left(\frac{1}{2},\ -\frac{5}{3}\right),\ (6,\ 2)$

$$m = \frac{2-\left(-\frac{5}{3}\right)}{6-\left(\frac{1}{2}\right)}$$

$$= \frac{\frac{11}{3}}{\frac{11}{2}} = \left(\frac{11}{3}\right)\cdot\left(\frac{2}{11}\right) = \frac{2}{3}$$

$$f(x) - 2 = \tfrac{2}{3}(x - 6)$$
$$f(x) - 2 = \tfrac{2}{3}x - 4$$
$$f(x) = \tfrac{2}{3}x - 2$$

(b)



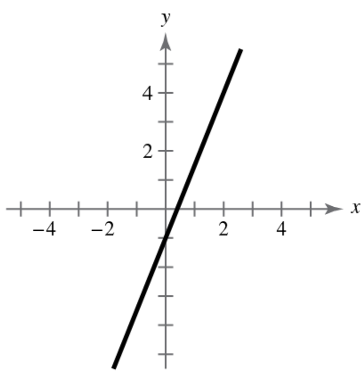
P.7.14.

(a) $f\left(\tfrac{3}{5}\right) = \tfrac{1}{2},\ f(4) = 9$

$\left(\tfrac{3}{5}, \tfrac{1}{2}\right), (4, 9)$

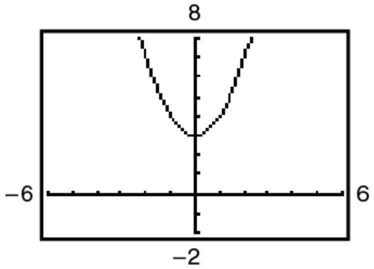
$$m = \frac{9 - \left(\frac{1}{2}\right)}{4 - \left(\frac{3}{5}\right)}$$
$$= \frac{\frac{17}{2}}{\frac{17}{5}} = \left(\tfrac{17}{2}\right) \cdot \left(\tfrac{5}{17}\right) = \tfrac{5}{2}$$
$$f(x) - 9 = \tfrac{5}{2}(x - 4)$$
$$f(x) - 9 = \tfrac{5}{2}x - 10$$
$$f(x) = \tfrac{5}{2}x - 1$$

(b)



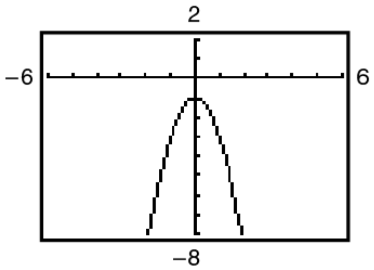
P.7.15.

$$g(x) = x^2 + 3$$



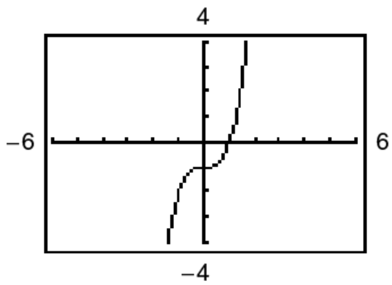
P.7.16.

$$g(x) = -2x^2 - 1$$



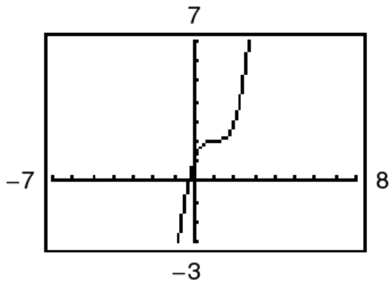
P.7.17.

$$f(x) = x^3 - 1$$



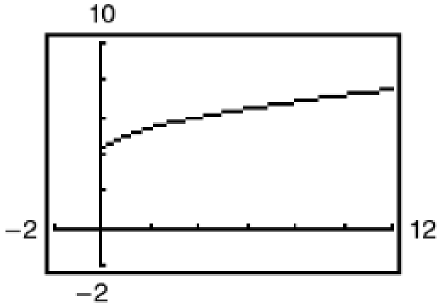
P.7.18.

$$f(x) = (x-1)^3 + 2$$



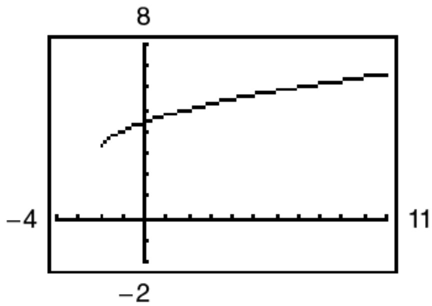
P.7.19.

$$f\left(x\right)=\sqrt{x}+4$$



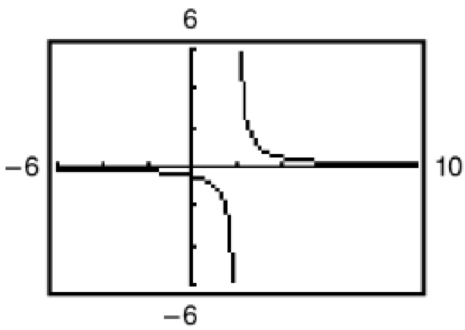
P.7.20.

$$h\left(x\right)=\sqrt{x+2}+3$$



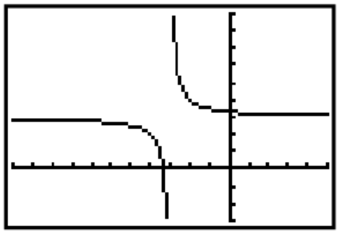
P.7.21.

$$f\left(x\right)=\frac{1}{x-2}$$



P.7.22.

$$k(x) = 3 + \frac{1}{x+3}$$



P.7.23.

$f(x) = [\![x]\!]$

(a) $f(2.1) = 2$

(b) $f(2.9) = 2$

(c) $f(-3.1) = -4$

(d) $f\left(\frac{7}{2}\right) = 3$

P.7.24.

$h(x) = [\![x+3]\!]$

(a) $h(-2) = [\![1]\!] = 1$

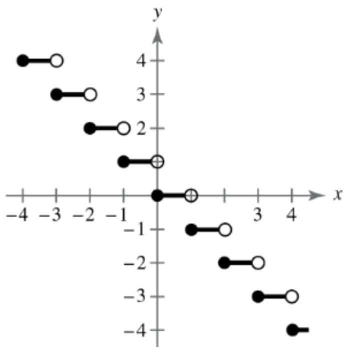
(b) $h\left(\frac{1}{2}\right) = [\![3.5]\!] = 3$

(c) $h(4.2) = [\![7.2]\!] = 7$

(d) $h(-21.6) = [\![-18.6]\!] = -19$

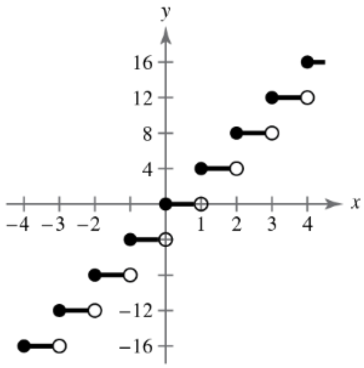
P.7.25.

$$g(x) = -[\![x]\!]$$



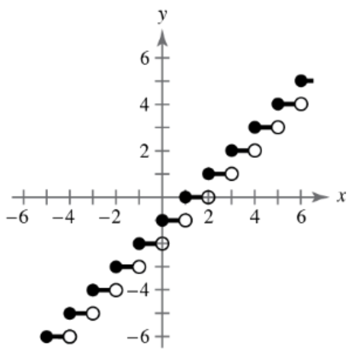
P.7.26.

$$g(x) = 4[\![x]\!]$$



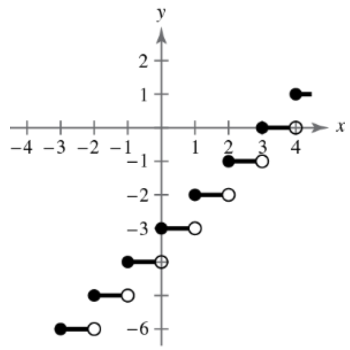
P.7.27.

$$g(x) = [\![x]\!] - 1$$



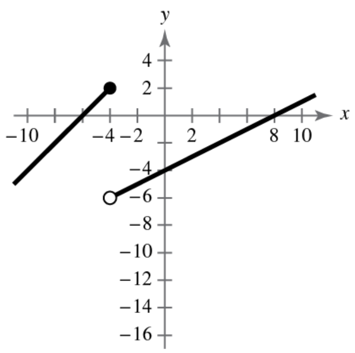
P.7.28.

$$g(x) = [\![x - 3]\!]$$



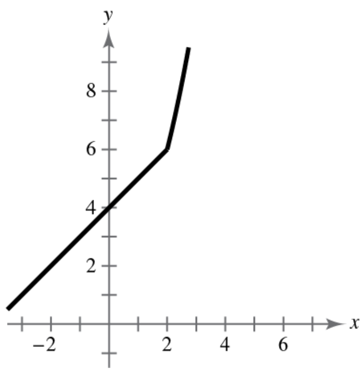
P.7.29.

$$g(x) = \begin{cases} x + 6, & x \le -4 \\ \frac{1}{2}x - 4, & x > -4 \end{cases}$$



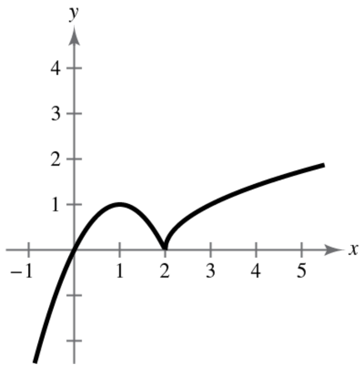
P.7.30.

$$f(x) = \begin{cases} 4 + x, & x \le 2 \\ x^2 + 2, & x > 2 \end{cases}$$



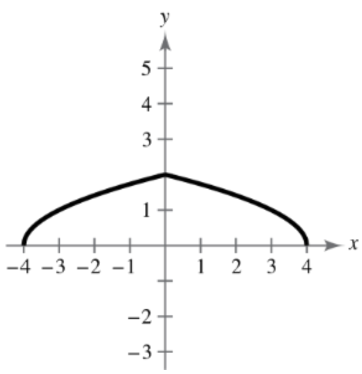
P.7.31.

$$f(x)=\begin{cases}1-(x-1)^2, & x \le 2\\ \sqrt{x-2}, & x > 2\end{cases}$$



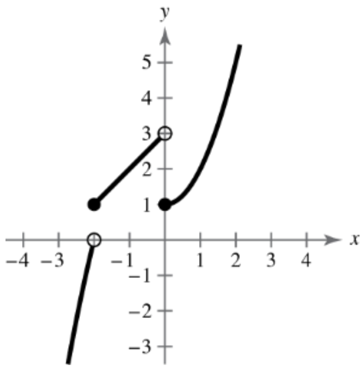
P.7.32.

$$f(x)=\begin{cases}\sqrt{4+x}, & x < 0\\ \sqrt{4-x}, & x \ge 0\end{cases}$$



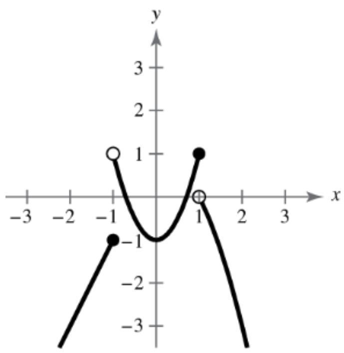
P.7.33.

$$h(x)=\begin{cases}4-x^2, & x < -2\\ 3+x, & -2 \le x < 0\\ x^2+1, & x \ge 0\end{cases}$$



P.7.34.

$$k(x) = \begin{cases} 2x+1, & x \le -1 \\ 2x^2 - 1, & -1 < x \le 1 \\ 1 - x^2, & x > 1 \end{cases}$$



P.7.35.

(a) $W(30) = 14(30) = 420$

$W(40) = 14(40) = 560$

$W(45) = 21(45-40) + 560 = 665$

$W(50) = 21(50-40) + 560 = 770$

(b) $W(h) = \begin{cases} 14h, & 0 < h \le 36 \\ 21(h-36) + 504, & h > 36 \end{cases}$

(c) $W(h) = \begin{cases} 16h, & 0 < h \le 40 \\ 24(h-40) + 640, & h > 40 \end{cases}$

P.7.36.

For the first two hours, the slope is 1. For the next six hours, the slope is 2. For the final hour, the slope is $\frac{1}{2}$.

$$f(t) = \begin{cases} t, & 0 \le t \le 2 \\ 2t - 2, & 2 < t \le 8 \\ \frac{1}{2}t + 10, & 8 < t \le 9 \end{cases}$$

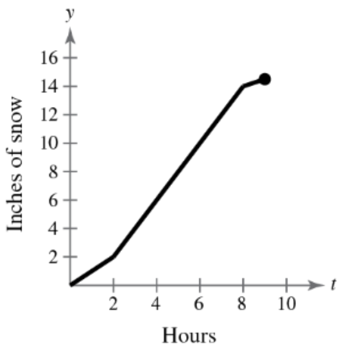
To find $f(t) = 2t - 2$, use $m = 2$ and $(2, 2)$.

$y - 2 = 2(t - 2) \Rightarrow y = 2t - 2$

To find $f(t) = \frac{1}{2}t + 10$, use $m = \frac{1}{2}$ and $(8, 14)$.

$y - 14 = \frac{1}{2}(t - 8) \Rightarrow y = \frac{1}{2}t + 10$

Total accumulation $= 14.5$ inches

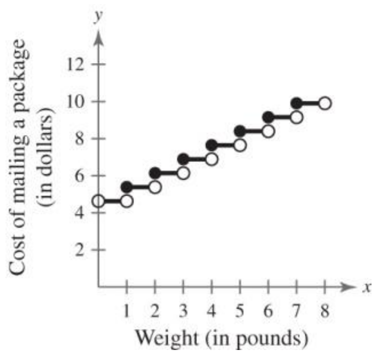


P.7.37.

(a) $C = 0.75[\![x]\!] + 4.63$

Note that $C(0.9) = 4.63$ and $C(1) = 0.75 + 4.63$.

(b)



P.7.38.

$f(x) = x^2$

(a) Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

(b) x-intercept: $(0, 0)$

$f(x) = x^3$

(a) Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

(b) x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

(c) Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$

(d) Even; the graph has y-axis symmetry.

y-intercept: $(0, 0)$

(c) Increasing: $(-\infty, \infty)$

(d) Odd; the graph has origin symmetry.

P.7.39.

False. A piecewise-defined function is a function that is defined by two or more equations over a specified domain. That domain may or may not include x- and y-intercepts.

P.7.40.

False. The vertical line $x = 2$ has an x-intercept at the point $(2, 0)$ but does not have a y-intercept. The horizontal line $y = 3$ has a y-intercept at the point $(0, 3)$ but does not have an x-intercept.

P.7.41.

According to the graph, the domains should be $x \le 3$ and $x > 3$.

P.7.42.

When $x = \frac{-4}{3}$, $\left[\!\left[\frac{-4}{3}\right]\!\right] = -2$, not -1. So, $f\left(\frac{-4}{3}\right) = -2 + 5 = 3$.

P.7.43.

(a) Yes. The amount that you pay in sales tax will increase as the price of the item purchased increases.

(b) No. The length of time that you study the night before an exam does not necessarily determine your score on the exam.

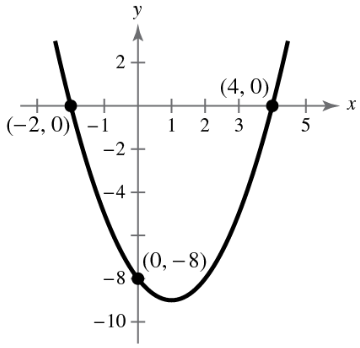
P.7.44.

(a) No. During the course of a year, for example, your salary may remain constant while your savings account balance may vary. That is, there may be two or more outputs (savings account balances) for one input (salary).

(b) Yes. The greater the height from which the acorn is dropped, the greater the speed with which the acorn will strike the ground.

P.7.45.

$$f(x) = x^2 - 2x - 8 = (x-4)(x+2)$$

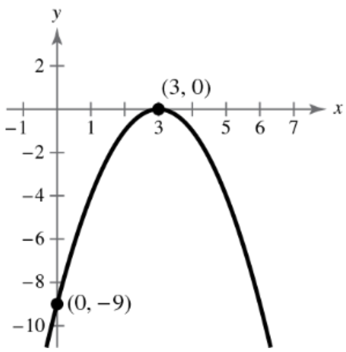


x-intercepts: $(4, 0)$, $(-2, 0)$

y-intercept: $(0, \ -8)$

P.7.46.

$$g(x) = -x^2 + 6x - 9 = -(x^2 - 6x + 9) = -(x-3)^2$$

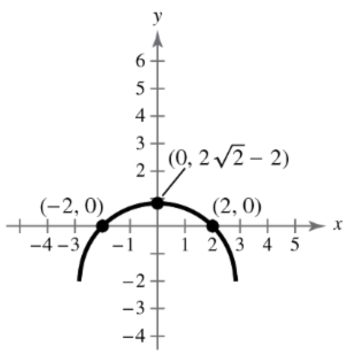


x-intercept: $(3, 0)$

y-intercept: $(0, -9)$

P.7.47.

$$p(x) = \sqrt{8 - x^2} - 2$$

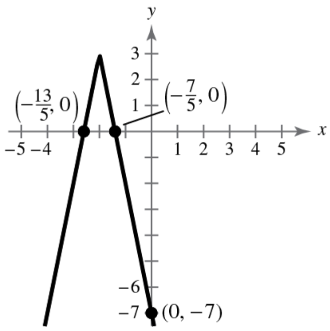


x-intercepts: $(2, 0), (-2, 0)$

y-intercept: $\left(0, 2\sqrt{2} - 2\right)$

P.7.48.

$$g(x) = -5\left| x + 2 \right| + 3$$



x-intercepts: $\left(-\frac{7}{5}, 0\right), \left(-\frac{13}{5}, 0\right)$

y-intercept: $(0, -7)$

P.7.49.

$$\frac{\sqrt[3]{(-x)^2} + \sqrt[3]{-x}}{-x} = -\frac{\sqrt[3]{x^2} + \sqrt[3]{x}}{x}$$
$$= \frac{-\left(x^{1/3} - 1\right)}{x^{2/3}}$$

P.7.50.

$$-\frac{\sqrt{(2-x)^3} + \sqrt{2-x}}{x-2} = \frac{\sqrt{2-x}\,(2-x+1)}{2-x}$$
$$= \frac{3-x}{\sqrt{2-x}}$$
$$= \frac{(3-x)\sqrt{2-x}}{(2-x)}$$

P.7.51.

$$(x-3)^2 - 3(x-3) + 2 = x^2 - 6x + 9 - 3x + 9 + 2$$
$$= x^2 - 9x + 20$$
$$= (x-5)(x-4)$$

P.7.52.

$$\begin{aligned}
3(x-1)^3 - 2(x-1)^2 - 3(x-1) + 2 &= (x-1)^2\left[3(x-1)-2\right] - \left[3(x-1)-2\right] \\
&= \left[(x-1)^2 - 1\right]\left[3(x-1)-2\right] \\
&= \left(x^2-2x\right)\left(3x-5\right) \\
&= 3x^3 - 11x^2 + 10x \\
&= x\left(3x^2 - 11x + 10\right) \\
&= x\left(3x-5\right)\left(x-2\right)
\end{aligned}$$

SECTION P.8 TRANSFORMATIONS OF FUNCTIONS

P.8.1.

$-f(x)$; $f(-x)$

P.8.2.

vertical stretch; vertical shrink

P.8.3.

The three types of rigid transformations are horizontal shifts, vertical shifts, and reflections.

P.8.4.

(a) iv

(b) ii

(c) iii

(d) i

P.8.5.

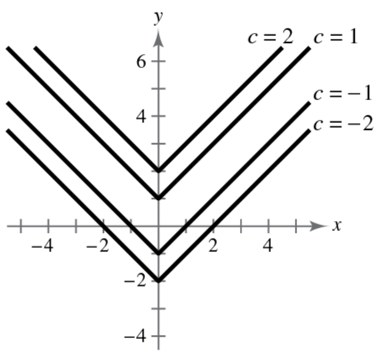
(a) $f(x) = |x| + c$ Vertical shifts

$c = -2: f(x) = |x| - 2$ 2 units down

$c = -1: f(x) = |x| - 1$ 1 unit down

$c = 1: f(x) = |x| + 1$ 1 unit up

$c = 2: f(x) = |x| + 2$ 2 units up



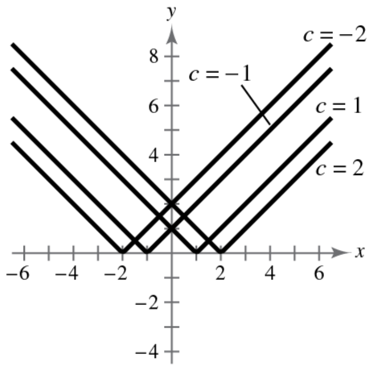
(b) $f(x) = |x - c|$ Horizontal shifts

$c = -2: f(x) = |x - (-2)| = |x + 2|$ 2 units left

$c = -1: f(x) = |x-(-1)| = |x+1|$ 1 unit left

$c = 1: f(x) = |x-(1)| = |x-1|$ 1 unit right

$c = 2: f(x) = |x-(2)| = |x-2|$ 2 units right



P.8.6.

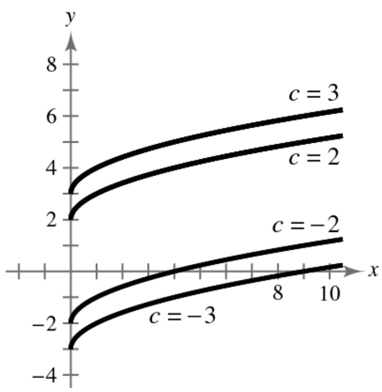
(a) $f(x) = \sqrt{x} + c$ Vertical shifts

$c = -3: f(x) = \sqrt{x} - 3$ 3 units down

$c = -2: f(x) = \sqrt{x} - 2$ 2 units down

$c = 2: f(x) = \sqrt{x} + 2$ 2 units up

$c = 3: f(x) = \sqrt{x} + 3$ 3 units up



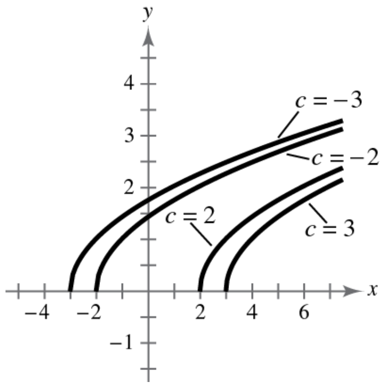
(b) $f(x) = \sqrt{x-c}$ Horizontal shifts

$c = -3: f(x) = \sqrt{x-(-3)} = \sqrt{x+3}$ 3 units left

$c = -2: f(x) = \sqrt{x-(-2)} = \sqrt{x+2}$ 2 units left

$c = 2:\ f(x) = \sqrt{x-2}$ 2 units right

$c = 3:\ f(x) = \sqrt{x-3}$ 3 units right



P.8.7.

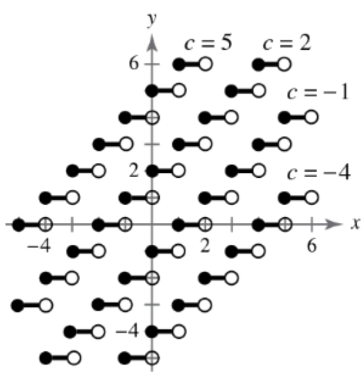
(a) $f(x) = [\![x]\!] + c$ Vertical shifts

$c = -4:\ f(x) = [\![x]\!] - 4$ 4 units down

$c = -1:\ f(x) = [\![x]\!] - 1$ 1 unit down

$c = 2:\ f(x) = [\![x]\!] + 2$ 2 units up

$c = 5:\ f(x) = [\![x]\!] + 5$ 5 units up



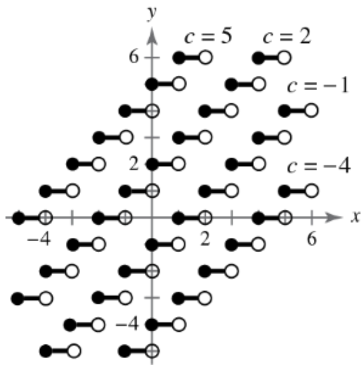
(b) $f(x) = [\![x + c]\!]$ Horizontal shifts

$c = -4:\ f(x) = [\![x - (-4)]\!] = [\![x + 4]\!]$ 4 units left

$c = -1:\ f(x) = [\![x - (-1)]\!] = [\![x + 1]\!]$ 1 unit left

$c = 2:\ f(x) = [\![x - (2)]\!] = [\![x - 2]\!]$ 2 units right

$c = 5:\ f(x) = [\![x - (5)]\!] = [\![x - 5]\!]$ 5 units right



P.8.8.

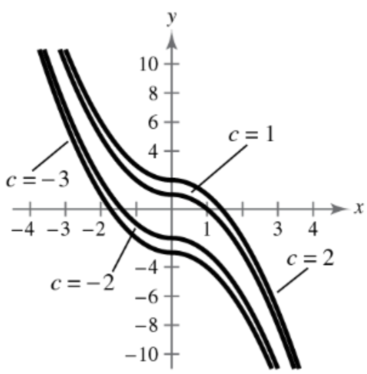
(a) $f(x) = \begin{cases} x^2 + c, & x < 0 \\ -x^2 + c, & x \ge 0 \end{cases}$ Vertical shifts

$c = -3:\ f(x) = \begin{cases} x^2 - 3, & x < 0 \\ -x^2 - 3, & x \ge 0 \end{cases}$ 3 units down

$c = -2:\ f(x) = \begin{cases} x^2 - 2, & x < 0 \\ -x^2 - 2, & x \ge 0 \end{cases}$ 2 units down

$c = 1:\ f(x) = \begin{cases} x^2 + 1, & x < 0 \\ -x^2 + 1, & x \ge 0 \end{cases}$ 1 unit up

$c = 2:\ f(x) = \begin{cases} x^2 + 2, & x < 0 \\ -x^2 + 2, & x \ge 0 \end{cases}$ 2 units up



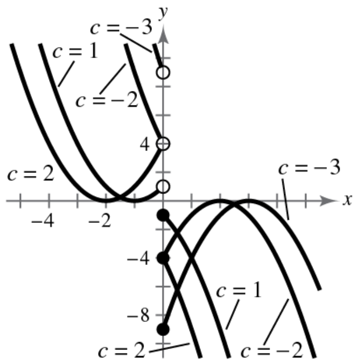
(b) $f(x) = \begin{cases} (x + c)^2, & x < 0 \\ -(x + c)^2, & x \ge 0 \end{cases}$ Horizontal shifts

$c = -3:\ f(x) = \begin{cases} (x - 3)^2, & x < 0 \\ -(x - 3)^2, & x \ge 0 \end{cases}$ 3 units right

$c = -2: \; f(x) \begin{cases} (x-2)^2, & x < 0 \\ -(x-2)^2, & x \geq 0 \end{cases}$ 2 units right

$c = 1: \; f(x) = \begin{cases} (x+1)^2, & x < 0 \\ -(x+1)^2, & x \geq 0 \end{cases}$ 1 unit left

$c = 2: \; f(x) = \begin{cases} (x+2)^2, & x < 0 \\ -(x+2)^2, & x \geq 0 \end{cases}$ 2 units left



P.8.9.

Parent function: $f(x) = x^2$

(a) Vertical shift 1 unit downward

$g(x) = x^2 - 1$

(b) Reflection in the x-axis, horizontal shift 1 unit to the left, and a vertical shift 1 unit upward

$g(x) = -(x+1)^2 + 1$

P.8.10.

Parent function: $f(x) = x^3$

(a) Shifted upward 1 unit

$g(x) = x^3 + 1$

(b) Reflection in the x-axis, shifted to the left 3 units and down 1 unit

$g(x) = -(x+3)^3 - 1$

P.8.11.

Parent function: $f(x) = |x|$

(a) Reflection in the x-axis and a horizontal shift 3 units to the left
$g(x) = -|x+3|$

(b) Horizontal shift 2 units to the right and a vertical shift 4 units downward
$g(x) = |x-2| - 4$

P.8.12.

Parent function: $f(x) = \sqrt{x}$

(a) A vertical shift 7 units downward and a horizontal shift 1 unit to the left
$g(x) = \sqrt{x+1} - 7$

(d) Reflection in the x- and y-axis and shifted to the right 3 units and downward 4 units
$g(x) = -\sqrt{-x+3} - 4$

P.8.13.

Parent function: $f(x) = x^3$

Horizontal shift 2 units to the right

$y = (x-2)^3$

P.8.14.

Parent function: $y = [\![x]\!]$

Vertical shift 4 units upward

$y = [\![x]\!] + 4$

P.8.15.

Parent function: $f(x) = \sqrt{x}$

Reflection in the x-axis and a vertical shift 1 unit upward

$y = -\sqrt{x} + 1$

P.8.16.

Parent function: $y = |x|$

Horizontal shift 2 units to the left

$y = |x+2|$

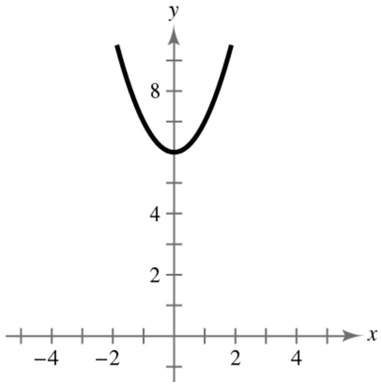
P.8.17.

$g(x) = x^2 + 6$

(a) Parent function: $f(x) = x^2$

(b) A vertical shift 6 units upward

(c)



(d) $g(x) = f(x) + 6$

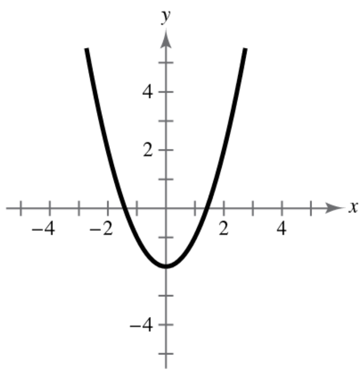
P.8.18.

$g(x) = x^2 - 2$

(a) Parent function: $f(x) = x^2$

(b) A vertical shift 2 units downward

(c)



(d) $g(x) = f(x) - 2$

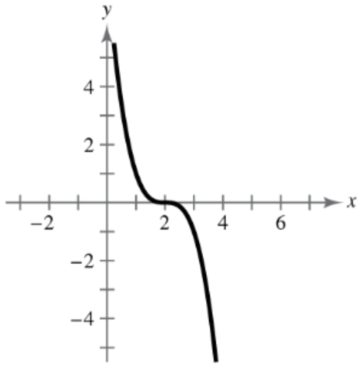
P.8.19.

$g(x) = -(x-2)^3$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift of 2 units to the right and a reflection in the x-axis

(c)



(d) $g(x) = -f(x-2)$

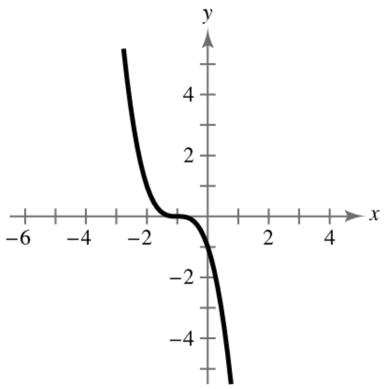
P.8.20.

$g(x) = -(x+1)^3$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift 1 unit to the left and a reflection in the x-axis

(c)



(d) $g(x) = -f(x+1)$

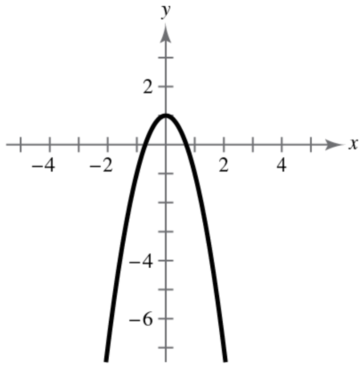
P.8.21.

$g(x) = -2x^2 + 1$

(a) Parent function: $f(x) = x^2$

(b) A vertical stretch, reflection in the x-axis and a vertical shift 1 unit upward

(c)



(d) $g(x) = -2f(x) + 1$

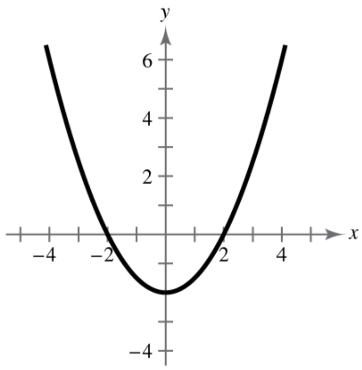
P.8.22.

$g(x) = \frac{1}{2}x^2 - 2$

(a) Parent function: $f(x) = x^2$

(b) A vertical shrink and a vertical shift 2 units downward

(c)



(d) $g(x) = \frac{1}{2}f(x) - 2$

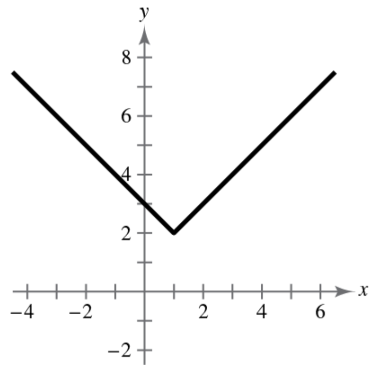
P.8.23.

$g(x) = |x - 1| + 2$

(a) Parent function: $f(x) = |\ x\ |$

(b) A horizontal shift 1 unit right and a vertical shift 2 units upward

(c)



(d) $g(x) = f(x-1)+2$

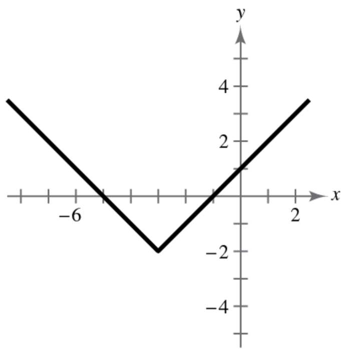
P.8.24.

$g(x) = |x+3| - 2$

(a) Parent function: $f(x) = |x|$

(b) A horizontal shift 3 units left and a vertical shift 2 units downward

(c)



(d) $g(x) = f(x+3)-2$

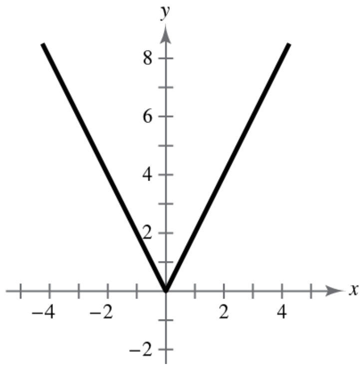
P.8.25.

$g(x) = |2x|$

(a) Parent function: $f(x) = |x|$

(b) A horizontal shrink

(c)



(d) $g(x) = f(2x)$

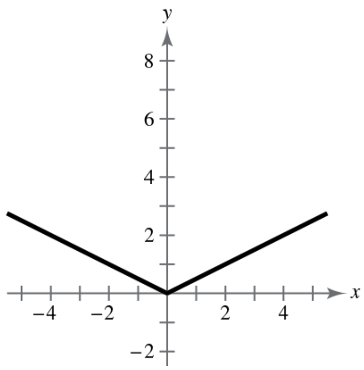
P.8.26.

$g(x) = \left| \frac{1}{2}x \right|$

(a) Parent function: $f(x) = |x|$

(b) A horizontal stretch

(c)



(d) $g(x) = f\left(\frac{1}{2}x\right)$

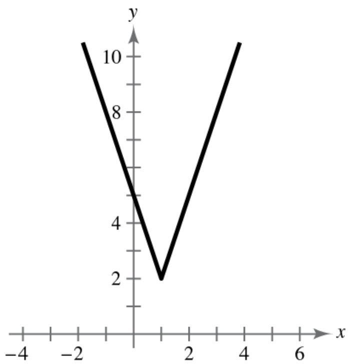
P.8.27.

$g(x) = 3|x - 1| + 2$

(a) Parent function: $f(x) = |x|$

(b) A horizontal shift of 1 unit to the right, a vertical stretch, and a vertical shift 2 units upward

(c)



(d) $g(x) = 3f(x-1) + 2$

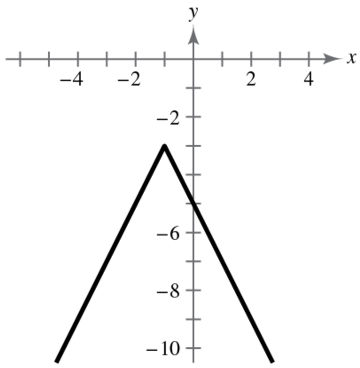
P.8.28.

$g(x) = -2|x+1| - 3$

(a) Parent function: $f(x) = |x|$

(b) A reflection in the x-axis, a vertical stretch, a horizontal shift 1 unit to the left, and a vertical shift 3 units downward

(c)



(d) $g(x) = -2f(x+1) - 3$

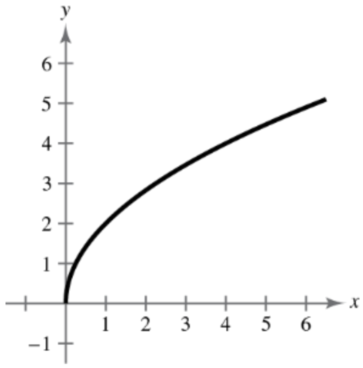
P.8.29.

$g(x) = 2\sqrt{x}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) A vertical stretch (each y value is multiplied by 2)

(c)



(d) $g(x) = 2f(x)$

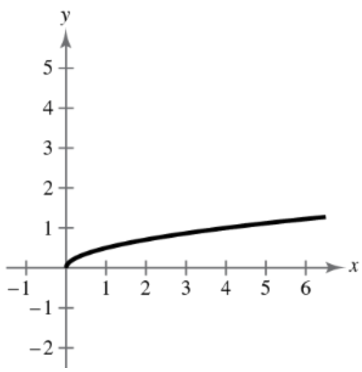
P.8.30.

$g(x) = \frac{1}{2}\sqrt{x}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) A vertical shrink (each y value is multiplied by $\frac{1}{2}$)

(c)



(d) $g(x) = \frac{1}{2}f(x)$

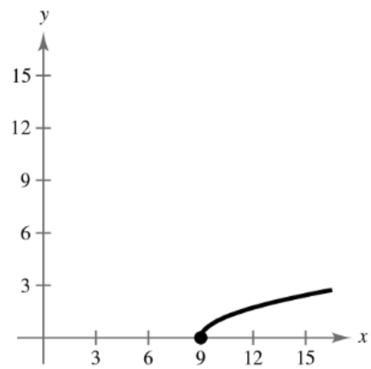
P.8.31.

$g(x) = \sqrt{x-9}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) horizontal shift 9 units to the right

(c)



(d) $g(x) = f(x-9)$

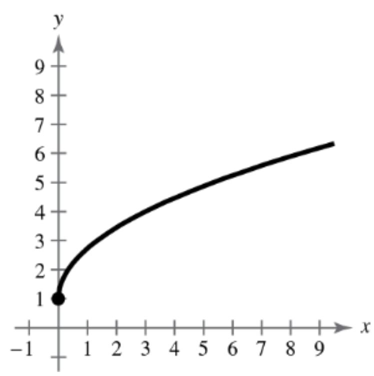
P.8.32.

$g(x) = \sqrt{3x} + 1$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shrink $\left(\text{each } x\text{-value is multiplied by } \frac{1}{3}\right)$, vertical shift of 1 unit upward

(c)



(d) $g(x) = f(3x) + 1$

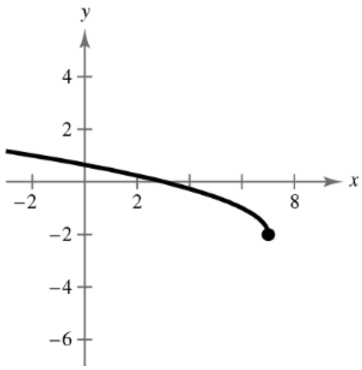
P.8.33.

$g(x) = \sqrt{7-x} - 2$ or $g(x) = \sqrt{-(x-7)} - 2$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Reflection in the y-axis, horizontal shift 7 units to the right, and a vertical shift 2 units downward

(c)



(d) $g(x) = f(7 - x) - 2$

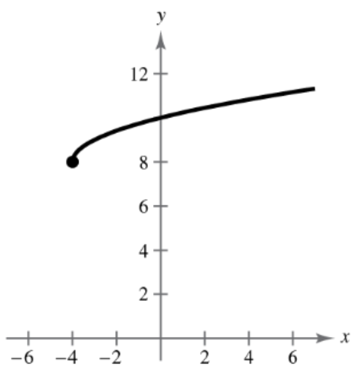
P.8.34.

$g(x) = \sqrt{x+4} + 8$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shift of 4 units to the left, vertical shift of 8 units upward

(c)



(d) $g(x) = f(x+4) + 8$

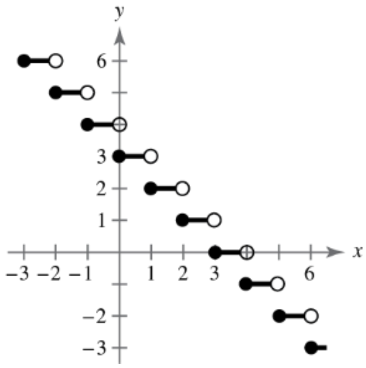
P.8.35.

$g(x) = 3 - [\![x]\!]$

(a) Parent function: $f(x) = [\![x]\!]$

(b) Reflection in the x-axis and a vertical shift 3 units upward

(c)



(d) $g(x) = 3 - f(x)$

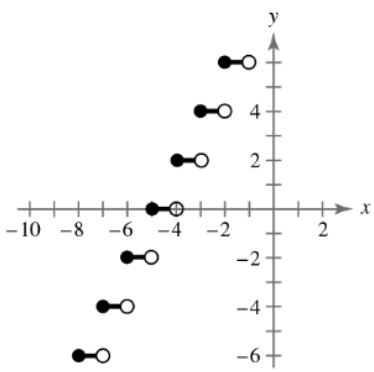
P.8.36.

$g(x) = 2[\![x+5]\!]$

(a) Parent function: $f(x) = [\![x]\!]$

(b) Horizontal shift 5 units to the left, vertical stretch (each y-value is multiplied by 2)

(c)



(d) $g(x) = 2f(x+5)$

P.8.37.

$f(x) = x^2$ but shifted three units to the right and seven units down

$g(x) = (x-3)^2 - 7$

P.8.38.

$f(x) = x^2$ but shifted two units to the left, nine units up, and then reflected in the x-axis

$g(x) = -(x+2)^2 + 9$

P.8.39.

$f(x) = \sqrt{x}$ moved 6 units to the left and reflected in both the x- and y-axes

$g(x) = -\sqrt{-x+6}$

P.8.40.

$f(x) = \sqrt{x}$ moved 9 units downward and reflected in both the x-axis and the y-axis

$$g(x) = -\left(\sqrt{-x} - 9\right)$$
$$= -\sqrt{-x} + 9$$

P.8.41.

$f(x) = x^2$

(a) Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 3)

$g(x) = -3x^2$

(b) Vertical shift 3 units upward and a vertical stretch (each y-value is multiplied by 4)

$g(x) = 4x^2 + 3$

P.8.42.

$f(x) = x^3$

(a) Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$

$g(x) = \frac{1}{4}x^3$

(b) Reflection in the x-axis and a vertical stretch $\left(\text{each } y\text{-value is multiplied by } 2\right)$

$g(x) = -2x^3$

P.8.43.

$f(x) = |x|$

(a) Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = -\frac{1}{2}|x|$

(b) Vertical stretch (each y-value is multiplied by 3) and a vertical shift 3 units downward

$g(x) = 3|x| - 3$

P.8.44.

$f(x) = \sqrt{x}$

(a) Vertical stretch (each y-value is multiplied by 8)

$g(x) = 8\sqrt{x}$

(b) Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$

$g(x) = -\frac{1}{4}\sqrt{x}$

P.8.45.

Parent function: $f(x) = x^3$

Vertical stretch (each y-value is multiplied by 2)

$g(x) = 2x^3$

P.8.46.

Parent function: $y = [\![x]\!]$

Horizontal stretch (each x-value is multiplied by 2)

$g(x) = [\![\frac{1}{2}x]\!]$

P.8.47.

Parent function: $f(x) = \sqrt{x}$

Reflection in the y-axis, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = \frac{1}{2}\sqrt{-x}$

P.8.48.

Parent function: $f(x) = |x|$

Reflection in the x-axis, vertical shift of 2 units downward, vertical stretch (each y-value is multiplied by 2)

$g(x) = -2|x| - 2$

P.8.49.

Parent function: $f(x) = x^3$

Reflection in the x-axis, horizontal shift 2 units to the right and a vertical shift 2 units upward

$g(x) = -(x-2)^3 + 2$

P.8.50.

Parent function: $f(x) = |x|$

Horizontal shift of 4 units to the left and a vertical shift of 2 units downward

$g(x) = |x+4| - 2$

P.8.51.

Parent function: $f(x) = \sqrt{x}$

Reflection in the x-axis and a vertical shift 3 units downward

$g(x) = -\sqrt{x} - 3$

P.8.52.

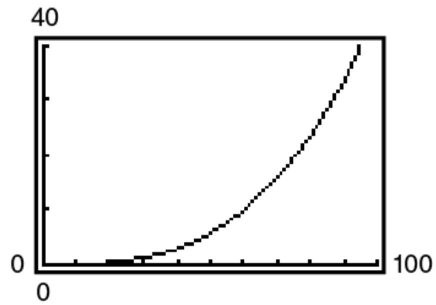
Parent function: $f(x) = x^2$

Horizontal shift of 2 units to the right and a vertical shift of 4 units upward

$g(x) = (x-2)^2 + 4$

P.8.53.

(a)



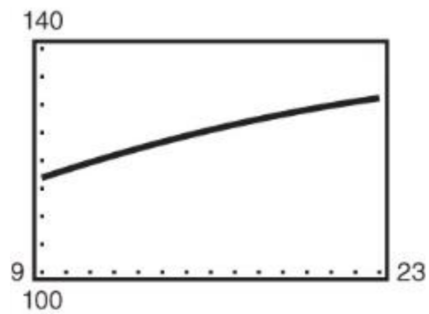
(b) $H(x) = 0.00004636x^3$

$$H\left(\frac{x}{1.6}\right) = 0.00004636\left(\frac{x}{1.6}\right)^3$$
$$= 0.00004636\left(\frac{x^3}{4.096}\right)$$
$$= 0.0000113184x^3 = 0.00001132x^3$$

The graph of $H\left(\frac{x}{1.6}\right)$ is a horizontal stretch of the graph of $H(x)$.

P.8.54.

(a) The transformation consists of a reflection in the x-axis, vertical shrink (each y-value is multiplied by 0.02399), horizontal shift to the right 33.643 units, and a vertical shift 134.533 units upward.



(b) $\dfrac{N(23)-N(9)}{23-9}=\dfrac{131.25-116.93}{14}\approx 1.023$

The number of households in the United States increased by an average rate of about 1,023,000 households per year from 2009 to 2023.

(c) $N(19)=-0.02899(19-33.643)^2+134.533\approx 128.3$

There were about 128.3 million households in 2019.

P.8.55.

False. $y=f(-x)$ is a reflection in the y-axis.

P.8.56.

False. $y=-f(x)$ is a reflection in the x-axis.

P.8.57.

True. Because $|x|=|-x|$, the graphs of $f(x)=|x|+6$ and $f(x)=|-x|+6$ are identical.

P.8.58.

False. The point $(-2,-61)$ lies on the transformation.

P.8.59.

$y=f(x+2)-1$

Horizontal shift 2 units to the left and a vertical shift 1 unit downward

$(0, 1)\to(0-2, 1-1)=(-2, 0)$
$(1, 2)\to(1-2, 2-1)=(-1, 1)$
$(2, 3)\to(2-2, 3-1)=(0, 2)$

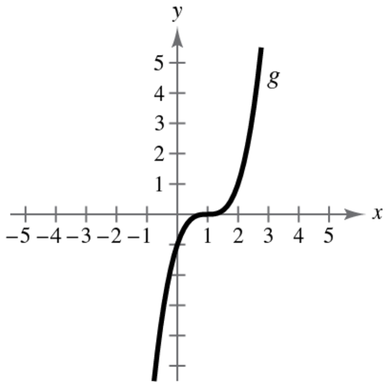
P.8.60.

(a) Answers will vary. *Sample answer*: To graph $f(x) = 3x^2 - 4x + 1$ use the point-plotting method since it is not written in a form that is easily identified by a sequence of translations of the parent function $y = x^2$.

(b) Answers will vary. *Sample answer*: To graph $f(x) = 2(x-1)^2 - 6$ use the method of translating the parent function $y = x^2$ since it is written in a form such that a sequence of translations is easily identified.

P.8.61.

Since the graph of $g(x)$ is a horizontal shift one unit to the right of $f(x) = x^3$, the equation should be $g(x) = (x-1)^3$ and not $g(x) = (x+1)^3$.



P.8.62.

(a) Increasing on the interval $(-2, 1)$ and decreasing on the intervals $(-\infty, -2)$ and $(1, \infty)$

(b) Increasing on the interval $(-1, 2)$ and decreasing on the intervals $(-\infty, -1)$ and $(2, \infty)$

(c) Increasing on the intervals $(-\infty, -1)$ and $(2, \infty)$ and decreasing on the interval $(-1, 2)$

(d) Increasing on the interval $(0, 3)$ and decreasing on the intervals $(-\infty, 0)$ and $(3, \infty)$

(e) Increasing on the intervals $(-\infty, 1)$ and $(4, \infty)$ and decreasing on the interval $(1, 4)$

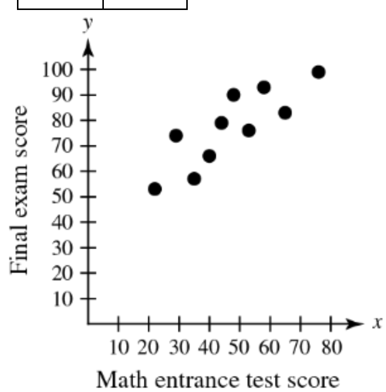
P.8.63.

No. $g(x) = -x^4 - 2$. Yes. $h(x) = -(x-3)^4$

P.8.64.

(a)

x	y
22	53
29	74
35	57
40	66
44	79
48	90
53	76
58	93
65	83
76	99



(b) The point $(65, 83)$ represents an entrance exam score of 65.

(c) No. There are many variables that well affect the final exam score.

P.8.65.

$$(2x+1)+(x^2+2x-1)=x^2+4x$$

P.8.66.

$$\begin{aligned}(2x+1)-(x^2+2x-1)&=2x+1-x^2-2x+1\\&=-x^2+2\end{aligned}$$

P.8.67.

$$\begin{aligned}(3x^2+x-1)-(1-x)&=3x^2+x-1-1+x\\&=3x^2+2x-2\end{aligned}$$

P.8.68.

$$(3x^2+x-1)+(1-x)=3x^2$$

P.8.69.

$$x^2(x-3)=x^3-3x^2$$

P.8.70.

$$x^2(1-x)=x^2-x^3=-x^3+x^2$$

P.8.71.

$$\begin{aligned}-2x(0.1x+17)&=(-2x)(0.1x)+(-2x)(17)\\&=-0.2x^2-34x\end{aligned}$$

P.8.72.

$$\begin{aligned}6y\left(5-\tfrac{3}{8}y\right)&=(6y)(5)-(6y)\left(\tfrac{3}{8}y\right)\\&=30y-\tfrac{9}{4}y^2\\&=-\tfrac{9}{4}y^2+30y\end{aligned}$$

P.8.73.

$$\begin{aligned}(3x+5)\div(6x^2+10x)&=\frac{3x+5}{6x^2+10x}=\frac{3x+5}{2x(3x+5)}\\&=\frac{1}{2x},\quad x\neq\frac{-5}{3}\end{aligned}$$

P.8.74.

$$(20x^2-43x)\div x=\frac{20x^2-43x}{x}=20x-43,\quad x\neq 0$$

Section P.9 Combinations of Functions: Composite Functions

P.9.1.

addition; subtraction; multiplication; division

P.9.2.

composition

P.9.3.

Since $(fg)(x) = 2x(x^2 + 1)$ and $f(x) = x^2 + 1$, $g(x) = 2x$, and $(fg)(x) = (gf)(x) = (2x)f(x)$.

P.9.4.

Since $(f \circ g)(x) = f(g(x))$ and $(f \circ g)(x) = f(x^2 + 1)$, $g(x)$ must equal $x^2 + 1$.

P.9.5.

$f(x) = x + 2$, $g(x) = x - 2$

(a) $\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= (x+2) + (x-2)\\ &= 2x\end{aligned}$

(b) $\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= (x+2) - (x-2)\\ &= 4\end{aligned}$

(c) $\begin{aligned}(fg)(x) &= f(x) \cdot g(x)\\ &= (x+2)(x-2)\\ &= x^2 - 4\end{aligned}$

(d) $\left(\dfrac{f}{x}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x+2}{x-2}$

Domain: all real numbers x except $x = 2$

P.9.6.

$f(x) = 2x - 5$, $g(x) = 2 - x$

(a) $(f+g)(x) = 2x - 5 + 2 - x = x - 3$

(b) $(f-g)(x) = 2x-5-(2-x)$
$= 2x-5-2+x$
$= 3x-7$

(c) $(fg)(x) = (2x-5)(2-x)$
$= 4x-2x^2-10+5x$
$= -2x^2+9x-10$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x-5}{2-x}$

Domain: all real numbers x except $x=2$

P.9.7.

$f(x) = x^2,\ g(x) = 4x-5$

(a) $(f+g)(x) = f(x)+g(x)$
$= x^2+(4x-5)$
$= x^2+4x-5$

(b) $(f-g)(x) = f(x)-g(x)$
$= x^2-(4x-5)$
$= x^2-4x+5$

(c) $(fg)(x) = f(x)\cdot g(x)$
$= x^2(4x-5)$
$= 4x^3-5x^2$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$
$= \dfrac{x^2}{4x-5}$

Domain: all real numbers x except $x=\dfrac{5}{4}$

P.9.8.

$f(x) = 3x+1, g(x) = x^2-16$

(a) $(f+g)(x) = f(x)+g(x)$
$= 3x+1+x^2-16$
$= x^2+3x-15$

(b) $(f-g)(x) = f(x) - g(x)$
$= 3x + 1 - (x^2 - 16)$
$= -x^2 + 3x + 17$

(c) $(fg)(x) = f(x) \cdot g(x)$
$= (3x+1)(x^2 - 16)$
$= 3x^3 + x^2 - 48x - 16$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x+1}{x^2 - 16}$

Domain: all real numbers *x*, except $x \neq \pm 4$

P.9.9.

$f(x) = x^2 + 6, \;\; g(x) = \sqrt{1-x}$

(a) $(f+g)(x) = f(x) + g(x) = x^2 + 6 + \sqrt{1-x}$

(b) $(f-g)(x) = f(x) - g(x) = x^2 + 6 - \sqrt{1-x}$

(c) $(fg)(x) = f(x) \cdot g(x) = (x^2 + 6)\sqrt{1-x}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2+6}{\sqrt{1-x}} = \frac{(x^2+6)\sqrt{1-x}}{1-x}$

Domain: $x < 1$

P.9.10.

$f(x) = \sqrt{x^2 - 4}, g(x) = \sqrt{x+2}$

(a) $(f+g)(x) = \sqrt{x^2 - 4} + \sqrt{x+2}$

(b) $(f-g)(x) = \sqrt{x^2 - 4} - \sqrt{x+2}$

(c) $(fg)(x) = \sqrt{x^2 - 4}\,\sqrt{x+2} = (x+2)\sqrt{x-2}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x^2-4}}{\sqrt{x+2}} = \sqrt{x-2}$

Domain of $\sqrt{x^2 - 4}$: All real numbers *x* such that $x \leq -2$ and $x \geq 2$

Domain of $\sqrt{x+2}$: All real numbers *x* such that $x \geq -2$

Domain of $\frac{f}{g}$: All real numbers *x* such that $x \geq 2$

P.9.11.

$$f(x)=\frac{x}{x+1},\ \ g(x)=x^3$$

(a) $(f+g)(x)=\dfrac{x}{x+1}+x^3=\dfrac{x+x^4+x^3}{x+1}$

(b) $(f-g)(x)=\dfrac{x}{x+1}-x^3=\dfrac{x-x^4-x^3}{x+1}$

(c) $(fg)(x)=\dfrac{x}{x+1}\cdot x^3=\dfrac{x^4}{x+1}$

(d) $\left(\dfrac{f}{g}\right)(x)=\dfrac{x}{x+1}\div x^3=\dfrac{x}{x+1}\cdot\dfrac{1}{x^3}$

$$=\frac{1}{x^2(x+1)}$$

Domain: all real numbers x except $x=0$ and $x=-1$

P.9.12.

$$f(x)=\frac{2}{x},\ g(x)=\frac{1}{x^2-1}$$

(a) $(f+g)(x)=\dfrac{2}{x}+\dfrac{1}{x^2-1}=\dfrac{2(x^2-1)+x}{x(x^2-1)}=\dfrac{2x^2+x-2}{x(x^2-1)}$

(b) $(f-g)(x)=\dfrac{2}{x}-\dfrac{1}{x^2-1}=\dfrac{2(x^2-1)-x}{x(x^2-1)}=\dfrac{2x^2-x-2}{x(x^2-1)}$

(c) $(fg)(x)=\left(\dfrac{2}{x}\right)\left(\dfrac{1}{x^2-1}\right)=\dfrac{2}{x(x^2-1)}$

(d) $\left(\dfrac{f}{g}\right)(x)=\dfrac{2}{x}\div\dfrac{1}{x^2-1}=\dfrac{\frac{2}{x}}{\frac{1}{x^2-1}}=\left(\dfrac{2}{x}\right)(x^2-1)=\dfrac{2(x^2-1)}{x}$

Domain: all real numbers x, $x\neq 0, \pm 1$

P.9.13.

$$\begin{aligned}(f+g)(2)&=f(2)+g(2)\\&=(2+3)+(2^2-2)\\&=7\end{aligned}$$

P.9.14.

$$\begin{aligned}(f-g)(1) &= f(1)-g(1)\\ &= (1+3)-\left((1)^2-2\right)\\ &= 5\end{aligned}$$

P.9.15.

$$\begin{aligned}(f-g)(3t) &= f(3t)-g(3t)\\ &= \left((3t)+3\right)-\left((3t)^2-2\right)\\ &= 3t+3-\left(9t^2-2\right)\\ &= -9t^2+3t+5\end{aligned}$$

P.9.16.

$$\begin{aligned}(f+g)(t-2) &= f(t-2)+g(t-2)\\ &= \left((t-2)+3\right)+\left((t-2)^2-2\right)\\ &= t+1+\left(t^2-4t+4-2\right)\\ &= t^2-3t+3\end{aligned}$$

P.9.17.

$$\begin{aligned}(fg)(6) &= f(6)g(6)\\ &= \left((6)+3\right)\left((6)^2-2\right)\\ &= (9)(34)\\ &= 306\end{aligned}$$

P.9.18.

$$\begin{aligned}(fg)(-6) &= f(-6)g(-6)\\ &= \left((-6)+3\right)\left((-6)^2-2\right)\\ &= (-3)(34)\\ &= -102\end{aligned}$$

P.9.19.

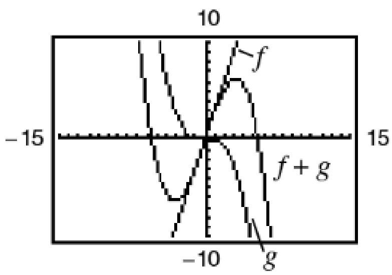
$$\begin{aligned}(f/g)(5) &= f(5)/g(5)\\ &= \left((5)+3\right)\Big/\left((5)^2-2\right)\\ &= \frac{8}{23}\end{aligned}$$

P.9.20.

$$(f/g)(0) = f(0)/g(0)$$
$$= ((0)+3)/((0)^2 - 2)$$
$$= -\frac{3}{2}$$

P.9.21.

$$f(x) = 3x,\ \ g(x) = -\frac{x^3}{10}$$
$$(f+g)(x) = 3x - \frac{x^3}{10}$$

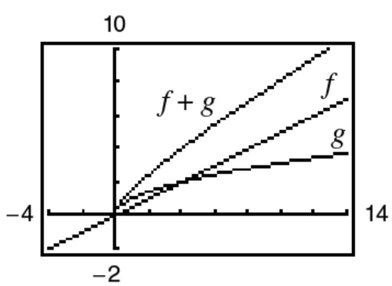


For $0 \le x \le 2$, $f(x)$ contributes most to the magnitude.

For $x > 6$, $g(x)$ contributes most to the magnitude.

P.9.22.

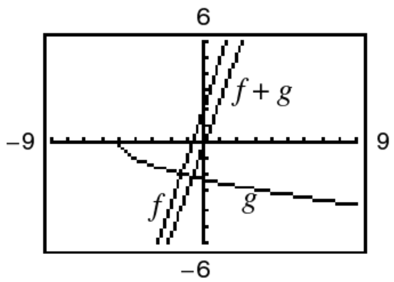
$$f(x) = \frac{x}{2},\ \ g(x) = \sqrt{x}$$
$$(f+g)(x) = \frac{x}{2} + \sqrt{x}$$



$g(x)$ contributes most to the magnitude of the sum for $0 \le x \le 2$. $f(x)$ contributes most to the magnitude of the sum for $x > 6$.

P.9.23.

$$f(x) = 3x + 2,\ \ g(x) = -\sqrt{x+5}$$
$$(f+g)x = 3x - \sqrt{x+5} + 2$$

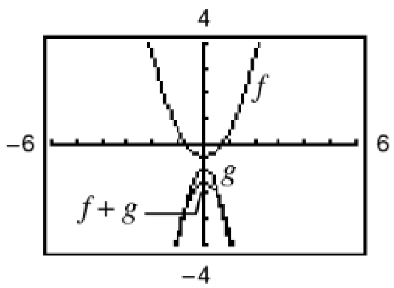


For $0 \le x \le 2, f(x)$ contributes most to the magnitude.

For $x > 6,\ f(x)$ contributes most to the magnitude.

P.9.24.

$$f(x) = x^2 - \tfrac{1}{2},\ \ g(x) = -3x^2 - 1$$
$$(f+g)(x) = -2x^2 - \tfrac{3}{2}$$



For $0 \le x \le 2,\ g(x)$ contributes most to the magnitude.

For $x > 6,\ g(x)$ contributes most to the magnitude.

P.9.25.

$$f(x) = x + 8,\ g(x) = x - 3$$

(a) $(f \circ g)(x) = f(g(x)) = f(x-3) = (x-3) + 8 = x + 5$

(b) $(g \circ f)(x) = g(f(x)) = g(x+8) = (x+8) - 3 = x + 5$

(c) $(g \circ g)(x) = g(g(x)) = g(x-3) = (x-3) - 3 = x - 6$

P.9.26.

$f(x) = 3x, \ g(x) = x^4$

(a) $(f \circ g)(x) = f(g(x)) = f(x^4) = 3(x^4) = 3x^4$

(b) $(g \circ f)(x) = g(f(x)) = g(3x) = (3x)^4 = 81x^4$

(c) $(g \circ g)(x) = g(g(x)) = g(x^4) = (x^4)^4 = x^{16}$

P.9.27.

$f(x) = \sqrt[3]{x-1}, \ g(x) = x^3 + 1$

(a) $$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x^3+1) \\ &= \sqrt[3]{(x^3+1)-1} \\ &= \sqrt[3]{x^3} = x\end{aligned}$$

(b) $$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g\left(\sqrt[3]{x-1}\right) \\ &= \left(\sqrt[3]{x-1}\right)^3 + 1 \\ &= (x-1)+1 = x\end{aligned}$$

(c) $$\begin{aligned}(g \circ g)(x) &= g(g(x)) \\ &= g(x^3+1) \\ &= (x^3+1)^3 + 1 \\ &= x^9 + 3x^6 + 3x^3 + 2\end{aligned}$$

P.9.28.

$f(x) = x^3, \ g(x) = \dfrac{1}{x}$

(a) $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x}\right) = \left(\dfrac{1}{x}\right)^3 = \dfrac{1}{x^3}$

(b) $(g \circ f)(x) = g(f(x)) = g(x^3) = \dfrac{1}{x^3}$

(c) $(g \circ g)(x) = g(g(x)) = g\left(\dfrac{1}{x}\right) = x$

P.9.29.

$f(x)=\sqrt{x+4}$ Domain: $x \geq -4$

$g(x)=x^2$ Domain: all real numbers x

(a) $(f \circ g)(x)=f(g(x))=f(x^2)=\sqrt{x^2+4}$

Domain: all real numbers x

(b) $(g \circ f)(x)=g(f(x))$

$=g(\sqrt{x+4})=(\sqrt{x+4})^2=x+4$

Domain: $x \geq -4$

P.9.30.

$f(x)=\sqrt[3]{x-5}$ Domain: all real numbers x

$g(x)=x^3+1$ Domain: all real numbers x

(a) $(f \circ g)(x)=f(g(x))$

$=f(x^3+1)$

$=\sqrt[3]{x^3+1-5}$

$=\sqrt[3]{x^3-4}$

Domain: all real numbers x

(b) $(g \circ f)(x)=g(f(x))$

$=g(\sqrt[3]{x-5})$

$=(\sqrt[3]{x-5})^3+1$

$=x-5+1=x-4$

Domain: all real numbers x

P.9.31.

$f(x)=|x|$ Domain: all real numbers x

$g(x)=x+6$ Domain: all real numbers x

(a) $(f \circ g)(x)=f(g(x))=f(x+6)=|x+6|$

Domain: all real numbers x

(b) $(g \circ f)(x)=g(f(x))=g(|x|)=|x|+6$

Domain: all real numbers x

P.9.32.

$f(x) = |x-4|$ Domain: all real numbers x

$g(x) = 3-x$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(3-x) = |(3-x)-4| = |-x-1|$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x)) = g(|x-4|) = 3-(|x-4|) = 3-|x-4|$

Domain: all real numbers x

P.9.33.

$f(x) = \dfrac{1}{x}$ Domain: all real numbers x except $x = 0$

$g(x) = x+3$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x+3) = \dfrac{1}{x+3}$

Domain: all real numbers x except $x = -3$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{1}{x}\right) = \dfrac{1}{x} + 3$

Domain: all real numbers x except $x = 0$

P.9.34.

$f(x) = \dfrac{3}{x^2-1}$ Domain: all real numbers x except $x = \pm 1$

$g(x) = x+1$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x+1) = \dfrac{3}{(x+1)^2-1} = \dfrac{3}{x^2+2x+1-1} = \dfrac{3}{x^2+2x}$

Domain: all real numbers x except $x = 0$ and $x = -2$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{3}{x^2-1}\right) = \dfrac{3}{x^2-1} + 1 = \dfrac{3+x^2-1}{x^2-1} = \dfrac{x^2+2}{x^2-1}$

Domain: all real numbers x except $x = \pm 1$

P.9.35.

(a) $(f+g)(3) = f(3) + g(3) = 2+1 = 3$

(b) $\left(\dfrac{f}{g}\right)(2) = \dfrac{f(2)}{g(2)} = \dfrac{0}{2} = 0$

P.9.36.

(a) $(f-g)(1) = f(1) - g(1) = 2 - 3 = -1$

(b) $(fg)(4) = f(4) \cdot g(4) = 4 \cdot 0 = 0$

P.9.37.

(a) $(f \circ g)(2) = f(g(2)) = f(2) = 0$

(b) $(g \circ f)(2) = g(f(2)) = g(0) = 4$

P.9.38.

(a) $(f \circ g)(1) = f(g(1)) = f(3) = 2$

(b) $(g \circ f)(3) = g(f(3)) = g(2) = 2$

P.9.39.

$h(x) = (2x+1)^2$

One possibility: Let $f(x) = x^2$ and $g(x) = 2x+1$, then $(f \circ g)(x) = h(x)$.

P.9.40.

$h(x) = (1-x)^3$

One possibility: Let $g(x) = 1-x$ and $f(x) = x^3$, then $(f \circ g)(x) = h(x)$.

P.9.41.

$h(x) = \sqrt[3]{x^2 - 4}$

One possibility: Let $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - 4$, then $(f \circ g)(x) = h(x)$.

P.9.42.

$h(x) = \sqrt{9-x}$

One possibility: Let $g(x) = 9-x$ and $f(x) = \sqrt{x}$, then $(f \circ g)(x) = h(x)$.

P.9.43.

$h(x) = \dfrac{1}{x+2}$

One possibility: Let $f(x) = 1/x$ and $g(x) = x+2$, then $(f \circ g)(x) = h(x)$.

P.9.44.

$$h(x)=\frac{4}{(5x+2)^2}$$

One possibility: Let $g(x)=5x+2$ and $f(x)=\frac{4}{x^2}$, then $(f\circ g)(x)=h(x)$.

P.9.45.

$$h(x)=\frac{-x^2+3}{4-x^2}$$

One possibility: Let $f(x)=\frac{x+3}{4+x}$ and $g(x)=-x^2$, then $(f\circ g)(x)=h(x)$.

P.9.46.

$$h(x)=\frac{27x^3+6x}{10-27x^3}$$

One possibility: Let $g(x)=x^3$ and $f(x)=\frac{27x+6\sqrt[3]{x}}{10-27x}$, then $(f\circ g)(x)=h(x)$.

P.9.47.

(a) $c(t)=\frac{b(t)-d(t)}{p(t)}\times 100=\text{percent change}$

(b) $c(24)$ represents the percent change in the population due to births and deaths in the year 2024.

P.9.48.

(a) $p(t)=d(t)+c(t)$

(b) $p(24)$ represents the number of dogs and cats in 2024.

(c) $h(t)=\frac{p(t)}{n(t)}=\frac{d(t)+c(t)}{n(t)}$

$h(t)$ represents the number of dogs and cats per capita.

P.9.49.

(a) $r(x)=\frac{x}{2}$

(b) $A(r)=\pi r^2$

(c) $(A \circ r)(x) = A(r(x)) = A\left(\frac{x}{2}\right) = \pi\left(\frac{x}{2}\right)^2$

$(A \circ r)(x)$ represents the area of the circular base of the tank on the square foundation with side length *x*.

P.9.50.

(a) $N(T(t)) = N(3t+2)$

$$\begin{aligned} &= 10(3t+2)^2 - 20(3t+2) + 600 \\ &= 10(9t^2 + 12t + 4) - 60t - 40 + 600 \\ &= 90t^2 + 60t + 600 \\ &= 30(3t^2 + 2t + 20),\ 0 \le t \le 6 \end{aligned}$$

This represents the number of bacteria in the food as a function of time.

(b) Use $t = 0.5$.

$N(T(0.5)) = 30\left(3(0.5)^2 + 2(0.5) + 20\right) = 652.5$

After half an hour, there will be about 653 bacteria.

(c) $30(3t^2 + 2t + 20) = 1500$

$3t^2 + 2t + 20 = 50$

$3t^2 + 2t - 30 = 0$

By the Quadratic Formula, $t \approx -3.513$ or 2.846.
Choosing the positive value for *t*, you have $t \approx 2.846$ hours.

P.9.51.

False. $(f \circ g)(x) = 6x + 1$ and $(g \circ f)(x) = 6x + 6$

P.9.52.

True. $(f \circ g)(c)$ is defined only when $g(c)$ is in the domain of *f*.

P.9.53.

(a) Answer not unique. *Sample answer*:

$f(x) = x + 3,\ g(x) = x + 2$

$(f \circ g)(x) = f(g(x)) = (x+2) + 3 = x + 5$

$(g \circ f)(x) = g(f(x)) = (x+3) + 2 = x + 5$

(b) Answer not unique. *Sample answer*: $f(x) = x^2,\ g(x) = x^3$

$$(f \circ g)(x) = f(g(x)) = (x^3)^2 = x^6$$
$$(g \circ f)(x) = g(f(x)) = (x^2)^3 = x^6$$

P.9.54.

(a) $f(p)$: matches L_2; For example, an original price of $p = \$15.00$ corresponds to a sale price of $S = \$7.50$.

(b) $g(p)$: matches L_1; For example an original price of $p = \$20.00$ corresponds to a sale price of $S = \$15.00$.

(c) $(g \circ f)(p)$: matches L_4; This function represents applying a 50% discount to the original price *p,* then subtracting a \$5 discount.

(d) $(f \circ g)(p)$ matches L_3; This function represents subtracting a \$5 discount from the original price *p,* then applying a 50% discount.

P.9.55.

Let $f(x)$ and $g(x)$ be two odd functions and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= [-f(x)][-g(x)] \quad \text{because } f \text{ and } g \text{ are odd} \\ &= f(x)g(x) \\ &= h(x). \end{aligned}$$

So, $h(x)$ is even.

Let $f(x)$ and $g(x)$ be two even functions and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= f(x)g(x) \quad \text{because } f \text{ and } g \text{ are even} \\ &= h(x). \end{aligned}$$

So, $h(x)$ is even.

P.9.56.

(a) $g(x) = \frac{1}{2}[f(x) + f(-x)]$

To determine if $g(x)$ is even, show $g(-x) = g(x)$.

$$g(-x) = \frac{1}{2}[f(-x) + f(-(-x))] = \frac{1}{2}[f(-x) + f(x)] = \frac{1}{2}[f(x) + f(-x)] = g(x) \ ✓$$

$$h(x) = \frac{1}{2}\left[f(x) - f(-x)\right]$$

To determine if $h(x)$ is odd show $h(-x) = -h(x)$.

$$h(-x) = \frac{1}{2}\left[f(-x) - f(-(-x))\right] = \frac{1}{2}\left[f(-x) - f(x)\right] = -\frac{1}{2}\left[f(x) - f(-x)\right] = -h(x) \checkmark$$

(b) $\frac{1}{2}\left[f(x) + f(-x)\right] + \frac{1}{2}\left[f(x) - f(-x)\right] = \frac{1}{2}\left[f(x) + f(-x) + f(x) - f(-x)\right]$

$$= \frac{1}{2}\left[2f(x)\right]$$

$$= f(x)$$

(c) $f(x) = x^2 - 2x + 1 = (x^2 + 1) + (-2x)$

$$k(x) = \frac{1}{x+1} = -\frac{1}{(x+1)(x-1)} + \frac{x}{(x+1)(x-1)}$$

P.9.57.

Let $f(x)$ be an odd function, $g(x)$ be an even function, and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= \left[-f(x)\right]g(x) \qquad \text{because } f \text{ is odd and } g \text{ is even} \\ &= -f(x)g(x) \\ &= -h(x). \end{aligned}$$

So, h is odd and the product of an odd function and an even function is odd.

P.9.58.

The expression $3(2x+1)+2$ is found using $(g \circ f)(x)$ instead of $(f \circ g)(x)$.

So, $(f \circ g)(x) = 2(3x+2)+1$

$$= 6x + 5.$$

P.9.59.

$y^2 = x - 5$

$y^2 = (-x) - 5 \Rightarrow y^2 = -x - 5 \Rightarrow$ No y-axis symmetry

$(-y)^2 = x - 5 \Rightarrow y^2 = x - 5 \Rightarrow$ x-axis symmetry

$(-y)^2 = (-x) - 5 \Rightarrow y^2 = -x - 5 \Rightarrow$ No origin symmetry

P.9.60.

$$2x+5y=17$$
$$2(-x)+5y=17 \Rightarrow -2x+5y=17 \Rightarrow \text{ No } y\text{-axis symmetry}$$
$$2x+5(-y)=17 \Rightarrow 2x-5y=17 \Rightarrow \text{ No } x\text{-axis symmetry}$$
$$2(-x)+5(-y)=17 \Rightarrow -2x-5y=17 \Rightarrow \text{ No origin symmetry}$$

P.9.61.

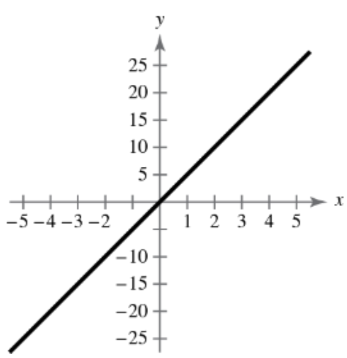
$$y=x^2+1$$
$$y=(-x)^2+1 \Rightarrow y=x^2+1 \Rightarrow y\text{-axis symmetry}$$
$$-y=x^2+1 \Rightarrow y=-x^2-1 \Rightarrow \text{ No } x\text{-axis symmetry}$$
$$-y=(-x)^2+1 \Rightarrow -y=x^2+1 \Rightarrow y=-x^2-1 \Rightarrow \text{ No origin symmetry}$$

P.9.62.

$$x^2+y^2=72$$
$$(-x)^2+y^2=72 \Rightarrow x^2+y^2=72 \Rightarrow y\text{-axis symmetry}$$
$$x^2+(-y)^2=72 \Rightarrow x^2+y^2=72 \Rightarrow x\text{-axis symmetry}$$
$$(-x)^2+(-y)^2=72 \Rightarrow x^2+y^2=72 \Rightarrow \text{ Origin symmetry}$$

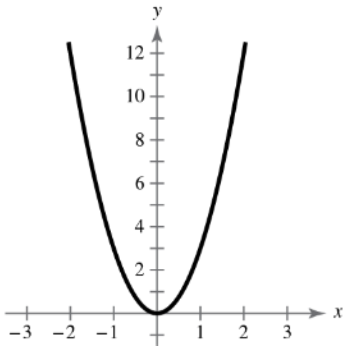
P.9.63.

$$y=5x$$
$$f(x)=5x$$
$$f(-x)=5(-x)=-5x=-f(x) \text{ Odd function}$$



P.9.64.

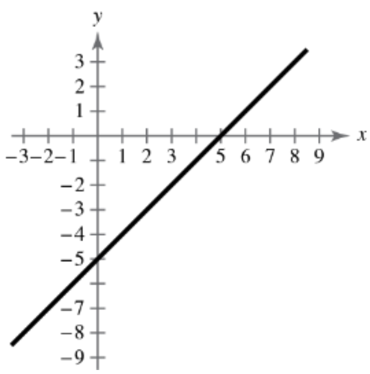
$$y = 3x^2$$
$$f(x) = 3x^2$$
$$f(-x) = 3(-x)^2 = 3x^2 = f(x)$$ Even function



P.9.65.

$$y = x - 5$$
$$f(x) = x - 5$$
$$f(-x) = -x - 5$$

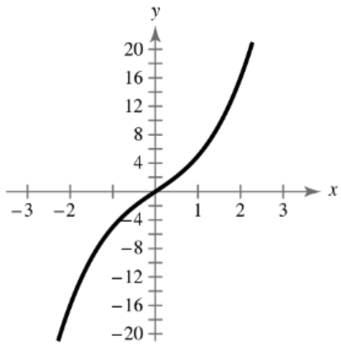
Neither even nor odd



P.9.66.

$$y = x^3 + 4x$$
$$f(x) = x^3 + 4x$$
$$f(-x) = (-x)^3 + 4(-x) = -(x^3 + 4x) = -f(x)$$

Odd function



P.9.67.

$$\begin{aligned} 2x + 3y &= 5 \\ 3y &= 5 - 2x \\ y &= \frac{5}{3} - \frac{2}{3}x \end{aligned}$$

P.9.68.

$$\begin{aligned} xy - 1 &= 3y + x \\ xy - 3y &= 1 + x \\ y(x - 3) &= 1 + x \\ y &= \frac{(1 + x)}{(x - 3)} \end{aligned}$$

P.9.69.

$$\begin{aligned} x &= \sqrt{y + 1} \\ x^2 &= y + 1 \\ y &= x^2 - 1, \quad x \geq 0 \end{aligned}$$

P.9.70.

$$\begin{aligned} x &= \frac{5 - y}{3y + 2} \\ 3xy + 2x &= 5 - y \\ 3xy + y &= 5 - 2x \\ y(3x + 1) &= 5 - 2x \\ y &= \frac{(5 - 2x)}{(3x + 1)} \end{aligned}$$

P.9.71.

$$\begin{aligned} A &= \pi\left(r+2\right)^2 - \pi r^2 \\ &= \pi\left[\left(r+2\right)^2 - r^2\right] \\ &= \pi\left[r^2 + 4r + 4 - r^2\right] \\ &= \pi\left(4r+4\right) \\ &= 4\pi\left(r+1\right) \end{aligned}$$

P.9.72.

$$\begin{aligned} A &= \tfrac{1}{2}\left[\tfrac{5}{4}\left(x+3\right)\right]\left(x+3\right) - \tfrac{1}{2}\left(5\right)\left(4\right) \\ &= \tfrac{5}{8}\left(x+3\right)^2 - 10 \\ &= \tfrac{5}{8}\left(x^2 + 6x + 9\right) - 10 \end{aligned}$$

SECTION P.10 INVERSE FUNCTIONS

P.10.1.

range; domain

P.10.2.

$y = x$

P.10.3.

To show that two function f and g are inverse functions, you must show that $f(g(x)) = x$ and $g(f(x)) = x$.

P.10.4.

If a function is one-to-one, no two x-values in the domain can correspond to the same y-value in the range. Therefore, a horizontal line can intersect the graph at most once.

P.10.5.

$$f(x) = 6x$$

$$f^{-1}(x) = \frac{x}{6} = \frac{1}{6}x$$

$$f(f^{-1}(x)) = f\left(\frac{x}{6}\right) = 6\left(\frac{x}{6}\right) = x$$

$$f^{-1}(f(x)) = f^{-1}(6x) = \frac{6x}{6} = x$$

P.10.6.

$$f(x) = \tfrac{1}{3}x$$

$$f^{-1}(x) = 3x$$

$$f(f^{-1}(x)) = f(3x) = \tfrac{1}{3}(3x) = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\tfrac{1}{3}x\right) = 3\left(\tfrac{1}{3}x\right) = x$$

P.10.7.

$$f(x) = 3x + 1$$

$$f^{-1}(x) = \frac{x-1}{3}$$

$$f(f^{-1}(x)) = f\left(\frac{x-1}{3}\right) = 3\left(\frac{x-1}{3}\right) + 1 = x$$

$$f^{-1}(f(x)) = f^{-1}(3x+1) = \frac{(3x+1)-1}{3} = x$$

P.10.8.

$$f(x)=\frac{x-3}{2}$$
$$f^{-1}(x)=2x+3$$
$$f\left(f^{-1}(x)\right)=f(2x+3)=\frac{(2x+3)-3}{3}=\frac{2x}{2}=x$$
$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{x-3}{2}\right)=2\left(\frac{x-3}{2}\right)+3$$
$$=(x-3)+3=x$$

P.10.9.

$$f(x)=x^3+1$$
$$f^{-1}(x)=\sqrt[3]{x-1}$$
$$f\left(f^{-1}(x)\right)=f\left(\sqrt[3]{x-1}\right)=\left(\sqrt[3]{x-1}\right)^3+1=(x-1)+1=x$$
$$f^{-1}\left(f(x)\right)=f^{-1}\left(x^3+1\right)=\sqrt[3]{\left(x^3+1\right)-1}=\sqrt[3]{x^3}=x$$

P.10.10.

$$f(x)=\frac{x^5}{4}$$
$$f^{-1}(x)=\sqrt[5]{4x}$$
$$f\left(f^{-1}(x)\right)=f\left(\sqrt[5]{4x}\right)=\frac{\left(\sqrt[5]{4x}\right)^5}{4}=\frac{4x}{4}=x$$
$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{x^5}{4}\right)=\sqrt[5]{4\left(\frac{x^5}{4}\right)}=\sqrt[5]{x^5}=x$$

P.10.11.

$$f(x)=x^2-4,\ x\geq 0$$
$$f^{-1}(x)=\sqrt{x+4}$$
$$f\left(f^{-1}(x)\right)=f\left(\sqrt{x+4}\right)=\left(\sqrt{x+4}\right)^2-4=(x+4)-4=x$$
$$f^{-1}\left(f(x)\right)=f^{-1}\left(x^2-4\right)=\sqrt{\left(x^2-4\right)+4}=\sqrt{x^2}=x$$

P.10.12.

$$f(x) = x^2 + 2,\ x \geq 0$$
$$f^{-1}(x) = \sqrt{x-2}$$
$$f\left(f^{-1}(x)\right) = f\left(\sqrt{x-2}\right) = \left(\sqrt{x-2}\right)^2 + 2 = (x-2) + 2 = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}\left(x^2+2\right) = \sqrt{\left(x^2+2\right)-2} = \sqrt{x^2} = x$$

P.10.13.

$$(f \circ g)(x) = f\left(g(x)\right) = f(4x+9) = \frac{4x+9-9}{4} = \frac{4x}{4} = x$$
$$(g \circ f)(x) = g\left(f(x)\right) = g\left(\frac{x-9}{4}\right) = 4\left(\frac{x-9}{4}\right) + 9 = x - 9 + 9 = x$$

P.10.14.

$$\begin{aligned} f\left(g(x)\right) &= f\left(-\frac{2x+8}{3}\right) = -\frac{3}{2}\left(-\frac{2x+8}{3}\right) - 4 \\ &= \frac{1}{2}\left(2(x+4)\right) - 4 \\ &= x + 4 - 4 = x \end{aligned}$$

$$\begin{aligned} g\left(f(x)\right) &= g\left(-\frac{3}{2}x - 4\right) = -\frac{2\left(-\frac{3}{2}x - 4\right) + 8}{3} \\ &= -\frac{-3x - 8 + 8}{3} \\ &= -\frac{-3x}{3} \\ &= x \end{aligned}$$

P.10.15.

$$\begin{aligned} f\left(g(x)\right) &= f\left(\sqrt[3]{4x}\right) = \frac{\left(\sqrt[3]{4x}\right)^3}{4} \\ &= \frac{4x}{4} \\ &= x \end{aligned}$$

$$\begin{aligned} g\left(f(x)\right) &= g\left(\frac{x^3}{4}\right) = \sqrt[3]{4\left(\frac{x^3}{4}\right)} \\ &= \sqrt[3]{x^3} \\ &= x \end{aligned}$$

P.10.16.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt[3]{x-5}\right) = \left(\sqrt[3]{x-5}\right)^3 + 5 = x - 5 + 5 = x$$
$$(g \circ f)(x) = g(f(x)) = g\left(x^3 + 5\right) = \sqrt[3]{x^3 + 5 - 5} = \sqrt[3]{x^3} = x$$

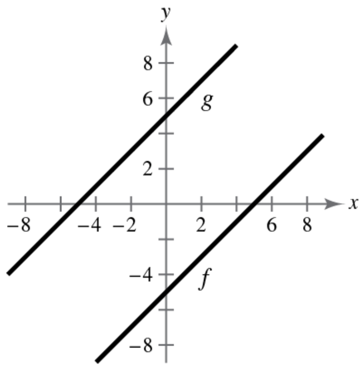
P.10.17.

$f(x) = x - 5, \;\; g(x) = x + 5$

(a) $f(g(x)) = f(x+5) = (x+5) - 5 = x$

$g(f(x)) = g(x-5) = (x-5) + 5 = x$

(b)



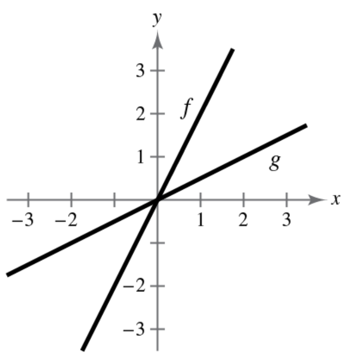
P.10.18.

$f(x) = 2x, \;\; g(x) = \dfrac{x}{2}$

(a) $f(g(x)) = f\left(\dfrac{x}{2}\right) = 2\left(\dfrac{x}{2}\right) = x$

$g(f(x)) = g(2x) = \dfrac{2x}{2} = x$

(b)



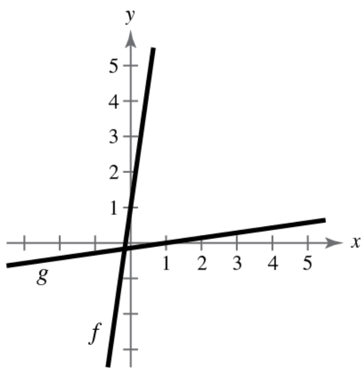
P.10.19.

$$f(x) = 7x + 1, \quad g(x) = \frac{x-1}{7}$$

(a) $f(g(x)) = f\left(\frac{x-1}{7}\right) = 7\left(\frac{x-1}{7}\right) + 1 = x$

$$g(f(x)) = g(7x+1) = \frac{(7x+1)-1}{7} = x$$

(b)



P.10.20.

$$f(x) = 3 - 4x, \quad g(x) = \frac{3-x}{4}$$

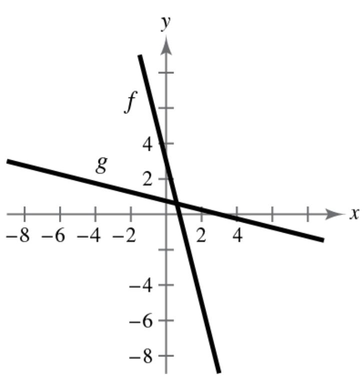
(a) $f(g(x)) = f\left(\frac{3-x}{4}\right) = 3 - 4\left(\frac{3-x}{4}\right)$

$$= 3 - (3 - x) = x$$

$$g(f(x)) = g(3-4x) = \frac{3-(3-4x)}{4} = \frac{4x}{4}$$

$$= x$$

(b)



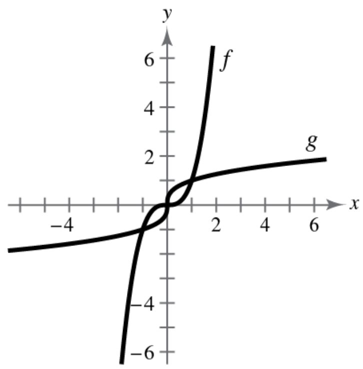
P.10.21.

$f(x) = x^3,\ g(x) = \sqrt[3]{x}$

(a) $f(g(x)) = f(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$

$g(f(x)) = g(x^3) = \sqrt[3]{(x^3)} = x$

(b)



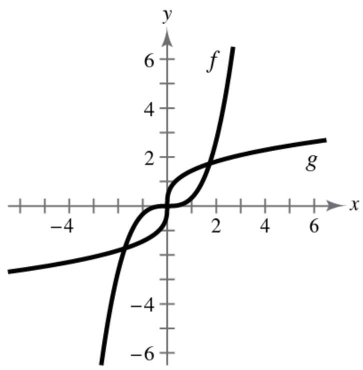
P.10.22.

$f(x) = \dfrac{x^3}{3},\ g(x) = \sqrt[3]{3x}$

(a) $f(g(x)) = f(\sqrt[3]{3x}) = \dfrac{(\sqrt[3]{3x})^3}{3} = \dfrac{3x}{3} = x$

$g(f(x)) = g\left(\dfrac{x^3}{3}\right) = \sqrt[3]{3\left(\dfrac{x^3}{3}\right)} = \sqrt[3]{x^3} = x$

(b)

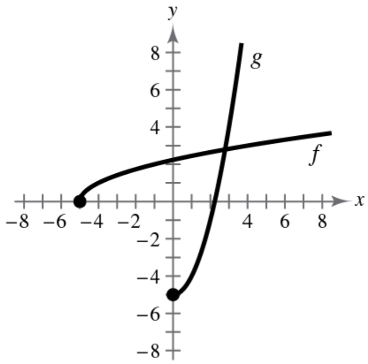


P.10.23.

$$f(x) = \sqrt{x+5},\ g(x) = x^2 - 5,\ x \ge 0$$

(a) $$f(g(x)) = f(x^2 - 5),\ x \ge 0$$
$$= \sqrt{(x^2 - 5) + 5} = x$$
$$g(f(x)) = g(\sqrt{x+5})$$
$$= (\sqrt{x+5})^2 - 5 = x$$

(b)

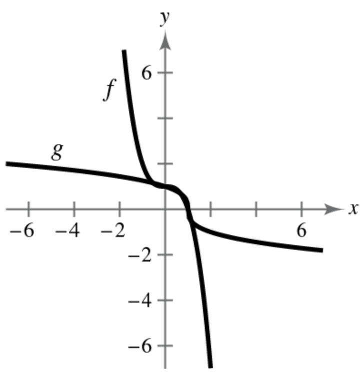


P.10.24.

$$f(x) = 1 - x^3,\ g(x) = \sqrt[3]{1-x}$$

(a) $$f(g(x)) = f(\sqrt[3]{1-x}) = 1 - (\sqrt[3]{1-x})^3$$
$$= 1 - (1-x) = x$$
$$g(f(x)) = g(1-x^3) = \sqrt[3]{1-(1-x^3)}$$
$$= \sqrt[3]{x^3} = x$$

(b)



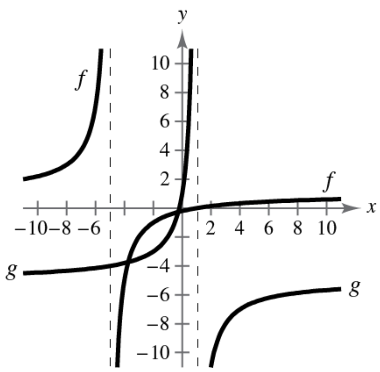
P.10.25.

$$f(x)=\frac{x-1}{x+5},\quad g(x)=-\frac{5x+1}{x-1}$$

(a) $$f(g(x))=f\left(-\frac{5x+1}{x-1}\right)=\frac{\left(-\dfrac{5x+1}{x-1}-1\right)}{\left(-\dfrac{5x+1}{x-1}+5\right)}\cdot\frac{x-1}{x-1}=\frac{-(5x+1)-(x-1)}{-(5x+1)+5(x-1)}=\frac{-6x}{-6}=x$$

$$g(f(x))=g\left(\frac{x-1}{x+5}\right)=-\frac{\left[5\left(\dfrac{x-1}{x+5}\right)+1\right]}{\left[\dfrac{x-1}{x+5}-1\right]}\cdot\frac{x+5}{x+5}=-\frac{5(x-1)+(x+5)}{(x-1)-(x+5)}=-\frac{6x}{-6}=x$$

(b)



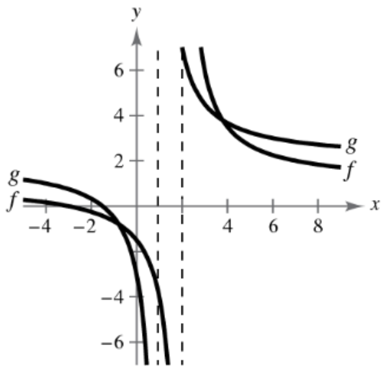
P.10.26.

$$f(x)=\frac{x+3}{x-2},\quad g(x)=\frac{2x+3}{x-1}$$

(a) $$f(g(x))=f\left(\frac{2x+3}{x-1}\right)=\frac{\dfrac{2x+3}{x-1}+3}{\dfrac{2x+3}{x-1}-2}=\frac{\dfrac{2x+3+3x-3}{x-1}}{\dfrac{2x+3-2x+2}{x-1}}=\frac{5x}{5}=x$$

$$g(f(x))=g\left(\frac{x+3}{x-2}\right)=\frac{2\left(\dfrac{x+3}{x-2}\right)+3}{\dfrac{x+3}{x-2}-1}=\frac{\dfrac{2x+6+3x-6}{x-2}}{\dfrac{x+3-x+2}{x-2}}=\frac{5x}{5}=x$$

(b)



P.10.27.

x	3	5	7	9	11	13
$f^{-1}(x)$	−1	0	1	2	3	4

P.10.28.

x	10	5	0	−5	−10	−15
$f^{-1}(x)$	−3	−2	−1	0	1	2

P.10.29.

Yes, because no horizontal line crosses the graph of f at more than one point, f *has* an inverse.

P.10.30.

No, because some horizontal lines intersect the graph of f twice, f *does not* have an inverse.

P.10.31.

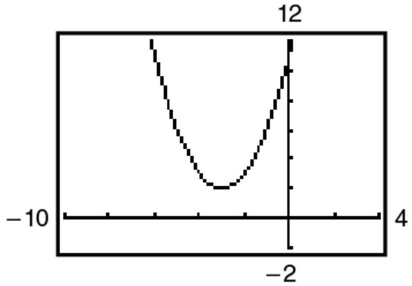
No, because some horizontal lines cross the graph of f twice, f *does not* have an inverse.

P.10.32.

Yes, because no horizontal lines intersect the graph of f at more than one point, f *has* an inverse.

P.10.33.

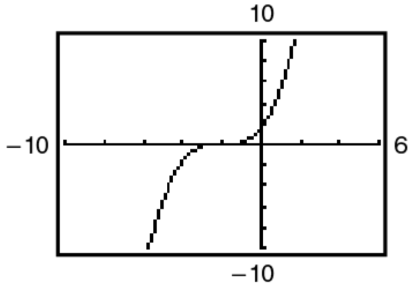
$g(x) = (x+3)^2 + 2$



g does not pass the Horizontal Line Test, so g *does not* have an inverse.

P.10.34.

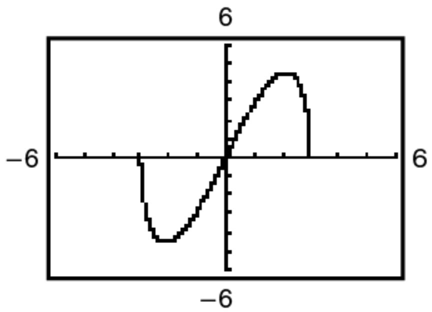
$f(x) = \frac{1}{5}(x+2)^3$



f does pass the Horizontal Line Test, so f *does* have an inverse.

P.10.35.

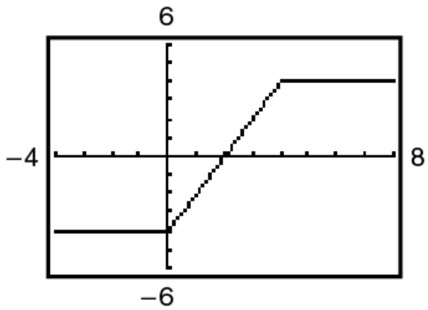
$f(x) = x\sqrt{9 - x^2}$



f does not pass the Horizontal Line Test, so f *does not* have an inverse.

P.10.36.

$$h(x) = |x| - |x-4|$$

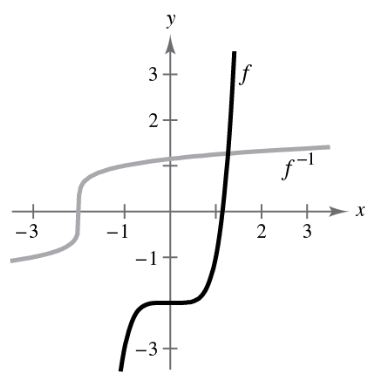


h does not pass the Horizontal Line Test, so h *does not* have an inverse.

P.10.37.

(a)
$$\begin{aligned} f(x) &= x^5 - 2 \\ y &= x^5 - 2 \\ x &= y^5 - 2 \\ y &= \sqrt[5]{x+2} \\ f^{-1}(x) &= \sqrt[5]{x+2} \end{aligned}$$

(b)



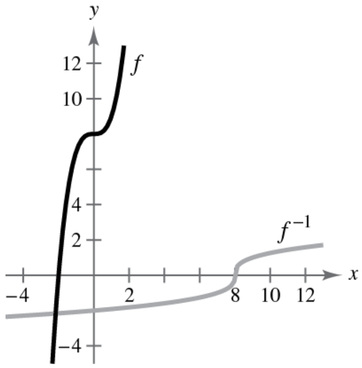
(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

P.10.38.

(a)
$$\begin{aligned} f(x) &= x^3 + 8 \\ y &= x^3 + 8 \\ x &= y^3 + 8 \\ x - 8 &= y^3 \\ \sqrt[3]{x-8} &= y \\ f^{-1}(x) &= \sqrt[3]{x-8} \end{aligned}$$

(b)



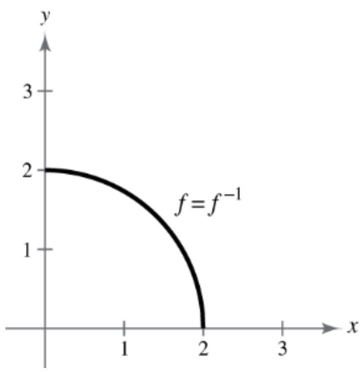
(c) The graph of f^{-1} is the reflection of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

P.10.39.

(a)
$$\begin{aligned} f(x) &= \sqrt{4 - x^2},\ \ 0 \le x \le 2 \\ y &= \sqrt{4 - x^2} \\ x &= \sqrt{4 - y^2} \\ x^2 &= 4 - y^2 \\ y^2 &= 4 - x^2 \\ y &= \sqrt{4 - x^2} \\ f^{-1}(x) &= \sqrt{4 - x^2},\ \ 0 \le x \le 2 \end{aligned}$$

(b)



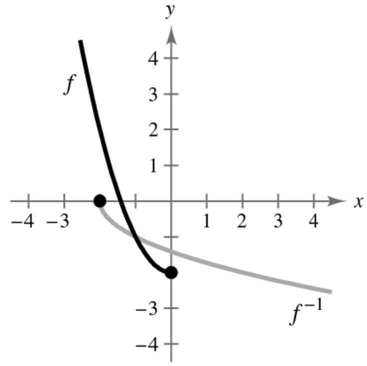
(c) The graph of f^{-1} is the same as the graph of f.

(d) The domains and ranges of f and f^{-1} are all real numbers x such that $0 \le x \le 2$.

P.10.40.

(a)
$$f(x) = x^2 - 2, \quad x \le 0$$
$$y = x^2 - 2$$
$$x = y^2 - 2$$
$$\pm\sqrt{x+2} = y$$
$$f^{-1}(x) = -\sqrt{x+2}$$

(b)



(c) The graph of f^{-1} is the reflection of f in the line $y = x$.

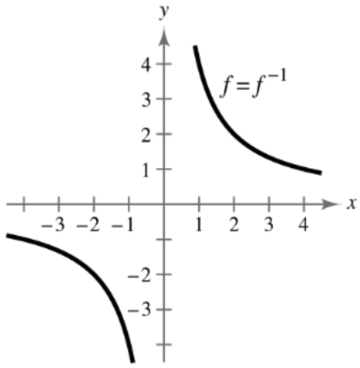
(d) $[-2, \infty)$ is the range of f and domain of f^{-1}.

$(-\infty, 0]$ is the domain of f and the range of f^{-1}.

P.10.41.

(a)
$$f(x) = \frac{4}{x}$$
$$y = \frac{4}{x}$$
$$x = \frac{4}{y}$$
$$xy = 4$$
$$y = \frac{4}{x}$$
$$f^{-1}(x) = \frac{4}{x}$$

(b)



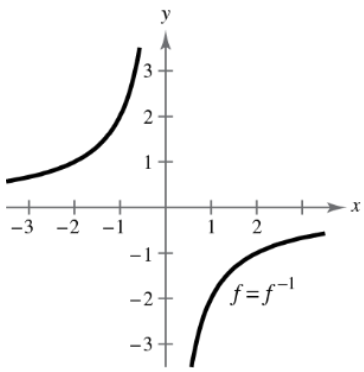
(c) The graph of f^{-1} is the same as the graph of *f*.

(d) The domains and ranges of *f* and f^{-1} are all real numbers except for 0.

P.10.42.

(a)
$$f(x) = -\frac{2}{x}$$
$$y = -\frac{2}{x}$$
$$x = -\frac{2}{y}$$
$$y = -\frac{2}{x}$$
$$f^{-1}(x) = -\frac{2}{x}$$

(b)



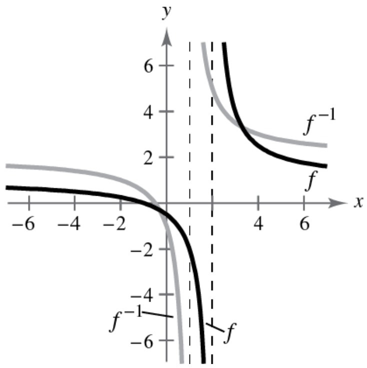
(c) The graphs are the same.

(d) The domains and ranges of *f* and f^{-1} are all real numbers except for 0.

P.10.43.

(a) $$f(x) = \frac{x+1}{x-2}$$
$$y = \frac{x+1}{x-2}$$
$$x = \frac{y+1}{y-2}$$
$$x(y-2) = y+1$$
$$xy - 2x = y+1$$
$$xy - y = 2x+1$$
$$y(x-1) = 2x+1$$
$$y = \frac{2x+1}{x-1}$$
$$f^{-1}(x) = \frac{2x+1}{x-1}$$

(b)



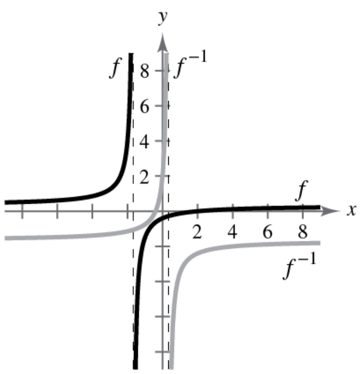
(c) The graph of f^{-1} is the reflection of graph of f in the line $y = x$.

(d) The domain of f and the range of f^{-1} is all real numbers except 2.
The range of f and the domain of f^{-1} is all real numbers except 1.

P.10.44.

(a)
$$f(x) = \frac{x-2}{3x+5}$$
$$y = \frac{x-2}{3x+5}$$
$$x = \frac{y-2}{3y+5}$$
$$3xy + 5x - y + 2 = 0$$
$$3xy - y = -5x - 2$$
$$y(3x-1) = -5x - 2$$
$$y = \frac{-5x-2}{3x-1}$$
$$y = \frac{-5x-2}{3x-1}$$
$$f^{-1}(x) = -\frac{5x+2}{3x-1}$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domain of f and the range of f^{-1} is all real numbers except $x = -\frac{5}{3}$.

The range of f and the domain of f^{-1} is all real numbers x except $x = \frac{1}{3}$.

P.10.45.

$$f(x) = x^4$$
$$y = x^4$$
$$x = y^4$$
$$y = \pm\sqrt[4]{x}$$

This does not represent y as a function of x. f does not have an inverse.

P.10.46.

$$f(x)=\frac{1}{x^2}$$
$$y=\frac{1}{x^2}$$
$$x=\frac{1}{y^2}$$
$$y^2=\frac{1}{x}$$
$$y=\pm\sqrt{\frac{1}{x}}$$

This does not represent y as a function of x. f does not have an inverse.

P.10.47.

$$g(x)=\frac{x+1}{6}$$
$$y=\frac{x+1}{6}$$
$$x=\frac{y+1}{6}$$
$$6x=y+1$$
$$y=6x-1$$

This is a function of x, so g has an inverse.

$$g^{-1}(x)=6x-1$$

P.10.48.

$$f(x)=3x+5$$
$$y=3x+5$$
$$x=3y+5$$
$$x-5=3y$$
$$\frac{x-5}{3}=y$$

This is a function of x, so f has an inverse.

$$f^{-1}(x)=\frac{x-5}{3}$$

P.10.49.

$$f(x) = \sqrt{2x+3} \Rightarrow x \ge -\frac{3}{2},\ \ y \ge 0$$
$$y = \sqrt{2x+3},\ \ x \ge -\frac{3}{2},\ \ y \ge 0$$
$$x = \sqrt{2y+3},\ \ y \ge -\frac{3}{2},\ \ x \ge 0$$
$$x^2 = 2y+3,\ \ x \ge 0,\ \ y \ge -\frac{3}{2}$$
$$y = \frac{x^2-3}{2},\ \ x \ge 0,\ \ y \ge -\frac{3}{2}$$

This is a function of x, so f has an inverse.

$$f^{-1}(x) = \frac{x^2-3}{2},\ x \ge 0$$

P.10.50.

$$f(x) = \sqrt{x-2} \Rightarrow x \ge 2,\ \ y \ge 0$$
$$y = \sqrt{x-2},\ \ x \ge 2,\ \ y \ge 0$$
$$x = \sqrt{y-2},\ \ y \ge 2,\ \ x \ge 0$$
$$x^2 = y-2,\ \ x \ge 0,\ \ y \ge 2$$
$$x^2+2 = y,\ \ x \ge 0,\ \ y \ge 2$$

This is a function of x, so f has an inverse.

$$f^{-1}(x) = x^2+2,\ x \ge 0$$

P.10.51.

$$f(x) = \frac{6x+4}{4x+5}$$
$$y = \frac{6x+4}{4x+5}$$
$$x = \frac{6y+4}{4y+5}$$
$$x(4y+5) = 6y+4$$
$$4xy+5x = 6y+4$$
$$4xy-6y = -5x+4$$
$$y(4x-6) = -5x+4$$
$$y = \frac{-5x+4}{4x-6}$$
$$= \frac{5x-4}{6-4x}$$

This is a function of x, so f has an inverse.

$$f^{-1}(x) = \frac{5x-4}{6-4x}$$

P.10.52.

The graph of f passes the Horizontal Line Test. So, you know f is one-to-one and has an inverse function.

$$\begin{aligned}
f(x) &= \frac{5x-3}{2x+5} \\
y &= \frac{5x-3}{2x+5} \\
x &= \frac{5y-3}{2y+5} \\
x(2y+5) &= 5y-3 \\
2xy+5x &= 5y-3 \\
2xy-5y &= -5x-3 \\
y(2x-5) &= -(5x+3) \\
y &= -\frac{5x+3}{2x-5} \\
f^{-1}(x) &= -\frac{5x+3}{2x-5}
\end{aligned}$$

P.10.53.

$$\begin{aligned}
f(x) &= (x+3)^2, \quad x \ge -3 \Rightarrow y \ge 0 \\
y &= (x+3)^2, \quad x \ge -3, \quad y \ge 0 \\
x &= (y+3)^2, \quad y \ge -3, \quad x \ge 0 \\
\sqrt{x} &= y+3, \quad y \ge -3, \quad x \ge 0 \\
y &= \sqrt{x}-3, \quad x \ge 0, \quad y \ge -3
\end{aligned}$$

This is a function of x, so f has an inverse.

$$f^{-1}(x) = \sqrt{x}-3, \quad x \ge 0$$

P.10.54.

$$\begin{aligned}
f(x) &= |x-2|, x \le 2 \Rightarrow y \ge 0 \\
y &= |x-2|, x \le 2, \quad y \ge 0 \\
x &= |y-2|, y \le 2, \quad x \ge 0
\end{aligned}$$

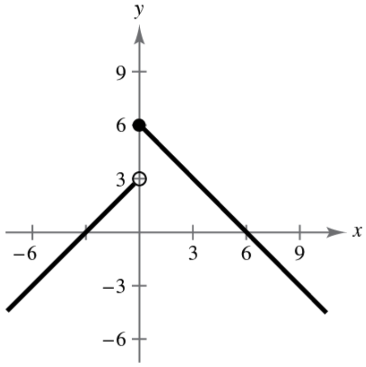
$$\begin{aligned}
x &= y-2 \quad \text{or} \quad -x = y-2 \\
2+x &= y \quad \text{or} \quad 2-x = y
\end{aligned}$$

The portion that satisfies the conditions $y \le 2$ and $x \ge 0$ is $2 - x = y$. This is a function of x, so f has an inverse.

$f^{-1}(x) = 2 - x,\ x \ge 0$

P.10.55.

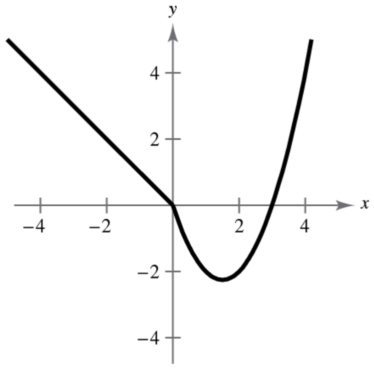
$$f(x) = \begin{cases} x + 3, & x < 0 \\ 6 - x, & x \ge 0 \end{cases}$$



This graph fails the Horizontal Line Test, so f does not have an inverse.

P.10.56.

$$f(x) = \begin{cases} -x, & x \le 0 \\ x^2 - 3x, & x > 0 \end{cases}$$



The graph fails the Horizontal Line Test, so f does not have an inverse.

P.10.57.

(a)
$$y = 10 + 0.75x$$
$$x = 10 + 0.75y$$
$$x - 10 = 0.75y$$
$$\frac{x-10}{0.75} = y$$

So, $f^{-1}(x) = \dfrac{x-10}{0.75}$.

$x =$ hourly wage, $y =$ number of units produced

(b) $y = \dfrac{24.25 - 10}{0.75} = 19$

So, 19 units are produced.

P.10.58.

(a) $y = 0.03x^2 + 245.50, \quad 0 < x < 100$

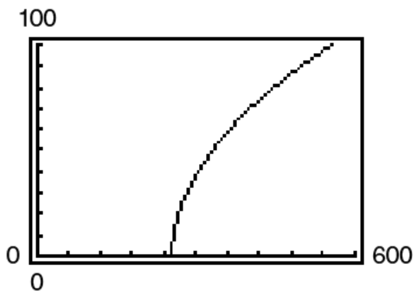
$\Rightarrow 245.50 < y < 545.50$

$$x = 0.03y^2 + 245.50$$
$$x - 245.50 = 0.03y^2$$
$$\frac{x - 245.50}{0.03} = y^2$$
$$\sqrt{\frac{x - 245.50}{0.03}} = y,\ 245.50 < x < 545.50$$
$$f^{-1}(x) = \sqrt{\frac{x - 245.50}{0.03}}$$

$x =$ temperature in degrees Fahrenheit

$y =$ percent load for a diesel engine

(b)



(c) $0.03x^2 + 245.50 \le 500$
$$0.03x^2 \le 254.50$$
$$x^2 \le 8483.33$$
$$x \le 92.10$$

Thus, $0 < x \le 92.10$.

In Exercises 59–62, $f(x) = x + 4$, $f^{-1}(x) = x - 4$, $g(x) = 2x - 5$, $g^{-1}(x) = \frac{x+5}{2}$.

P.10.59.

$$\begin{aligned}(g^{-1} \circ f^{-1})(x) &= g^{-1}(f^{-1}(x)) \\ &= g^{-1}(x-4) \\ &= \frac{(x-4)+5}{2} \\ &= \frac{x+1}{2}\end{aligned}$$

P.10.60.

$$\begin{aligned}(f^{-1} \circ g^{-1})(x) &= f^{-1}(g^{-1}(x)) \\ &= f^{-1}\left(\frac{x+5}{2}\right) \\ &= \frac{x+5}{2} - 4 \\ &= \frac{x+5-8}{2} \\ &= \frac{x-3}{2}\end{aligned}$$

P.10.61.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2x-5) \\ &= (2x-5)+4 \\ &= 2x-1\end{aligned}$$

$$(f \circ g)^{-1}(x) = \frac{x+1}{2}$$

Note: Comparing Exercises 59 and 61, $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$.

P.10.62.

$$\begin{aligned}(g \circ f)(x) &= g(f(x))\\ &= g(x+4)\\ &= 2(x+4)-5\\ &= 2x+8-5\\ &= 2x+3\\ y &= 2x+3\\ x &= 2y+3\\ x-3 &= 2y\\ \frac{x-3}{2} &= y\\ (g \circ f)^{-1}(x) &= \frac{x-3}{2}\end{aligned}$$

In Exercises 63–68, $f(x) = \frac{1}{8}x - 3$, $f^{-1}(x) = 8(x+3)$, $g(x) = x^3$, $g^{-1}(x) = \sqrt[3]{x}$.

P.10.63.

$$\begin{aligned}(f^{-1} \circ g^{-1})(1) &= f^{-1}(g^{-1}(1))\\ &= f^{-1}(\sqrt[3]{1})\\ &= 8(\sqrt[3]{1}+3) = 32\end{aligned}$$

P.10.64.

$$\begin{aligned}(g^{-1} \circ f^{-1})(-3) &= g^{-1}(f^{-1}(-3))\\ &= g^{-1}(8(-3+3))\\ &= g^{-1}(0) = \sqrt[3]{0} = 0\end{aligned}$$

P.10.65.

$$\begin{aligned}(f^{-1} \circ f^{-1})(4) &= f^{-1}(f^{-1}(4))\\ &= f^{-1}(8[4+3])\\ &= 8[8(4+3)+3]\\ &= 8[8(7)+3]\\ &= 8(59) = 472\end{aligned}$$

P.10.66.

$$\begin{aligned}\left(g^{-1}\circ g^{-1}\right)(-1) &= g^{-1}\left(g^{-1}(-1)\right)\\ &= g^{-1}\left(\sqrt[3]{-1}\right)\\ &= \sqrt[3]{\sqrt[3]{-1}}\\ &= \sqrt[3]{-1}\\ &= -1\end{aligned}$$

P.10.67.

$$\begin{aligned}(f\circ g)(x) = f\left(g(x)\right) = f\left(x^3\right) &= \tfrac{1}{8}x^3 - 3\\ y &= \tfrac{1}{8}x^3 - 3\\ x &= \tfrac{1}{8}y^3 - 3\\ x+3 &= \tfrac{1}{8}y^3\\ 8(x+3) &= y^3\\ \sqrt[3]{8(x+3)} &= y\\ (f\circ g)^{-1}(x) &= 2\sqrt[3]{x+3}\end{aligned}$$

P.10.68.

$$\begin{aligned}g^{-1}\circ f^{-1} &= g^{-1}\left(f^{-1}(x)\right)\\ &= g^{-1}\left(8(x+3)\right)\\ &= \sqrt[3]{8(x+3)}\\ &= 2\sqrt[3]{x+3}\end{aligned}$$

P.10.69.

False. $f(x) = x^2$ is even and does not have an inverse.

P.10.70.

True. If $f(x)$ has an inverse and it has a y-intercept at $(0, b)$, then the point $(b, 0)$, must be a point on the graph of $f^{-1}(x)$.

P.10.71.

Let $(f \circ g)(x) = y$. Then $x = (f \circ g)^{-1}(y)$. Also,

$$\begin{aligned}(f \circ g)(x) = y \Rightarrow f(g(x)) &= y\\ g(x) &= f^{-1}(y)\\ x &= g^{-1}(f^{-1}(y))\\ x &= (g^{-1} \circ f^{-1})(y).\end{aligned}$$

Because f and g are both one-to-one functions, $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

P.10.72.

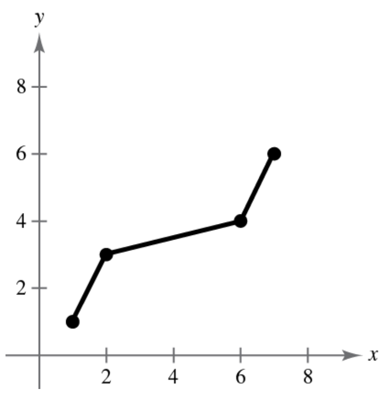
Let $f(x)$ be a one-to-one odd function. Then $f^{-1}(x)$ exists and $f(-x) = -f(x)$. Letting (x, y) be any point on the graph of $f(x) \Rightarrow (-x, -y)$ is also on the graph of $f(x)$ and $f^{-1}(-y) = -x = -f^{-1}(y)$. So, $f^{-1}(x)$ is also an odd function.

P.10.73.

(a)

x	1	3	4	6
f	1	2	6	7

x	1	2	6	7
$f^{-1}(x)$	1	3	4	6



(b)

x	−4	−2	0	3
f	3	4	0	−1

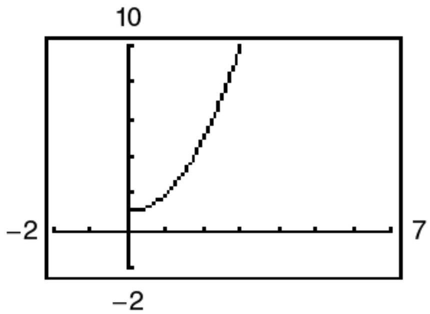
The graph does not pass the Horizontal Line Test, so $f^{-1}(x)$ does not exist.

P.10.74.

(a) $C(x)$ is represented by graph L_1 and $C^{-1}(x)$ is represented by graph L_2.

(b) $C(x)$ represents the cost of making x units of personalized T-shirts. $C^{-1}(x)$ represents the number of personalized T-shirts that can be made for a given cost.

P.10.75.



There is an inverse function $f^{-1}(x) = \sqrt{x-1}$ because the domain of f is equal to the range of f^{-1} and the range of f is equal to the domain of f^{-1}.

P.10.76.

x	–10	0	7	45
$f\left(f^{-1}(x)\right)$	–10	0	7	45
$f^{-1}\left(f(x)\right)$	–10	0	7	45

$f(x)$ and $f^{-1}(x)$ are inverses of each other.

P.10.77.

$$\begin{aligned} x^2 - 10x + 24 &= 0 \\ (x-6)(x-4) &= 0 \\ x &= 4,\ 6 \end{aligned}$$

P.10.78.

$$\begin{aligned} x^2 - 8x + 12 &= 0 \\ (x-6)(x-2) &= 0 \\ x &= 2,\ 6 \end{aligned}$$

P.10.79.

$$\begin{aligned} -\left(x - \frac{13}{2}\right)^2 + \frac{25}{4} &= 0 \\ \frac{25}{4} - \left(x - \frac{13}{2}\right)^2 &= 0 \\ \left[\frac{5}{2} + \left(x - \frac{13}{2}\right)\right]\left[\frac{5}{2} - \left(x - \frac{13}{2}\right)\right] &= 0 \\ (x - 4)(9 - x) &= 0 \\ x &= 4, 9 \end{aligned}$$

P.10.80.

$$\begin{aligned} 3\left(x + \frac{1}{4}\right)^2 - \frac{363}{16} &= 0 \\ 3\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) - \frac{363}{16} &= 0 \\ 3\left(x^2 + \frac{1}{2}x - \frac{120}{16}\right) &= 0 \\ 3\left(x^2 + \frac{1}{2}x - \frac{15}{2}\right) &= 0 \\ 3(x + 3)\left(x - \frac{5}{2}\right) &= 0 \\ x &= -3, \frac{5}{2} \end{aligned}$$

P.10.81.

The function is increasing on $(2, \infty)$ and decreasing on $(-\infty, 2)$.

P.10.82.

The function is increasing on $(-\infty, -2)$ and decreasing on $(-2, \infty)$.

P.10.83.

The function is increasing on $(-\infty, -1)$ and $(1, \infty)$ and decreasing on $(-1, 1)$.

P.10.84.

The function is increasing on $(-2, -1)$ and $(0, \infty)$ and decreasing on $(-\infty, -2)$ and $(-1, 0)$.

P.10.85.

Relative maximum at $(2, 0)$

Relative minimum at $(0, -3)$

P.10.86.

Relative minimum at $\left(-2, -\frac{1}{4}\right)$

No relative maxima

REVIEW EXERCISES FOR CHAPTER P

P.R.1.

$\left\{11,\ -14,\ -\frac{8}{9},\ -\frac{5}{2},\ \sqrt{6},\ 0.4\right\}$

(a) Natural numbers: 11

(b) Whole numbers: 0, 11

(c) Integers: $11,\ -14$

(d) Rational numbers: $11,\ -14,\ -\frac{8}{9},\ \frac{5}{2},\ 0.4$

(e) Irrational numbers: $\sqrt{6}$

P.R.2.

$\left\{\sqrt{15},\ -2,\ -\frac{10}{3},\ 0,\ 5.2,\ \frac{3}{7}\right\}$

(a) Natural numbers: none

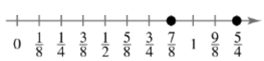
(b) Whole numbers: 0

(c) Integers: $-22,\ 0$

(d) Rational numbers: $-22,\ -\frac{10}{3},\ 0,\ 5.2,\ \frac{3}{7}$

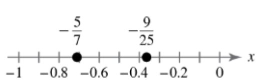
(e) Irrational numbers: $\sqrt{15}$

P.R.3.



$\frac{5}{4} > \frac{7}{8}$

P.R.4.



$-\frac{9}{25} > -\frac{5}{7}$

P.R.5.

$|5| = 5$

P.R.6.

$|-16| = 16$

P.R.7.

$|8-23| = |-15| = 15$

P.R.8.

$$|14-7| = |7| = 7$$

P.R.9.

$$-|9| - |-9| = -9 - 9 = -18$$

P.R.10.

$$|-10| - |-11| = 10 - 11 = -1$$

P.R.11.

$$d(-74,\ 48) = |48-(-74)| = 122$$

P.R.12.

$$d(-112,\ -6) = |-6-(-112)| = 106$$

P.R.13.

$-x^2 + x - 1$

(a) $-(1)^2 + 1 - 1 = -1$

(b) $-(-1)^2 + (-1) - 1 = -3$

P.R.14.

$\dfrac{x}{x-3}$

(a) $\dfrac{-3}{-3-3} = \dfrac{1}{2}$

(b) $\dfrac{3}{3-3}$ is undefined.

P.R.15.

$0 + (a-5) = a - 5$

Illustrates the Additive Identity Property

P.R.16.

$1 \bullet (3x+4) = 3x + 4$

Illustrates the Multiplicative Identity Property

P.R.17.

$$2x+\left(3x-10\right)=\left(2x+3x\right)-10$$

Illustrates the Associative Property of Addition

P.R.18.

$$4\left(t+2\right)=4\bullet t+4\bullet 2$$

Illustrates the Left Distributive Property of Multiplication Over Addition

P.R.19.

$$\left(t^2+1\right)+3=3+\left(t^2+1\right)$$

Illustrates the Commutative Property of Addition

P.R.20.

$$\frac{2}{y+4}\bullet\frac{y+4}{2}=1,\quad y\neq-4$$

Illustrates the Multiplicative Inverse Property

P.R.21.

$$\begin{aligned}2(x-2)&=2x-4\\2x-4&=2x-4\\0&=0\quad\text{Identity}\end{aligned}$$

All real numbers are solutions.

P.R.22.

$$\begin{aligned}2(x+3)&=2x-2\\2x+6&=2x-2\\6&=-2\quad\text{Contradiction}\end{aligned}$$

No solution

P.R.23.

$$\begin{aligned}3\left(x-2\right)+2x&=2\left(x+3\right)\\3x-6+2x&=2x+6\\3x&=12\\x&=4\end{aligned}$$

Conditional equation

P.R.24.

$$\begin{aligned} 5(x-1)-2x &= 3x-5 \\ 5x-5-2x &= 3x-5 \\ 3x-5 &= 3x-5 \\ 0 &= 0 \quad \text{Identity} \end{aligned}$$

All real numbers are solutions.

P.R.25.

$$\begin{aligned} 2(x+5)-7 &= x+9 \\ 2x+10-7 &= x+9 \\ 2x+3 &= x+9 \\ x &= 6 \end{aligned}$$

P.R.26.

$$\begin{aligned} 7(x-4) &= 1-(x+9) \\ 7x-28 &= 1-x-9 \\ 7x-28 &= -x-8 \\ 8x &= 20 \\ x &= \tfrac{5}{2} \end{aligned}$$

P.R.27.

$$\begin{aligned} \frac{x}{5}-3 &= \frac{x}{3}+1 \\ 15\left(\frac{x}{5}-3\right) &= \left(\frac{x}{3}+1\right)15 \\ 3x-45 &= 5x+15 \\ -2x &= 60 \\ x &= -30 \end{aligned}$$

P.R.28

$$
\begin{aligned}
3+\frac{2}{x-5} &= \frac{2x}{x-5}\\
\frac{3(x-5)+2}{x-5} &= \frac{2x}{x-5}\\
\frac{3x-15+2}{x-5} &= \frac{2x}{x-5}\\
\frac{3x-13}{x-5} &= \frac{2x}{x-5}\\
(x-5)\frac{3x-13}{x-5} &= \frac{2x}{x-5}(x-5)\\
3x-13 &= 2x\\
x &= 3
\end{aligned}
$$

P.R.29.

$$
\begin{aligned}
2x^2-x-28 &= 0\\
(2x+7)(x-4) &= 0\\
2x+7 = 0 &\Rightarrow x = -\tfrac{7}{2}\\
x-4 = 0 &\Rightarrow x = 4
\end{aligned}
$$

P.R.30.

$$
\begin{aligned}
6 &= 3x^2\\
2 &= x^2\\
\pm\sqrt{2} &= x
\end{aligned}
$$

P.R.31.

$$
\begin{aligned}
(x+13)^2 &= 25\\
x+13 &= \pm 5\\
x &= -13 \pm 5\\
x &= -18 \text{ or } x = -8
\end{aligned}
$$

P.R.32.

$$\begin{aligned} 9x^{23} - 12x &= 14 \\ 9x^2 - 12x - 14 &= 0 \\ x &= \frac{-12 \pm \sqrt{(-12)^2 - 4(9)(-14)}}{2(9)} \\ &= \frac{-12 \pm 18\sqrt{2}}{18} \\ &= \frac{2}{3} \pm \sqrt{2} \end{aligned}$$

P.R.33.

$$\begin{aligned} 5x^4 - 12x^3 &= 0 \\ x^3(5x - 12) &= 0 \\ x^3 = 0 \text{ or } 5x - 12 &= 0 \\ x = 0 \text{ or } \quad x &= \tfrac{12}{5} \end{aligned}$$

P.R.34.

$$\begin{aligned} x^3 + 8x^2 - 2x &= 16 \\ x^3 + 8x^2 - 2x - 16 &= 0 \\ x^2(x+8) - 2(x+8) &= 0 \\ (x^2 - 2)(x+8) &= 0 \\ x^2 - 2 = 0 &\Rightarrow x = \pm\sqrt{2} \\ x + 8 = 0 &\Rightarrow x = -8 \end{aligned}$$

P.R.35.

$$\begin{aligned} \sqrt{x+4} &= 3 \\ \left(\sqrt{x+4}\right)^2 &= (3)^2 \\ x + 4 &= 9 \\ x &= 5 \end{aligned}$$

P.R.36.

$$5\sqrt{x} - \sqrt{x-1} = 6$$
$$5\sqrt{x} = 6 + \sqrt{x-1}$$
$$25x = 36 + 12\sqrt{x-1} + x - 1$$
$$24x - 35 = 12\sqrt{x-1}$$
$$576x^2 - 1680x + 1225 = 144(x-1)$$
$$576x^2 - 1824x + 1369 = 0$$
$$x = \frac{-(-1824) \pm \sqrt{(-1824)^2 - 4(576)(1369)}}{2(576)} = \frac{1824 \pm \sqrt{172,800}}{1152} = \frac{1824 \pm 240\sqrt{3}}{1152}$$
$$x = \frac{38 + 5\sqrt{3}}{24}$$
$$x = \frac{38 - 5\sqrt{3}}{24}, \text{ extraneous}$$

P.R.37.

$$(x-1)^{2/3} - 25 = 0$$
$$(x-1)^{2/3} = 25$$
$$(x-1)^2 = 25^3$$
$$x - 1 = \pm\sqrt{25^3}$$
$$x = 1 \pm 125$$
$$x = 126 \text{ or } x = -124$$

P.R.38.

$$(x+2)^{3/4} = 27$$
$$x + 2 = 27^{4/3}$$
$$x + 2 = 81$$
$$x = 79$$

P.R.39.

$$|x - 5| = 10$$
$$x - 5 = -10 \text{ or } x - 5 = 10$$
$$x = -5 \qquad\qquad x = 15$$

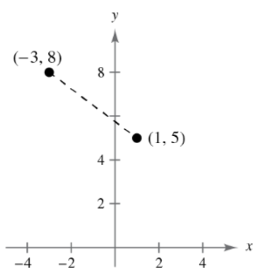
P.R.40.

$$\left| x^2 - 3 \right| = 2x \quad \text{or} \qquad x^2 - 3 = 2x$$
$$x^2 - 2x - 3 = 0 \qquad x^2 + 2x - 3 = 0$$
$$(x-3)(x+1) = 0 \qquad (x+3)(x-1) = 0$$
$$x = 3 \text{ or } x = -1 \qquad x = -3 \text{ or } x = 1$$

The only solutions of the original equation are $x = 3$ or $x = 1$.
$(x = 3$ and $x = -1$ are extraneous.$)$

P.R.41.

(a)

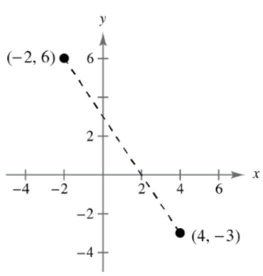


(b) $d = \sqrt{(-3-1)^2 + (8-5)^2} = \sqrt{16+9} = 5$

(c) Midpoint: $\left(\dfrac{-3+1}{2}, \dfrac{8+5}{2}\right) = \left(-1, \dfrac{13}{2}\right)$

P.R.42.

(a)



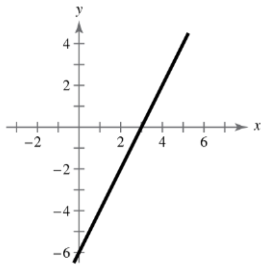
(b) $d = \sqrt{(-2-4)^2 + (6+3)^2}$
$= \sqrt{36+81} = \sqrt{117} = 3\sqrt{13}$

(c) Midpoint: $\left(\dfrac{-2+4}{2}, \dfrac{6-3}{2}\right) = \left(1, \dfrac{3}{2}\right)$

P.R.43.

$y = 2x - 6$

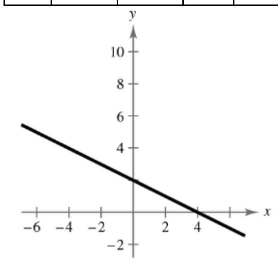
x	1	2	3	4	5
y	–4	–2	0	2	4



P.R.44.

$y = -\frac{1}{2}x + 2$

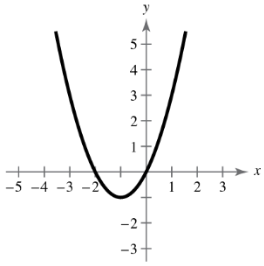
x	–4	–2	0	2	4
y	4	3	2	1	0



P.R.45.

$y = x^2 + 2x$

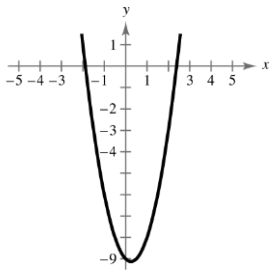
x	−3	−2	−1	0	1
y	3	0	−1	0	3



P.R.46.

$y = 2x^2 - x - 9$

x	–2	–1	0	1	2	3
y	1	–6	–9	–8	–3	6



P.R.47.

x-intercepts: $(1,\ 0),\ (5,\ 0)$

y-intercept: $(0,\ 5)$

P.R.48.

x-intercepts: $(-4,\ 0),\ (2,\ 0)$

y-intercept: $(0,\ -2)$

P.R.49.

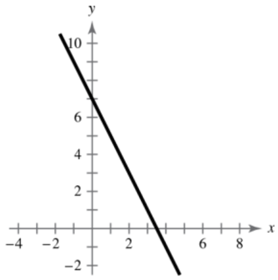
$y = -3x + 7$

Intercepts: $\left(\frac{7}{3},\ 0\right),\ (0, 7)$

$y = -3(-x) + 7 \Rightarrow y = 3x + 7 \Rightarrow$ No y-axis symmetry

$-y = -3x + 7 \Rightarrow y = 3x - 7 \Rightarrow$ No x-axis symmetry

$-y = -3(-x) + 7 \Rightarrow y = -3x - 7 \Rightarrow$ No origin symmetry



P.R.50.

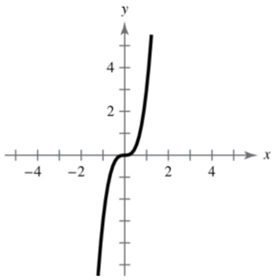
$y = 3x^3$

Intercepts: $(0,\ 0)$

$y = 3(-x)^3 \Rightarrow y = -3x^3 \Rightarrow$ No y-axis symmetry

$-y = 3x^3 \Rightarrow y = -3x^3 \Rightarrow$ No x-axis symmetry

$-y = 3(-x)^3 \Rightarrow y = 3x^3 \Rightarrow$ Original symmetry



P.R.51.

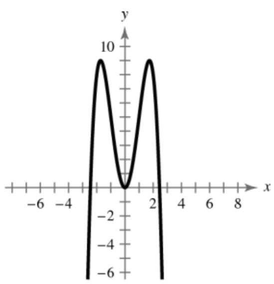
$y = -x^4 + 6x^2$

Intercept: $(0,\ 0)$

$y = -(-x)^4 + 6(-x)^2 \Rightarrow y = -x^4 + 6x^2 \Rightarrow y$-axis symmetry

$-y = -x^4 + 6x^2 \Rightarrow y = x^4 - 6x^2 \Rightarrow$ No x-axis symmetry

$-y = -(-x)^4 + 6(-x)^2 \Rightarrow y = x^4 - 6x^2 \Rightarrow$ No origin symmetry



P.R.52.

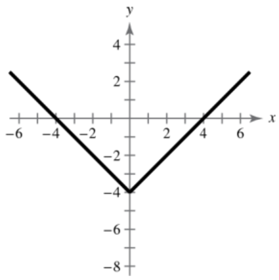
$y = |x| - 4$

Intercepts: $(\pm 4,\ 0),\ (0,\ -4)$

$y = |-x| - 4 \Rightarrow y = |x| - 4 \Rightarrow y$-axis symmetry

$-y = |x| - 4 \Rightarrow y = -|x| + 4 \Rightarrow$ No x-axis symmetry

$-y = |-x| - 4 \Rightarrow y = -|x| + 4 \Rightarrow$ No origin symmetry



P.R.53.

Endpoints of a diameter: $(0,\ 0)$ and $(4,\ -6)$

Center: $\left(\dfrac{0+4}{2},\ \dfrac{0+(-6)}{2}\right)=(2,\ -3)$

Radius: $r=\sqrt{(2-0)^2+(-3-0)^2}=\sqrt{4+9}=\sqrt{13}$

Standard form: $(x-2)^2+(y-(-3))^2=\left(\sqrt{13}\right)^2$

$$(x-2)^2+(y+3)^2=13$$

P.R.54.

Endpoints of a diameter: $(-2,\ -3)$ and $(4,\ -10)$

Center: $\left(\dfrac{-2+4}{2},\ \dfrac{-3+(-10)}{2}\right)=\left(1,\ -\dfrac{13}{2}\right)$

Radius: $r=\sqrt{(1-(-2))^2+\left(-\dfrac{13}{2}-(-3)\right)^2}=\sqrt{9+\dfrac{49}{4}}=\sqrt{\dfrac{85}{4}}$

Standard form: $(x-1)^2+\left(y-\left(-\dfrac{13}{2}\right)\right)^2=\left(\sqrt{\dfrac{85}{4}}\right)^2$

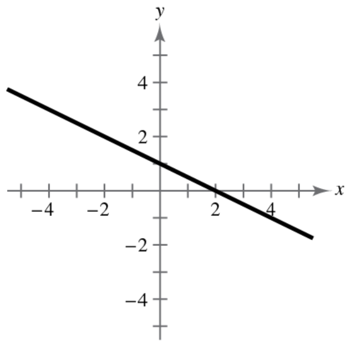
$$(x-1)^2+\left(y+\dfrac{13}{2}\right)^2=\dfrac{85}{4}$$

P.R.55.

$y=-\dfrac{1}{2}x+1$

Slope: $m=-\dfrac{1}{2}$

y-intercept: $(0,1)$

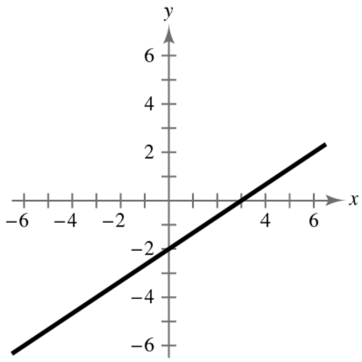


P.R.56.

$$2x - 3y = 6$$
$$-3y = -2x + 6$$
$$y = \frac{2}{3}x - 2$$

Slope: $m = \frac{2}{3}$

y-intercept: $(0, -2)$

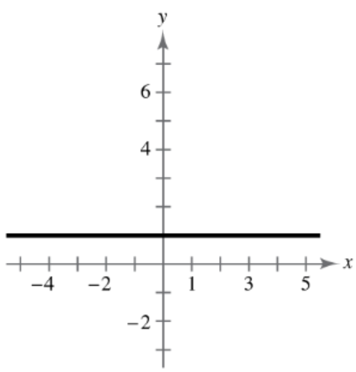


P.R.57.

$y = 1$

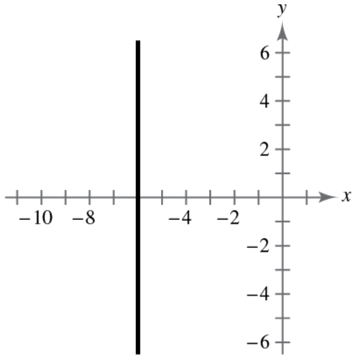
Slope: $m = 0$

y-intercept: $(0, 1)$



P.R.58.

$x = -6$
Slope is undefined.
y-intercept: none



P.R.59.

$$(5, -2),\ (-1, 4)$$

$$m = \frac{4-(-2)}{-1-5} = \frac{6}{-6} = -1$$

P.R.60.

$$(-1, 6), (3, -2)$$

$$m = \frac{-2-6}{3-(-1)} = \frac{-8}{4} = -2$$

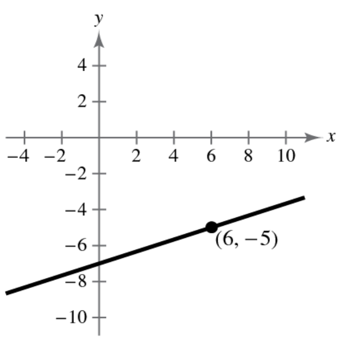
P.R.61.

$$(6, -5), m = \tfrac{1}{3}$$

$$y-(-5) = \frac{1}{3}(x-6)$$

$$y+5 = \frac{1}{3}x - 2$$

$$y = \frac{1}{3}x - 7$$



P.R.62.

$$\left(-4,-2\right),\ m=-\frac{3}{4}$$

$$y-\left(-2\right)=-\frac{3}{4}\left(x-\left(-4\right)\right)$$

$$y+2=-\frac{3}{4}\left(x+4\right)$$

$$y+2=-\frac{3}{4}x-3$$

$$y=-\frac{3}{4}x-5$$

P.R.63.

$$\left(-6,4\right),\left(4,9\right)$$

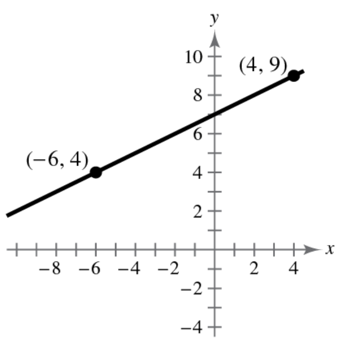
$$m=\frac{9-4}{4-\left(-6\right)}=\frac{5}{10}=\frac{1}{2}$$

$$y-4=\frac{1}{2}\left(x-\left(-6\right)\right)$$

$$y-4=\frac{1}{2}\left(x+6\right)$$

$$y-4=\frac{1}{2}x+3$$

$$y=\frac{1}{2}x+7$$

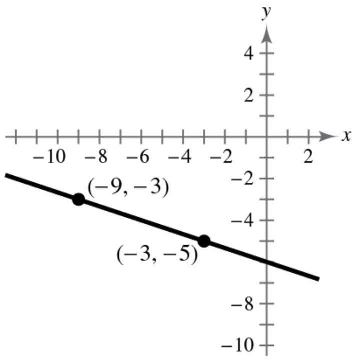


P.R.64.

$$\left(-9,-3\right),\left(-3,-5\right)$$

$$m=\frac{-5-\left(-3\right)}{-3-\left(-9\right)}=\frac{-2}{6}=-\frac{1}{3}$$

$$y-(-3)=-\frac{1}{3}\left(x-(-9)\right)$$
$$y+3=-\frac{1}{3}(x+9)$$
$$y+3=-\frac{1}{3}x-3$$
$$y=-\frac{1}{3}x-6$$



P.R.65.

Point: $(3,-2)$

$5x-4y=8$

$$y=\frac{5}{4}x-2$$

(a) Parallel slope: $m=\frac{5}{4}$

$$y-(-2)=\frac{5}{4}(x-3)$$
$$y+2=\frac{5}{4}x-\frac{15}{4}$$
$$y=\frac{5}{4}x-\frac{23}{4}$$

(b) Perpendicular slope: $m=-\frac{4}{5}$

$$y-(-2)=-\frac{4}{5}(x-3)$$
$$y+2=-\frac{4}{5}x+\frac{12}{5}$$
$$y=-\frac{4}{5}x+\frac{2}{5}$$

P.R.66.

Point: $(-8, 3)$, $2x + 3y = 5$

$$3y = 5 - 2x$$

$$y = \frac{5}{3} - \frac{2}{3}x$$

(a) Parallel slope: $m = -\frac{2}{3}$

$$\begin{aligned} y - 3 &= -\frac{2}{3}(x + 8) \\ 3y - 9 &= -2x - 16 \\ 3y &= -2x - 7 \\ y &= -\frac{2}{3}x - \frac{7}{3} \end{aligned}$$

(b) Perpendicular slope: $m = \frac{3}{2}$

$$\begin{aligned} y - 3 &= \frac{3}{2}(x + 8) \\ 2y - 6 &= 3x + 24 \\ 2y &= 3x + 30 \\ y &= \frac{3}{2}x + 15 \end{aligned}$$

P.R.67.

Verbal Model: Sale price $=$ (List price) $-$ (Discount)

Labels: Sale price $= S$
List price $= L$
Discount $=$ 20% of $L = 0.2L$

Equation: $S = L - 0.2L$
$S = 0.8L$

P.R.68.

Verbal Model: Amount earned $=$ (starting fee) $+$ (per page rate) $\cdot$ (number of *pages*)

Labels: Hourly wage $= A$
Starting fee $= 50$
Per page rate $= 2.50$
Number of pages $= p$

Equation: $A = 2.5p + 50$

P.R.69.

$$16x - y^4 = 0$$
$$y^4 = 16x$$
$$y = \pm 2\sqrt[4]{x}$$

No, y is not a function of x. Some x-values correspond to two y-values.

P.R.70.

$$2x - y - 3 = 0$$
$$2x - 3 = y$$

Yes, the equation represents y as a function of x.

P.R.71.

$$y = \sqrt{1-x}$$

Yes, the equation represents y as a function of x. Each x-value, $x \le 1$, corresponds to only one y-value.

P.R.72.

$|y| = x + 2$ corresponds to $y = x + 2$ or $-y = x + 2$.

No, y is not a function of x. Some x-values correspond to two y-values.

P.R.73.

$$g(x) = x^{4/3}$$

(a) $g(8) = 8^{4/3} = 2^4 = 16$

(b) $g(t+1) = (t+1)^{4/3}$

(c) $(-27)^{4/3} = (-3)^4 = 81$

(d) $g(-x) = (-x)^{4/3} = x^{4/3}$

P.R.74.

$$h(x) = |x - 2|$$

(a) $h(-4) = |-4 - 2| = |-6| = 6$

(b) $h(-2) = |-2 - 2| = |-4| = 4$

(c) $h(0) = |0 - 2| = |-2| = 2$

(d) $h(-x+2) = |-x + 2 - 2| = |-x| = |x|$

P.R.75.

$f(x) = \sqrt{25 - x^2}$

Domain: $25 - x^2 \geq 0$

$(5 + x)(5 - x) \geq 0$

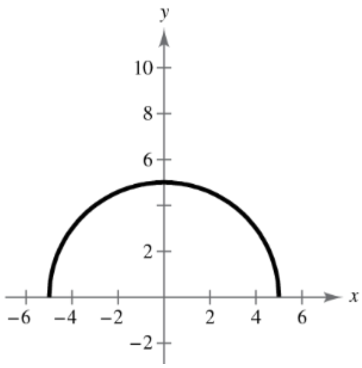
Critical numbers: $x = \pm 5$

Test intervals: $(-\infty, -5), (-5, 5), (5, \infty)$

Test: Is $25 - x^2 \geq 0$?

Solution set: $-5 \leq x \leq 5$

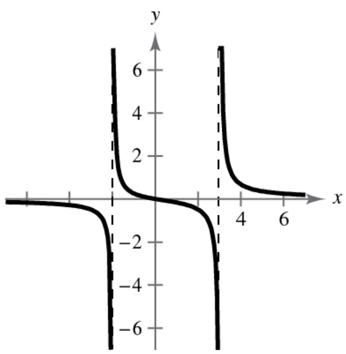
Domain: all real numbers x such that $-5 \leq x \leq 5$, or $[-5, 5]$



P.R.76.

$$h(x) = \frac{x}{x^2 - x - 6}$$
$$= \frac{x}{(x + 2)(x - 3)}$$

Domain: All real numbers x except $x = -2, 3$



P.R.77.

$v(t) = -32t + 48$

$v(1) = 16$ feet per second

P.R.78.

$$0 = -32t + 48$$
$$t = \frac{48}{32} = 1.5 \text{ seconds}$$

P.R.79.

$$f(x) = 2x^2 + 3x - 1$$
$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{\left[2(x+h)^2 + 3(x+h) - 1\right] - (2x^2 + 3x - 1)}{h}\\ &= \frac{2x^2 + 4xh + 2h^2 + 3x + 3h - 1 - 2x^2 - 3x + 1}{h}\\ &= \frac{h(4x + 2h + 3)}{h}\\ &= 4x + 2h + 3,\ h \neq 0\end{aligned}$$

P.R.80.

$$\begin{aligned}f(x) &= x^3 - 5x^2 + x\\ f(x+h) &= (x+h)^3 - 5(x+h)^2 + (x+h)\\ &= x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h\\ \frac{f(x+h) - f(x)}{h} &= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h - x^3 + 5x^2 - x}{h}\\ &= \frac{3x^2h + 3xh^2 + h^3 - 10xh - 5h^2 + h}{h}\\ &= \frac{h(3x^2 + 3xh + h^2 - 10x - 5h + 1)}{h}\\ &= 3x^2 + 3xh + h^2 - 10x - 5h + 1,\ h \neq 0\end{aligned}$$

P.R.81.

$$y = (x-3)^2$$

A vertical line intersects the graph no more than once, so *y is* a function of *x*.

P.R.82.

$$x = -|4 - y|$$

A vertical line intersects the graph more than once, so *y is not* a function of *x*.

P.R.83.

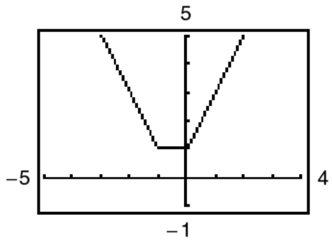
$$f(x) = \sqrt{2x+1}$$
$$\sqrt{2x+1} = 0$$
$$2x+1 = 0$$
$$2x = -1$$
$$x = -\tfrac{1}{2}$$

P.R.84.

$$f(x) = \frac{x^3 - x^2}{2x+1}$$
$$\frac{x^3 - x^2}{2x+1} = 0$$
$$x^3 - x^2 = 0$$
$$x^2(x-1) = 0$$
$$x^2 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = 0 \qquad\qquad x = 1$$

P.R.85.

$$f(x) = |x| + |x+1|$$



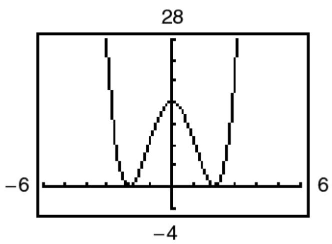
f is increasing on $(0, \infty)$.

f is decreasing on $(-\infty, -1)$.

f is constant on $(-1, 0)$.

P.R.86.

$$f(x) = \left(x^2 - 4\right)^2$$



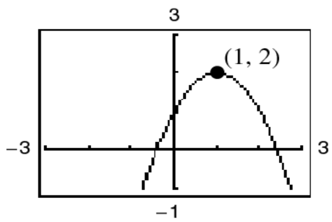
f is increasing on $(-2, 0)$ and $(2, \infty)$.

f is decreasing on $(-\infty, -2)$ and $(0, 2)$.

P.R.87.

$f(x) = -x^2 + 2x + 1$

Relative maximum: $(1, 2)$

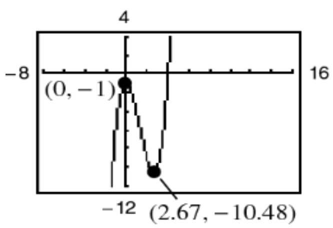


P.R.88.

$f(x) = x^3 - 4x^2 - 1$

Relative minimum: $(2.67, -10.48)$

Relative maximum: $(0, -1)$



P.R.89.

$f(x) = -x^2 + 8x - 4$

$$\frac{f(4) - f(0)}{4 - 0} = \frac{12 - (-4)}{4} = 4$$

The average rate of change of f from $x_1 = 0$ to $x_2 = 4$ is 4.

P.R.90.

$f(x) = x^3 + 2x + 1$

$$\frac{f(3) - f(1)}{3 - 1} = \frac{34 - 4}{2} = \frac{30}{2} = 15$$

The average rate of change of f from $x_1 = 1$ to $x_2 = 3$ is 15.

P.R.91.

$$f(x) = x^4 - 20x^2$$
$$f(-x) = (-x)^4 - 20(-x)^2 = x^4 - 20x^2 = f(x)$$

The function is even, so the graph has y-axis symmetry.

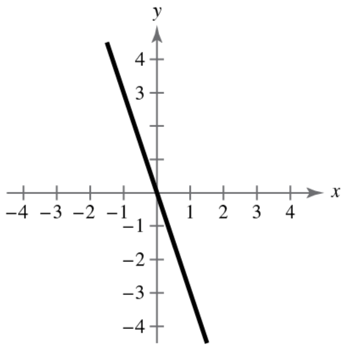
P.R.92.

$$f(x) = 2x\sqrt{x^2+3}$$
$$\begin{aligned} f(-x) &= 2(-x)\sqrt{(-x)^2+3} \\ &= -2x\sqrt{x^2+3} \\ &= -f(x) \end{aligned}$$

The function is odd, so the graph has origin symmetry.

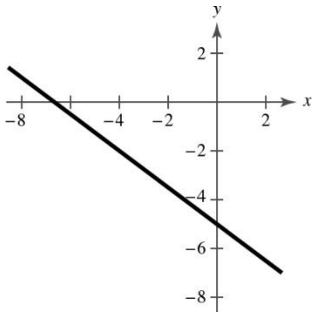
P.R.93.

$$f(x) = -3x$$



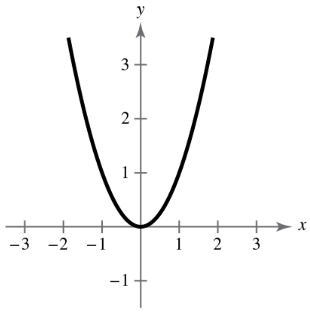
P.R.94.

$$f(x) = -\frac{3}{4}x - 5$$



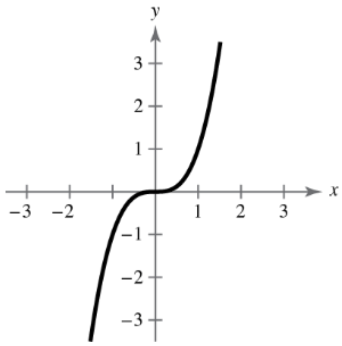
P.R.95.

$$f(x) = x^2$$



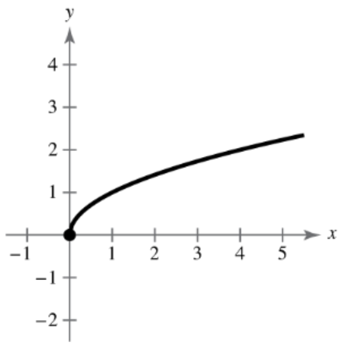
P.R.96.

$$f(x) = x^3$$



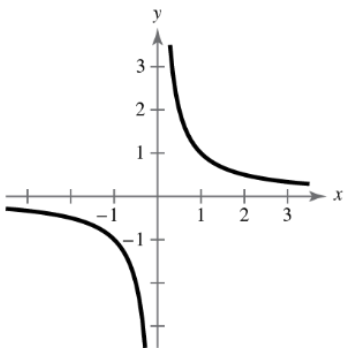
P.R.97.

$$f(x) = \sqrt{x}$$



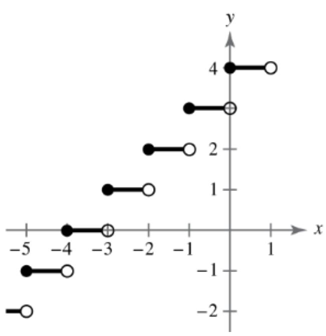
P.R.98.

$$f(x)=\frac{1}{x}$$



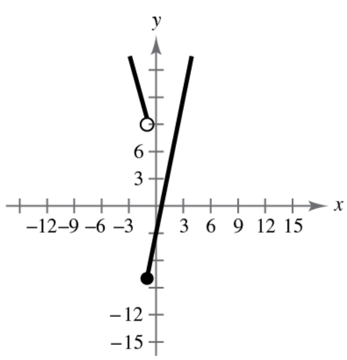
P.R.99.

$$g(x)=[\![x+4]\!]$$



P.R.100.

$$f(x)=\begin{cases}5x-3, & x\ge -1\\ -4x+5, & x<-1\end{cases}$$



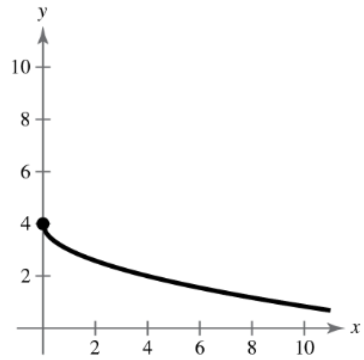
P.R.101.

(a) $f(x) = \sqrt{x}$

(b) $h(x) = -\sqrt{x} + 4$

Reflection in the x-axis and a vertical shift 4 units upward

(c)



(d) $h(x) = -f(x) + 4$

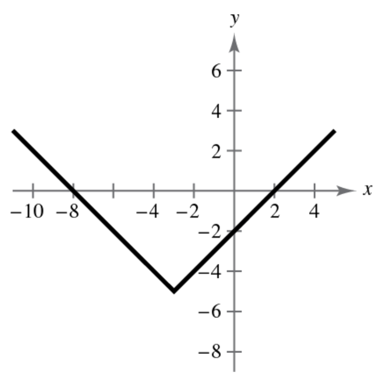
P.R.102.

(a) $f(x) = |x|$

(b) $h(x) = |x + 3| - 5$

Horizontal shift 3 units to the left and a vertical shift 5 units downward

(c)



(d) $h(x) = f(x + 3) - 5$

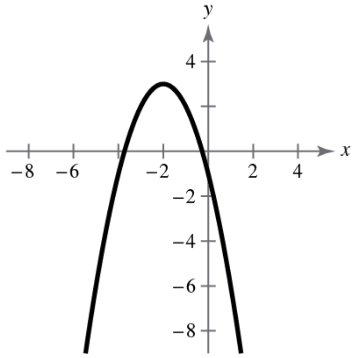
P.R.103.

(a) $f(x) = x^2$

(b) $h(x) = -(x + 2)^2 + 3$

Reflection in the x-axis, a horizontal shift 2 units to the left, and a vertical shift 3 units upward

(c)



(d) $h(x) = -f(x+2)+3$

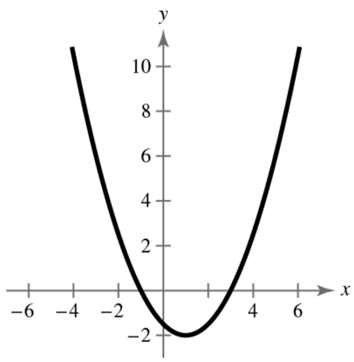
P.R.104.

(a) $f(x) = x^2$

(b) $h(x) = \frac{1}{2}(x-1)^2 - 2$

Horizontal shift one unit to the right, vertical shrink, and a vertical shift 2 units downward

(c)



(d) $h(x) = \frac{1}{2}f(x-1) - 2$

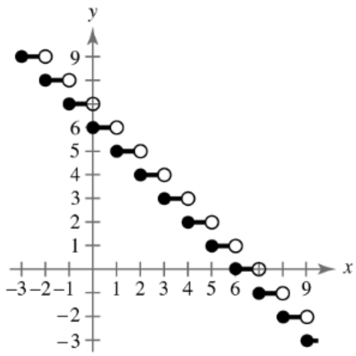
P.R.105.

(a) $f(x) = [\![x]\!]$

(b) $h(x) = -[\![x]\!] + 6$

Reflection in the x-axis and a vertical shift 6 units upward

(c)



(d) $h(x) = -f(x) + 6$

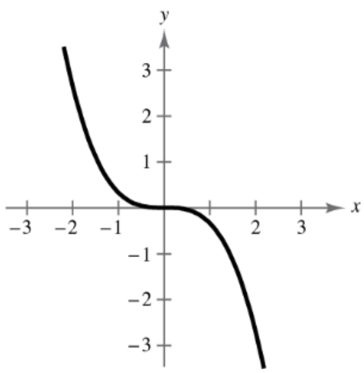
P.R.106.

(a) $f(x) = x^3$

(b) $h(x) = -\frac{1}{3}x^3$

Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{3}\right)$

(c)



(d) $h(x) = -\frac{1}{3}f(x)$

P.R.107.

$f(x) = x^2 + 3, \quad g(x) = 2x - 1$

(a) $(f+g)(x) = (x^2+3) + (2x-1) = x^2 + 2x + 2$

(b) $(f-g)(x) = (x^2+3) - (2x-1) = x^2 - 2x + 4$

(c) $(fg)(x) = (x^2+3)(2x-1) = 2x^3 - x^2 + 6x - 3$

(d) $\left(\frac{f}{g}\right)(x) = \frac{x^2+3}{2x-1}$, Domain: $x \neq \frac{1}{2}$

P.R.108.

$f(x) = x^2 - 4, \; g(x) = \sqrt{3-x}$

(a) $(f+g)(x) = f(x) + g(x) = x^2 - 4 + \sqrt{3-x}$

(b) $(f-g)(x) = f(x) - g(x) = x^2 - 4 - \sqrt{3-x}$

(c) $(fg)(x) = f(x)g(x) = (x^2-4)(\sqrt{3-x})$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2-4}{\sqrt{3-x}}$, Domain: $x < 3$

P.R.109.

$f(x) = \frac{1}{3}x - 3, \; g(x) = 3x + 1$

The domains of f and g are all real numbers.

(a)
$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(3x+1) \\ &= \frac{1}{3}(3x+1) - 3 \\ &= x + \frac{1}{3} - 3 \\ &= x - \frac{8}{3}\end{aligned}$$

Domain: all real numbers

(b)
$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g\left(\frac{1}{3}x - 3\right) \\ &= 3\left(\frac{1}{3}x - 3\right) + 1 \\ &= x - 9 + 1 \\ &= x - 8\end{aligned}$$

Domain: all real numbers

P.R.110.

$f(x) = \sqrt{x+1}, \; g(x) = x^2$

The domain of f is all real numbers x such that $x \ge -1$. The domain of g is all real numbers.

(a) $$\begin{aligned}(f\circ g)(x) &= f(g(x))\\ &= f(x^2)\\ &= \sqrt{x^2+1}\end{aligned}$$

Domain: all real numbers

(b) $$\begin{aligned}(g\circ f)(x) &= g(f(x))\\ &= g\left(\sqrt{x+1}\right)\\ &= \left(\sqrt{x+1}\right)^2\\ &= x+1\end{aligned}$$

Domain: all real numbers x such that $x \geq -1$

In Exercises 111 and 112, use the following functions: $f(x) = x - 100$, $g(x) = 0.95x$.

P.R.111.

$(f\circ g)(x) = f(0.95x) = 0.95x - 100$ represents the sale price if first the 5% discount is applied and then the \$100 rebate.

P.R.112.

$(g\circ f)(x) = g(x-100) = 0.95(x-100) = 0.95x - 95$ represents the sale price if first the \$100 rebate is applied and then the 5% discount.

P.R.113.

$$\begin{aligned}f(x) &= 3x+8\\ y &= 3x+8\\ x &= 3y+8\\ x-8 &= 3y\\ y &= \frac{x-8}{3}\\ y &= \frac{1}{3}(x-8)\end{aligned}$$

So, $f^{-1}(x) = \frac{1}{3}(x-8) = \frac{x-8}{3}$.

$$f\left(f^{-1}(x)\right) = f\left(\frac{1}{3}(x-8)\right) = 3\left(\frac{1}{3}(x-8)\right) + 8 = x - 8 + 8 = x$$

$$f^{-1}(f(x)) = f^{-1}(3x+8) = \frac{1}{3}(3x+8-8) = \frac{1}{3}(3x) = x$$

P.R.114.

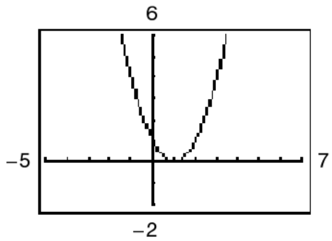
$$f(x) = \frac{x-4}{5}$$
$$y = \frac{x-4}{5}$$
$$x = \frac{y-4}{5}$$
$$5x = y - 4$$
$$y = 5x + 4$$

So, $f^{-1}(x) = 5x + 4$.

$$f\left(f^{-1}(x)\right) = f(5x+4) = \frac{5x+4-4}{5} = \frac{5x}{5} = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{x-4}{5}\right) = 5\left(\frac{x-4}{5}\right) + 4 = x - 4 + 4 = x$$

P.R.115.

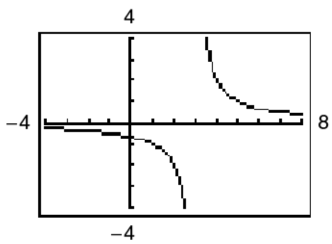
$$f(x) = (x-1)^2$$



The function is not one-to-one. Some horizontal lines intersect the graph more than once.

P.R.116.

$$h(t) = \frac{2}{t-3}$$

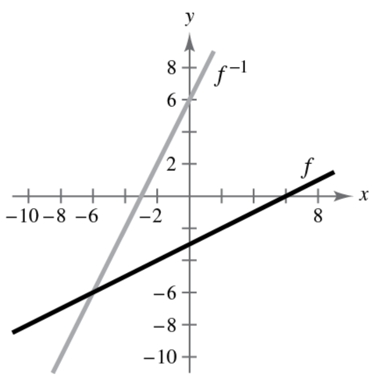


The function is one-to-one. Horizontal lines intersect the graph at most once.

P.R.117.

(a)
$$f(x) = \tfrac{1}{2}x - 3$$
$$y = \tfrac{1}{2}x - 3$$
$$x = \tfrac{1}{2}y - 3$$
$$x + 3 = \tfrac{1}{2}y$$
$$2(x+3) = y$$
$$f^{-1}(x) = 2x + 6$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

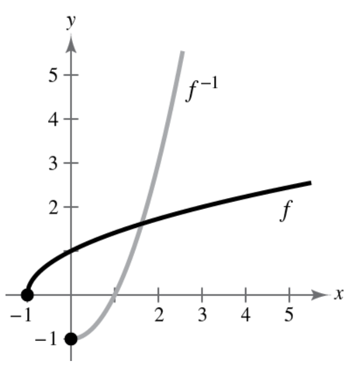
(d) The domains and ranges of f and f^{-1} are the set of all real numbers.

P.R.118.

(a)
$$f(x) = \sqrt{x+1}$$
$$y = \sqrt{x+1}$$
$$x = \sqrt{y+1}$$
$$x^2 = y + 1$$
$$x^2 - 1 = y$$
$$f^{-1}(x) = x^2 - 1, \qquad x \ge 0$$

Note: The inverse must have a restricted domain.

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domain of f and the range of f^{-1} is $[-1, \infty)$.

The range of f and the domain of f^{-1} is $[0, \infty)$.

P.R.119.

$f(x) = 2(x-4)^2$ is increasing on $(4, \infty)$.

Let $f(x) = 2(x-4)^2$, $x > 4$ and $y > 0$.

$$\begin{aligned} y &= 2(x-4)^2 \\ x &= 2(y-4)^2, \quad x > 0, \quad y > 4 \\ \frac{x}{2} &= (y-4)^2 \\ \sqrt{\frac{x}{2}} &= y - 4 \\ \sqrt{\frac{x}{2}} + 4 &= y \\ f^{-1}(x) &= \sqrt{\frac{x}{2}} + 4, \quad x > 0 \end{aligned}$$

P.R.120.

$f(x) = |x-2|$ is increasing on $(2, \infty)$.

Let $f(x) = x - 2$, $x > 2$, $y > 0$.

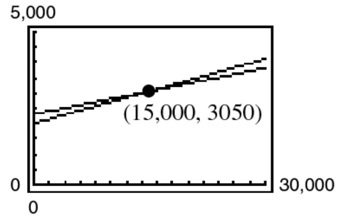
$$\begin{aligned} y &= x - 2 \\ x &= y - 2, x > 0, y > 2 \\ x + 2 &= y, x > 0, y > 2 \\ f^{-1}(x) &= x + 2, x > 0 \end{aligned}$$

PROBLEM SOLVING FOR CHAPTER P

P.PS.1.

(a) $W_1 = 0.07S + 2000$

(b) $W_2 = 0.05S + 2300$

(c) 

Point of intersection: $(15,000, 3050)$

Both jobs pay the same, \$3050, if you sell \$15,000 per month.

(d) No. If you think you can sell \$20,000 per month, keep your current job with the higher commission rate. For sales over \$15,000 it pays more than the other job.

P.PS.2.

Mapping numbers onto letters is *not* a function. Each number between 2 and 9 is mapped to more than one letter.

$$\{(2, A), (2, B), (2, C), (3, D), (3, E), (3, F), (4, G), (4, H), (4, I),$$
$$(5, J), (5, K), (5, L), (6, M), (6, N), (6, O), (7, P), (7, Q), (7, R),$$
$$(7, S), (8, T), (8, U), (8, V), (9, W), (9, X), (9, Y), (9, Z)\}$$

Mapping letters onto numbers *is* a function. Each letter is only mapped to one number.

$$\{(A, 2), (B, 2), (C, 2), (D, 3), (E, 3), (F, 3), (G, 4), (H, 4), (I, 4),$$
$$(J, 5), (K, 5), (L, 5), (M, 6), (N, 6), (O, 6), (P, 7), (Q, 7), (R, 7),$$
$$(S, 7), (T, 8), (U, 8), (V, 8), (W, 9), (X, 9), (Y, 9), (Z, 9)\}$$

P.PS.3.

(a) Let $f(x)$ and $g(x)$ be two even functions.

Then define $h(x) = f(x) \pm g(x)$.

$$\begin{aligned} h(-x) &= f(-x) \pm g(-x) \\ &= f(x) \pm g(x) \quad \text{because } f \text{ and } g \text{ are even} \\ &= h(x) \end{aligned}$$

So, $h(x)$ is also even.

(b) Let $f(x)$ and $g(x)$ be two odd functions.

Then define $h(x) = f(x) \pm g(x)$.

$$\begin{aligned} h(-x) &= f(-x) \pm g(-x) \\ &= -f(x) \pm g(x) \quad \text{because } f \text{ and } g \text{ are odd} \\ &= -h(x) \end{aligned}$$

So, $h(x)$ is also odd. $\left(\text{If } f(x) \neq g(x)\right)$

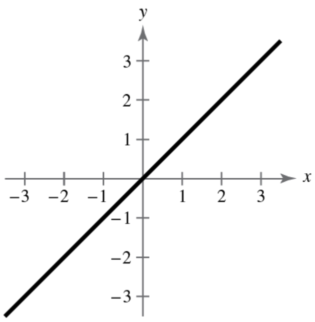
(c) Let $f(x)$ be odd and $g(x)$ be even. Then define $h(x) = f(x) \pm g(x)$.

$$\begin{aligned} h(-x) &= f(-x) \pm g(-x) \\ &= -f(x) \pm g(x) \quad \text{because } f \text{ is odd and } g \text{ is even} \\ &\neq h(x) \\ &\neq -h(x) \end{aligned}$$

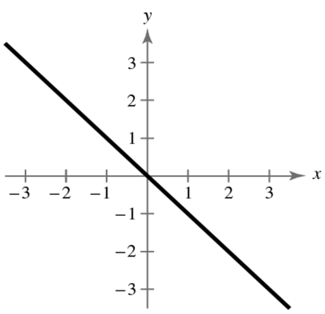
So, $h(x)$ is neither odd nor even.

P.PS.4.

$f(x) = x$



$g(x) = -x$



$(f \circ f)(x) = x$ and $(g \circ g)(x) = x$

These are the only two linear functions that are their own inverse functions since m has to equal $1/m$ for this to be true. General formula: $y = -x + c$

P.PS.5.

$$f(x) = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$$
$$f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$$
$$= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 = f(x)$$

So, $f(x)$ is even.

P.PS.6.

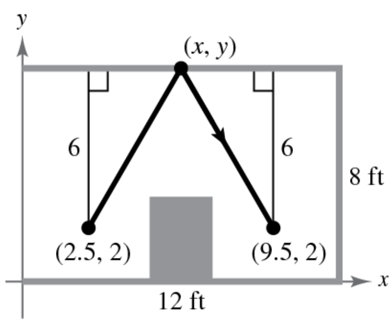
It appears, from the drawing, that the triangles are equal; thus $(x, y) = (6, 8)$.

The line between $(2.5, 2)$ and $(6, 8)$ is $y = \frac{12}{7}x - \frac{16}{7}$.

The line between $(9.5, 2)$ and $(6, 8)$ is $y = -\frac{12}{7}x + \frac{128}{7}$.

The path of the ball is:

$$f(x) = \begin{cases} \frac{12}{7}x - \frac{16}{7}, & 2.5 \le x \le 6 \\ -\frac{12}{7}x + \frac{128}{7}, & 6 < x \le 9.5 \end{cases}$$



P.PS.7.

(a) April 11: 10 hours
April 12: 24 hours
April 13: 24 hours
April 14: $23\frac{2}{3}$ hours
Total: $81\frac{2}{3}$ hours

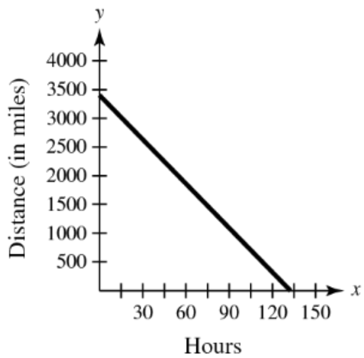
(b) $\text{Speed} = \dfrac{\text{distance}}{\text{time}} = \dfrac{2100}{81\frac{2}{3}} = \dfrac{180}{7} = 25\dfrac{5}{7}$ mph

(c) $D = -\dfrac{180}{7}t + 3400$

Domain: $0 \le t \le \dfrac{1190}{9}$

Range: $0 \le D \le 3400$

(d)



P.PS.8.

(a) $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}=\dfrac{f(2)-f(1)}{2-1}=\dfrac{1-0}{1}=1$

(b) $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}=\dfrac{f(1.5)-f(1)}{1.5-1}=\dfrac{0.75-0}{0.5}=1.5$

(c) $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}=\dfrac{f(1.25)-f(1)}{1.25-1}=\dfrac{0.4375-0}{0.25}=1.75$

(d) $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}=\dfrac{f(1.125)-f(1)}{1.125-1}=\dfrac{0.234375-0}{0.125}=1.875$

(e) $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}=\dfrac{f(1.0625)-f(1)}{1.0625-1}=\dfrac{0.12109375-0}{0.625}=1.9375$

(f) Yes, the average rate of change appears to be approaching 2.

(g) a. $(1, 0), (2, 1), m=1, y=x-1$

b. $(1, 0), (1.5, 0.75), m=\dfrac{0.75}{0.5}=1.5, y=1.5x-1.5$

c. $(1, 0), (1.25, 0.4375), m=\dfrac{0.4375}{0.25}=1.75, y=1.75x-1.75$

d. $(1, 0), (1.125, 0.234375), m=\dfrac{0.234375}{0.125}=1.875, y=1.875x-1.875$

e. $(1, 0), (1.0625, 0.12109375), m=\dfrac{0.12109375}{0.0625}=1.9375,$

$y=1.9375x-1.9375$

(h) $(1, f(1))=(1, 0), m\to 2, y=2(x-1), y=2x-2$

P.PS.9.

(a)–(d) Use $f(x) = 4x$ and $g(x) = x + 6$.

(a) $(f \circ g)(x) = f(x+6) = 4(x+6) = 4x + 24$

(b) $(f \circ g)^{-1}(x) = \dfrac{x-24}{4} = \dfrac{1}{4}x - 6$

(c) $f^{-1}(x) = \dfrac{1}{4}x$

$g^{-1}(x) = x - 6$

(d) $(g^{-1} \circ f^{-1})(x) = g^{-1}\left(\dfrac{1}{4}x\right) = \dfrac{1}{4}x - 6$

$f(x) = x^3 + 1$ and $g(x) = 2x$

$(f \circ g)(x) = f(2x) = (2x)^3 + 1 = 8x^3 + 1$

$(f \circ g)^{-1}(x) = \sqrt[3]{\dfrac{x-1}{8}} = \dfrac{1}{2}\sqrt[3]{x-1}$

$f^{-1}(x) = \sqrt[3]{x-1}$

$g^{-1}(x) = \dfrac{1}{2}x$

(e) $(g^{-1} \circ f^{-1})(x) = g^{-1}\left(\sqrt[3]{x-1}\right) = \dfrac{1}{2}\sqrt[3]{x-1}$

(f) Answers will vary.

(g) Conjecture: $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$

P.PS.10.

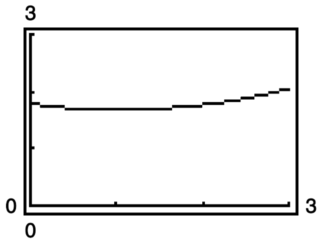
(a) The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3-x)^2}$.

The total time is

$$T(x) = \frac{\sqrt{4+x^2}}{2} + \frac{\sqrt{1+(3-x)^2}}{4}$$
$$= \frac{1}{2}\sqrt{4+x^2} + \frac{1}{4}\sqrt{x^2 - 6x + 10}.$$

(b) Domain of $T(x)$: $0 \le x \le 3$

(c)

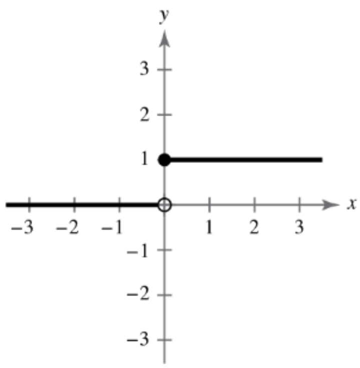


(d) $T(x)$ is a minimum when $x = 1$.

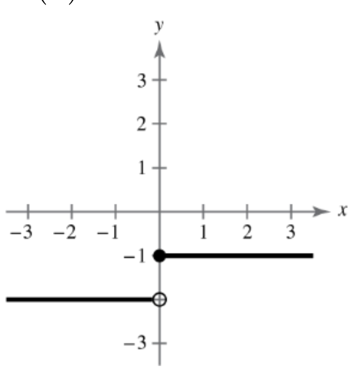
(e) Answers will vary. *Sample answer:* To reach point Q in the shortest amount of time, you should row to a point one mile down the coast, and then walk the rest of the way. The distance $x = 1$ yields a time of 1.68 hours.

P.PS.11.

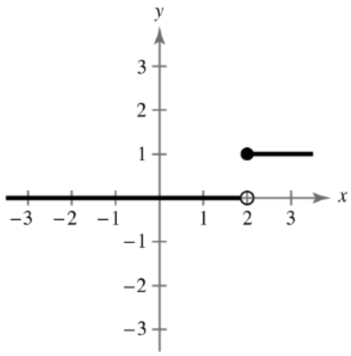
$$H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



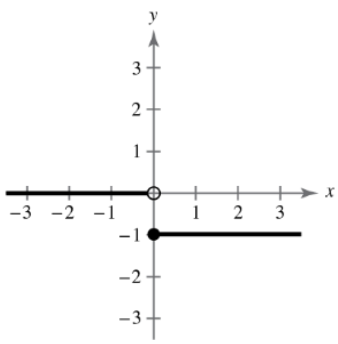
(a) $H(x) - 2$



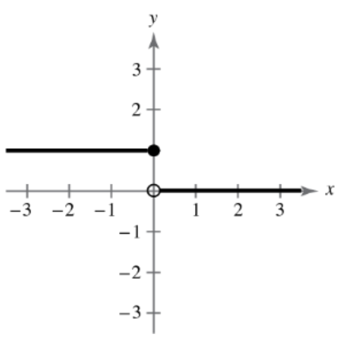
(b) $H(x-2)$



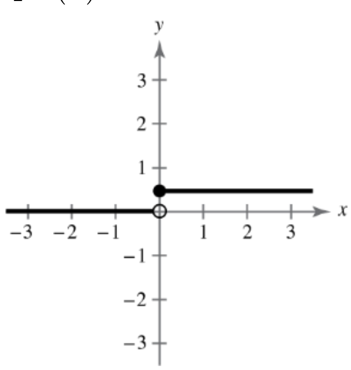
(c) $-H(x)$



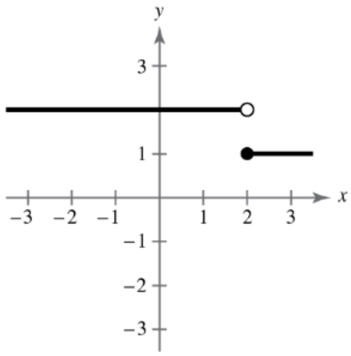
(d) $H(-x)$



(e) $\frac{1}{2}H(x)$



(f) $-H(x-2)+2$



P.PS.12.

$$f(x) = y = \frac{1}{1-x}$$

(a) Domain: all real numbers x except $x = 1$
Range: all real numbers y except $y = 0$

(b) $f(f(x)) = f\left(\frac{1}{1-x}\right)$

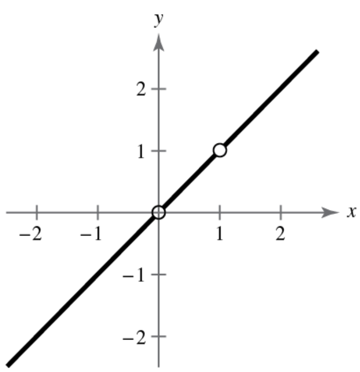
$$= \frac{1}{1-\left(\frac{1}{1-x}\right)} = \frac{1}{\frac{1-x-1}{1-x}}$$

$$= \frac{1-x}{-x} = \frac{x-1}{x}$$

Domain: all real numbers x except $x = 0$ and $x = 1$

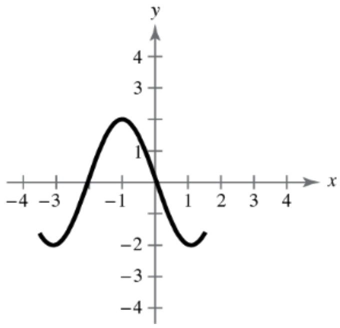
(c) $f(f(f(x))) = f\left(\frac{x-1}{x}\right) = \frac{1}{1-\left(\frac{x-1}{x}\right)} = \frac{1}{\frac{1}{x}} = x$

The graph is not a line. It has holes at $(0, 0)$ and $(1, 1)$.

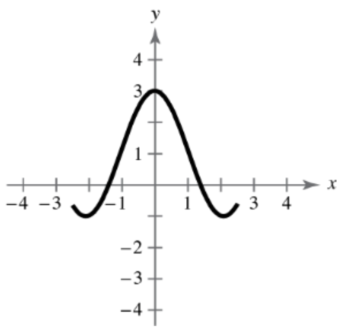


P.PS.13.

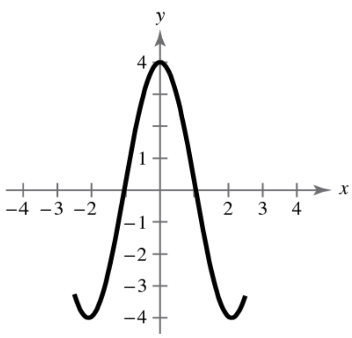
(a) $f(x+1)$



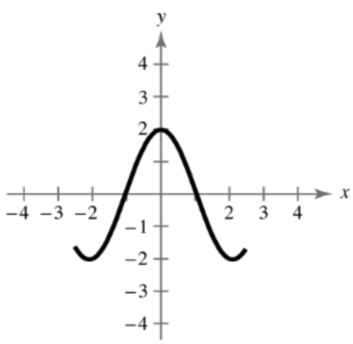
(b) $f(x)+1$



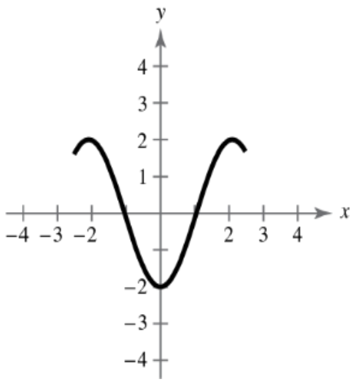
(c) $2f(x)$



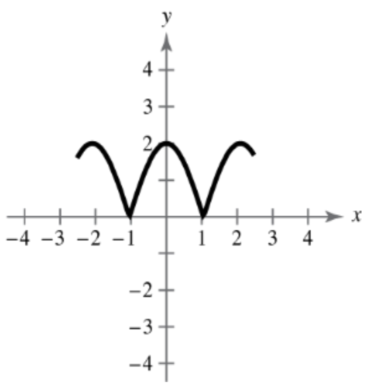
(d) $f(-x)$



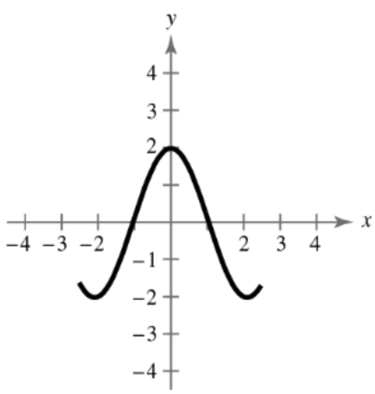
(e) $-f(x)$



(f) $\left|f\left(x\right)\right|$



(g) $f\left(\left|x\right|\right)$



P.PS.14.

x	$f(x)$	$f^{-1}(x)$
-4	—	2
-3	4	1
-2	1	0
-1	0	—
0	-2	-1
1	-3	-2
2	-4	—
3	—	—
4	—	-3

(a)

x	$f(f^{-1}(x))$
-4	$f(f^{-1}(-4)) = f(2) = -4$
-2	$f(f^{-1}(-2)) = f(0) = -2$
0	$f(f^{-1}(0)) = f(-1) = 0$
4	$f(f^{-1}(4)) = f(-3) = 4$

(b)

x	$(f + f^{-1})(x)$
-3	$f(-3) + f^{-1}(-3) = 4 + 1 = 5$
-2	$f(-2) + f^{-1}(-2) = 1 + 0 = 1$
0	$f(0) + f^{-1}(0) = -2 + (-1) = -3$
1	$f(1) + f^{-1}(1) = -3 + (-2) = -5$

(c)

x	$(f \cdot f^{-1})(x)$
-3	$f(-3)f^{-1}(-3) = (4)(1) = 4$
-2	$f(-2)f^{-1}(-2) = (1)(0) = 0$
0	$f(0)f^{-1}(0) = (-2)(-1) = 2$
1	$f(1)f^{-1}(1) = (-3)(-2) = 6$

(d)

x	$\lvert f^{-1}(x)\rvert$
-4	$\lvert f^{-1}(-4)\rvert = \lvert 2\rvert = 2$
-3	$\lvert f^{-1}(-3)\rvert = \lvert 1\rvert = 1$
0	$\lvert f^{-1}(0)\rvert = \lvert -1\rvert = 1$
4	$\lvert f^{-1}(4)\rvert = \lvert -3\rvert = 3$

Solution and Answer Guide

LARSON, TRIGONOMETRY, 12E, 2027, 9798214407593; CHAPTER 1: TRIGONOMETRY

TABLE OF CONTENTS

Section 1.1 Radian and Degree Measure

1.1.1.

coterminal

1.1.2.

radian

1.1.3.

Two complementary angles sum to $\frac{\pi}{2}$.

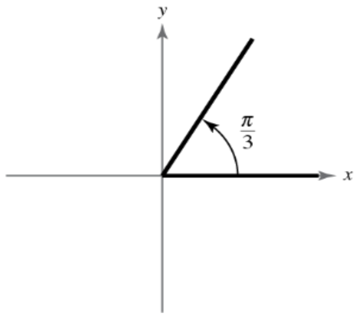
Two supplementary angles sum to π.

1.1.4.

You can add or subtract 2π or a multiple of 2π.

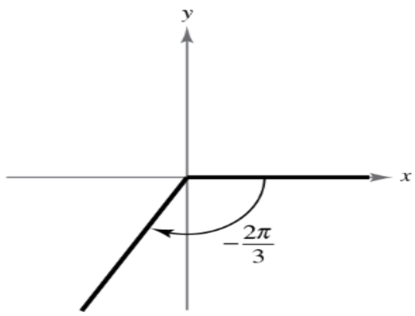
1.1.5.

(a) $\frac{\pi}{3}$



The angle is not quadrantal. It lies in Quadrant I.

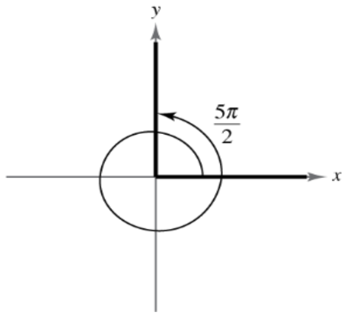
(b) $-\frac{2\pi}{3}$



The angle is not quadrantal. It lies in Quadrant III.

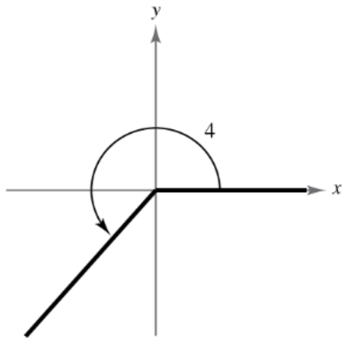
1.1.6.

(a) $\frac{5\pi}{2}$



The angle is quadrantal.

(b) 4



The angle is not quadrantal. It lies in Quadrant III.

1.1.7.

Sample answers:

(a) $\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$

$\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$

(b) $-\frac{5\pi}{6} + 2\pi = \frac{7\pi}{6}$

$-\frac{5\pi}{6} - 2\pi = -\frac{17\pi}{6}$

1.1.8.

Sample answers:

(a) $\frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}$

$\frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3}$

(b) $-\frac{9\pi}{4} + 2\pi = -\frac{\pi}{4}$

$-\frac{9\pi}{4} + 4\pi = \frac{7\pi}{4}$

1.1.9.

(a) Complement: $\frac{\pi}{2} - \frac{\pi}{12} = \frac{5\pi}{12}$

Supplement: $\pi - \frac{\pi}{12} = \frac{11\pi}{12}$

(b) Complement: Not possible, $\frac{11\pi}{12}$ is greater than $\frac{\pi}{2}$.

Supplement: $\pi - \frac{11\pi}{12} = \frac{\pi}{12}$

1.1.10.

(a) Complement: $\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

Supplement: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

(b) Complement: $\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

Supplement: $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

1.1.11.

(a) Complement: $\frac{\pi}{2} - 1 \approx 0.57$

Supplement: $\pi - 1 \approx 2.14$

(b) Complement: Not possible, 2 is greater than $\frac{\pi}{2}$.

Supplement: $\pi - 2 \approx 1.14$

1.1.12.

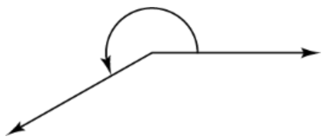
(a) Complement: Not possible, 3 is greater than $\frac{\pi}{2}$.

Supplement: $\pi - 3 \approx 0.14$

(b) Complement: $\frac{\pi}{2} - 1.5 \approx 0.07$

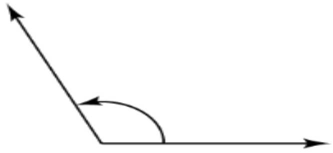
Supplement: $\pi - 1.5 \approx 1.64$

1.1.13.



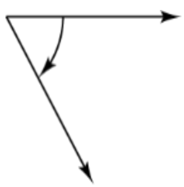
The angle shown is approximately $210°$.

1.1.14.



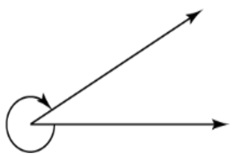
The angle shown is approximately $120°$.

1.1.15.



The angle shown is approximately $-60°$.

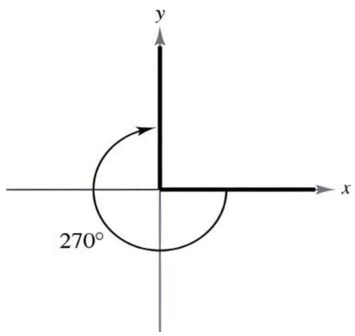
1.1.16.



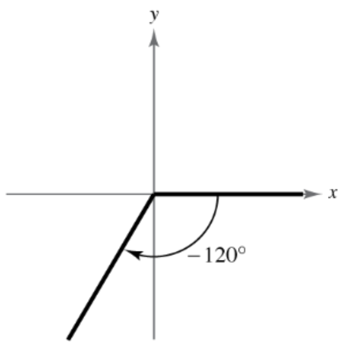
The angle shown is approximately $-330°$.

1.1.17.

(a) 270°

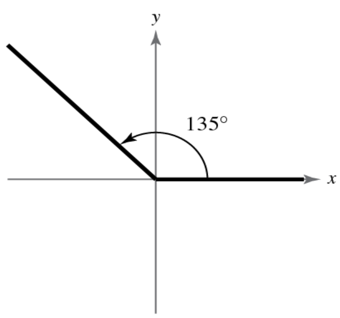


(b) –120°

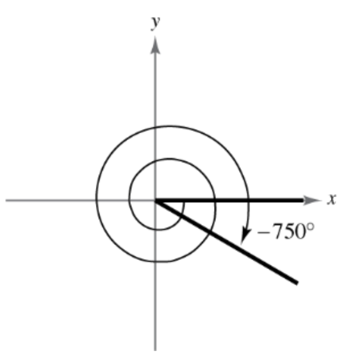


1.1.18.

(a) 135°



(b) –750°



1.1.19.

(a) Coterminal angles for $120°$

$120° + 360° = 480°$

$120° - 360° = -240°$

(b) Coterminal angles for $-210°$

$-210° + 360° = 150°$

$-210° - 360° = -570°$

1.1.20.

(a) Coterminal angles for $45°$

$45° + 360° = 405°$

$45° - 360° = -315°$

(b) Coterminal angles for $-420°$

$-420° + 720° = 300°$

$-420° + 360° = -60°$

1.1.21.

(a) Complement: $90° - 18° = 72°$

Supplement: $180° - 18° = 162°$

(b) Complement: $90° - 85° = 5°$

Supplement: $180° - 85° = 95°$

1.1.22.

(a) Complement: $90° - 46° = 44°$

Supplement: $180° - 46° = 134°$

(b) Complement: Not possible. $93°$ is greater than $90°$.

Supplement: $180° - 93° = 87°$

1.1.23.

(a) Complement: $90° - 24° = 66°$

Supplement: $180° - 24° = 156°$

(b) Complement: Not possible. $126°$ is greater than $90°$.

Supplement: $180° - 126° = 54°$

1.1.24.

(a) Complement: Not possible, $130°$ is greater than $90°$.

Supplement: $180° - 130° = 50°$

(b) Complement: Not possible, $170°$ is greater than $90°$.

Supplement: $180° - 170° = 10°$

1.1.25.

(a) $120° = 120°\left(\dfrac{\pi}{180°}\right) = -\dfrac{2\pi}{3}$

(b) $-20° = -20°\left(\dfrac{\pi}{180°}\right) = -\dfrac{\pi}{9}$

1.1.26.

(a) $-60° = -60°\left(\dfrac{\pi}{180°}\right) = -\dfrac{\pi}{3}$

(b) $144° = 144°\left(\dfrac{\pi}{180°}\right) = \dfrac{4\pi}{5}$

1.1.27.

(a) $\dfrac{3\pi}{2} = \dfrac{3\pi}{2}\left(\dfrac{180°}{\pi}\right) = 270°$

(b) $-\dfrac{7\pi}{6} = -\dfrac{7\pi}{6}\left(\dfrac{180°}{\pi}\right) = -210°$

1.1.28.

(a) $-\dfrac{7\pi}{12} = -\dfrac{7\pi}{12}\left(\dfrac{180°}{\pi}\right) = -105°$

(b) $\dfrac{5\pi}{4} = \dfrac{5\pi}{4}\left(\dfrac{180°}{\pi}\right) = 225°$

1.1.29.

$45° = 45°\left(\dfrac{\pi}{180°}\right) \approx 0.785$ radian

1.1.30.

$-48.27° = -48.27°\left(\dfrac{\pi}{180°}\right) \approx -0.842$ radian

1.1.31.

$-0.54° = -0.54°\left(\dfrac{\pi}{180°}\right) \approx -0.009$ radian

1.1.32.

$$345° = 345°\left(\frac{\pi}{180°}\right) \approx 6.021 \text{ radians}$$

1.1.33.

$$\frac{5\pi}{11} = \frac{5\pi}{11}\left(\frac{180°}{\pi}\right) \approx 81.818°$$

1.1.34.

$$\frac{15\pi}{8} = \frac{15\pi}{8}\left(\frac{180°}{\pi}\right) = 337.5°$$

1.1.35.

$$-4.2\pi = -4.2\pi\left(\frac{180°}{\pi}\right) = -756°$$

1.1.36.

$$-0.57 = -0.57\left(\frac{180°}{\pi}\right) \approx -32.659°$$

1.1.37.

(a) $54°45' = 54° + \left(\frac{45}{60}\right)^{\circ} = 54.75°$

(b) $-128°\ 30' = -128° - \left(\frac{30}{60}\right)^{\circ} = -128.5°$

1.1.38.

(a) $$\begin{aligned}-135°\ 10'\ 36'' &= 135° + \left(\frac{10}{60}\right)^{\circ} + \left(\frac{36}{3600}\right)^{\circ} \\ &\approx 135° + 0.1667° + 0.01° \approx 135.177°\end{aligned}$$

(b) $$\begin{aligned}-408°\ 16'\ 20'' &= -\left(408° + \left(\frac{16}{60}\right)^{\circ} + \left(\frac{20}{3600}\right)^{\circ}\right) \\ &\approx -\left(408° + 0.2667° + 0.0056°\right) \\ &\approx -408.272°\end{aligned}$$

1.1.39.

(a) $240.6° = 240° + 0.6(60)' = 240°\ 36'$

(b) $-145.8° = -\left[145° + 0.8(60')\right] = -145°\ 48'$

1.1.40.

(a) $$\begin{aligned} 345.12^\circ &= 345^\circ + (0.12)(60') \\ &= 345^\circ + 7.2' \\ &= 345^\circ + 7' + 0.2(60'') \\ &= 345^\circ\ 7'\ 12'' \end{aligned}$$

(b) $$\begin{aligned} -3.58^\circ &= -\left(3^\circ + (0.58)(60')\right) \\ &= -\left(3^\circ + 34' \ + 0.8(60'')\right) \\ &= -3^\circ\ 34'\ 48'' \end{aligned}$$

1.1.41.

$$r = 15 \text{ inches}, \theta = 120^\circ$$

$$s = r\theta$$

$$\begin{aligned} s = 15(120^\circ)\left(\frac{\pi}{180^\circ}\right) &= 10\pi \text{ inches} \\ &\approx 31.42 \text{ inches} \end{aligned}$$

1.1.42.

$$r = 3 \text{ meters}, \ \theta = 150^\circ$$

$$s = r\theta$$

$$\begin{aligned} s = 3(150^\circ)\left(\frac{\pi}{180^\circ}\right) &= \frac{5\pi}{2} \text{ meters} \\ &\approx 7.85 \text{ meters} \end{aligned}$$

1.1.43.

$$r = 6 \text{ inches}, \theta = \frac{\pi}{3}$$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right) = 6\pi \approx 18.85 \text{ square inches}$$

1.1.44.

$$A = \frac{1}{2}r^2\theta$$

$$\begin{aligned} A &= \frac{1}{2}(2.5)^2(225)\left(\frac{\pi}{180}\right) \\ &\approx 12.27 \text{ square feet} \end{aligned}$$

1.1.45.

The angle in degrees should be multiplied by $\dfrac{\pi}{180^\circ}$.

$$20^\circ = (20^\circ)\left(\frac{\pi \text{ rad}}{180^\circ}\right) = \frac{\pi}{9} \text{ radians.}$$

1.1.46.

The central angle should be converted to radians first to be used in the calculations.

$$s = r\theta$$

$$s = (6)\left(72^\circ \frac{\pi}{180^\circ}\right)$$

$$= \frac{12\pi}{5}$$

$$\approx 7.54 \text{ millimeters}$$

1.1.47.

$$\theta = 41^\circ\ 15'\ 50'' - 32^\circ\ 47'\ 9'' \approx 8.47806^\circ \approx 0.14797 \text{ radian}$$

$$s = r\theta \approx 4000(0.14782) \approx 592 \text{ miles}$$

1.1.48.

$$\theta = 47^\circ\ 37'\ 18'' - 37^\circ\ 47'\ 36'' \approx 9^\circ\ 49'\ 42'' = 9.82833^\circ \approx 0.17154 \text{ radian}$$

$$r = 4000 \text{ miles}$$

$$s = r\theta = 4000(0.17154) \approx 686 \text{ miles}$$

1.1.49.

$$\theta = \frac{s}{r} = \frac{2.5}{6} = \frac{25}{60} = \frac{5}{12} \text{ radian} \approx 23.87^\circ$$

1.1.50.

(a) $\text{Angular speed} = \dfrac{(5200)(2\pi)\ \text{radians}}{1\ \text{minute}} = 10,400\pi \text{ radians per minute}$

$\approx 32,672.56 \text{ radians per minute}$

(b) $\text{Linear speed} = \dfrac{\left(\frac{7.25}{2}\text{ in.}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)(5200)(2\pi)\text{ feet}}{1\text{ minute}} = \dfrac{9425\pi}{3}\ \text{ feet per minute}$

$\approx 9869.84\ \text{ feet per minute}$

1.1.51.

(a) $4\ \text{rpm} = 4(2\pi)\ \text{radians per minute} = 8\pi \approx 25\ \text{radians per minute}$

(b) $r = 25$ feet

$\dfrac{r\theta}{t} = 200\pi \text{ feet per minute}$

Linear speed $\approx 25(25.13)$ feet per minute ≈ 628.3 feet per minute

1.1.52.

(a) $$\text{Road speed (linear speed)} = \frac{\left(\frac{25}{2}\text{ in.}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)(480)(2\pi)}{1\text{ minute}\left(\frac{1\text{ hour}}{60\text{ minutes}}\right)}$$
$$\approx 35.70\text{ miles per hour}$$

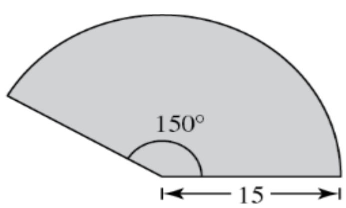
(b) $$\frac{55\text{ mi}}{1\text{ h}} \times \frac{5280\text{ ft}}{1\text{ mi}} \times \frac{12\text{ in.}}{1\text{ ft}} \times \frac{1\text{ h}}{60\text{ min}} = \frac{58{,}080\text{ in.}}{1\text{ min}}$$

The circumference of the machine is $C = 2\pi\left(\frac{25}{2}\right) = 25\pi$ inches.

The number of revolutions per minute is $r = 58{,}080/25\pi \approx 739.50$.

1.1.53.

$$\begin{aligned} A &= \frac{1}{2}r^2\theta \\ &= \frac{1}{2}(15)^2(150^\circ)\left(\frac{\pi}{180^\circ}\right) \\ &= 93.75\pi \\ &\approx 294.52\text{ square meters} \end{aligned}$$



1.1.54.

(a) Arc length of larger sprocket in feet:

$s = r\theta$

$s = \frac{1}{3}(2\pi) = \frac{2\pi}{3}$ feet

Therefore, the chain moves $\frac{2\pi}{3}$ feet as does the smaller rear sprocket.

Thus, the angle θ of the smaller sprocket is

$$\theta = \frac{s}{r} = \frac{\frac{2\pi}{3}\text{ ft}}{\frac{2}{12}\text{ ft}} = 4\pi \quad \left(r = 2\text{ inches} = \frac{2}{12}\text{ feet}\right)$$

and the arc length of the tire in feet is:

$$s = \theta r$$

$$s = (4\pi)\left(\frac{14}{12}\right) = \frac{14\pi}{3}\text{ feet}$$

$$\text{Speed} = \frac{s}{t} = \frac{\frac{14\pi}{3}}{1\text{ sec}} = \frac{14\pi}{3}\text{ feet per second}$$

$$\frac{14\pi\text{ feet}}{3\text{ seconds}} \times \frac{3600\text{ seconds}}{1\text{ hour}} \times \frac{1\text{ mile}}{5280\text{ feet}} \approx 10\text{ miles per hour}$$

(b) Since the arc length of the tire is $\frac{14\pi}{3}$ feet and the cyclist is pedaling at a rate of one revolution per second, we have:

$$\begin{aligned}\text{Distance} &= \left(\frac{14\pi}{3}\ \frac{\text{feet}}{\text{revolutions}}\right)\left(\frac{1\text{ mile}}{5280\text{ feet}}\right)(n\text{ revolutions})\\ &= \frac{7\pi}{7920}n\text{ miles}\end{aligned}$$

(c) $$\begin{aligned}\text{Distance} &= \text{Rate}\cdot\text{Time}\\ &= \left(\frac{14\pi}{3}\text{ feet/second}\right)\left(\frac{1\text{ mile}}{5280\text{ feet}}\right)(t\text{ seconds})\\ &= \frac{7\pi}{7920}t\text{ miles}\end{aligned}$$

The functions are both linear.

1.1.55.

False. $\frac{180^\circ}{\pi}$ is in degree measure.

1.1.56.

False. A measurement of 4π radians corresponds to two complete revolutions from the initial to the terminal side of an angle.

1.1.57.

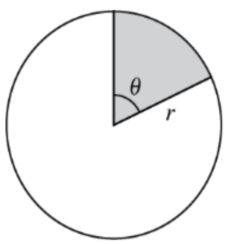
True. If α and β are coterminal angles, then $\alpha = \beta + n(360^\circ)$ or $\alpha = \beta + n(2\pi)$, where n is an integer. The difference between α and β is $\alpha - \beta = n(360^\circ)$, or $\alpha - \beta = n(2\pi)$ if expressed in radians.

1.1.58.

False. 1 radian $= \left(\dfrac{180°}{\pi}\right) \approx 57.3°$, so one radian is much larger than one degree.

1.1.59.

$$\text{Area of circle} = \pi r^2$$
$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Measure of central angle of sector}}{\text{Measure of central angle of circle}}$$
$$\frac{\text{Area of sector}}{\pi r^2} = \frac{\theta}{2\pi}$$
$$\text{Area of sector} = \left(\pi r^2\right)\left(\frac{\theta}{2\pi}\right) = \frac{1}{2}r^2\theta$$



1.1.60.

Angles B and C are coterminal with Angle A, since they have the same initial and terminal side as Angle A.

1.1.61.

$$x^2 = 7^2 + 24^2$$
$$x^2 = 625$$
$$x = 25$$

1.1.62.

$$x^2 = 6^2 + 4^2$$
$$x^2 = 52$$
$$x = \sqrt{52} = 2\sqrt{13}$$

1.1.63.

$$\frac{x^2 - 9}{x^2 - 2x - 3} = \frac{(x+3)(x-3)}{(x-3)(x+1)} = \frac{x+3}{x+1},\ x \neq 3$$

1.1.64.

$$\frac{x^2 - 4}{x^2 - x - 6} = \frac{(x+2)(x-2)}{(x-3)(x+2)} = \frac{x-2}{x-3},\ x \neq -2$$

1.1.65.

$$\begin{aligned}\frac{x^2+11x+30}{x^2+x-30} &= \frac{(x+5)(x+6)}{(x+6)(x-5)} \\ &= \frac{x+5}{x-5},\ x \neq -6\end{aligned}$$

1.1.66.

$$\begin{aligned}\frac{2x^2-3x-9}{2x^2-5x-12} &= \frac{(x-3)(2x+3)}{(2x+3)(x-4)} \\ &= \frac{x-3}{x-4},\ x \neq -\tfrac{3}{2}\end{aligned}$$

Section 1.2 Trigonometric Functions: The Unit Circle

1.2.1.

unit circle

1.2.2.

periodic

1.2.3.

odd; even

1.2.4.

The period is the smallest positive number c for which $f(t+c)=f(t)$ for all t in the domain of f.

1.2.5.

$$x=\frac{12}{13},\quad y=\frac{5}{13}$$

$$\sin t=y=\frac{5}{13} \qquad \csc t=\frac{1}{y}=\frac{13}{5}$$
$$\cos t=x=\frac{12}{13} \qquad \sec t=\frac{1}{x}=\frac{13}{12}$$
$$\tan t=\frac{y}{x}=\frac{5}{12} \qquad \cot t=\frac{x}{y}=\frac{12}{5}$$

1.2.6.

$$x=-\frac{8}{17},\quad y=\frac{15}{17}$$

$$\sin t=y=\frac{15}{17} \qquad \csc t=\frac{1}{y}=\frac{17}{15}$$
$$\cos t=x=-\frac{8}{17} \qquad \sec t=\frac{1}{x}=-\frac{17}{8}$$
$$\tan t=\frac{y}{x}=-\frac{15}{8} \qquad \cot t=\frac{x}{y}=-\frac{8}{15}$$

1.2.7.

$$x=-\frac{4}{5},\, y=-\frac{3}{5}$$

$$\sin\ t = y = -\frac{3}{5} \qquad \csc\ t = \frac{1}{y} = -\frac{5}{3}$$
$$\cos\ t = x = -\frac{4}{5} \qquad \sec\ t = \frac{1}{x} = -\frac{5}{4}$$
$$\tan\ t = \frac{y}{x} = \frac{3}{4} \qquad \cot\ t = \frac{x}{y} = \frac{4}{3}$$

1.2.8.

$$x = \frac{12}{13},\ y = -\frac{5}{13}$$

$$\sin t = y = -\frac{5}{13} \qquad \csc t = \frac{1}{y} = -\frac{13}{5}$$
$$\cos t = x = \frac{12}{13} \qquad \sec\ t = \frac{1}{x} = \frac{13}{12}$$
$$\tan t = \frac{y}{x} = -\frac{5}{12} \qquad \cot\ t = \frac{x}{y} = -\frac{12}{5}$$

1.2.9.

$t = \dfrac{\pi}{2}$ corresponds to the point $(x, y) = (0, 1)$.

1.2.10.

$t = \dfrac{\pi}{4}$ corresponds to the point $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$.

1.2.11.

$t = \dfrac{5\pi}{6}$ corresponds to the point $(x, y) = \left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$.

1.2.12.

$t = \dfrac{4\pi}{3}$ corresponds to the point $(x, y) = \left(-\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$.

1.2.13.

$t = \dfrac{\pi}{4}$ corresponds to the point $(x, y) = \left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$.

$$\sin\frac{\pi}{4} = y = \frac{\sqrt{2}}{2}$$
$$\cos\frac{\pi}{4} = x = \frac{\sqrt{2}}{2}$$
$$\tan\frac{\pi}{4} = \frac{y}{x} = 1$$

1.2.14.

$t = \dfrac{\pi}{3}$ corresponds to the point $(x,\ y) = \left(\dfrac{1}{2},\ \dfrac{\sqrt{3}}{2}\right)$.

$$\sin\frac{\pi}{3} = y = \frac{\sqrt{3}}{2}$$
$$\cos\frac{\pi}{3} = x = \frac{1}{2}$$
$$\tan\frac{\pi}{3} = \frac{y}{x} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

1.2.15.

$t = -\dfrac{\pi}{6}$ corresponds to the point $\left(\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right)$.

$$\sin -\frac{\pi}{6} = y = -\frac{1}{2}$$
$$\cos -\frac{\pi}{6} = x = \frac{\sqrt{3}}{2}$$
$$\tan -\frac{\pi}{6} = \frac{y}{x} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

1.2.16.

$t = -\dfrac{\pi}{4}$ corresponds to the point $(x, y) = \left(\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$.

$$\sin\left(-\frac{\pi}{4}\right) = y = -\frac{\sqrt{2}}{2}$$
$$\cos\left(-\frac{\pi}{4}\right) = x = \frac{\sqrt{2}}{2}$$
$$\tan\left(-\frac{\pi}{4}\right) = \frac{y}{x} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$$

1.2.17.

$t = \frac{11\pi}{6}$ corresponds to the point $(x, y) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$$\sin\frac{11\pi}{6} = y = -\frac{1}{2}$$
$$\cos\frac{11\pi}{6} = x = \frac{\sqrt{3}}{2}$$
$$\tan\frac{11\pi}{6} = \frac{y}{x} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

1.2.18.

$t = \frac{5\pi}{3}$ corresponds to the point $(x, y) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$$\sin\frac{5\pi}{3} = y = -\frac{\sqrt{3}}{2}$$
$$\cos\frac{5\pi}{3} = x = \frac{1}{2}$$
$$\tan\frac{5\pi}{3} = \frac{y}{x} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$$

1.2.19.

$t = -\frac{3\pi}{2}$ corresponds to the point $(x, y) = (0, 1)$.

$$\sin\left(-\frac{3\pi}{2}\right) = y = 1$$
$$\cos\left(-\frac{3\pi}{2}\right) = x = 0$$
$$\tan\left(-\frac{3\pi}{2}\right) = \frac{y}{x} \text{ is undefined.}$$

1.2.20.

$t = -2\pi$ corresponds to the point $(x, y) = (1, 0)$.

$$\sin(-2\pi) = y = 0$$
$$\cos(-2\pi) = x = 1$$
$$\tan(-2\pi) = \frac{y}{x} = \frac{0}{1} = 0$$

1.2.21.

$t = \frac{2\pi}{3}$ corresponds to the point $(x, y) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

$$\sin\frac{2\pi}{3} = y = \frac{\sqrt{3}}{2} \qquad \csc\frac{2\pi}{3} = \frac{1}{y} = \frac{2\sqrt{3}}{3}$$

$$\cos\frac{2\pi}{3} = x = -\frac{1}{2} \qquad \sec\frac{2\pi}{3} = \frac{1}{x} = -2$$

$$\tan\frac{2\pi}{3} = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3} \qquad \cot\frac{2\pi}{3} = \frac{x}{y} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{3}$$

1.2.22.

$t = \frac{5\pi}{6}$ corresponds to the point $(x, y) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

$$\sin\frac{5\pi}{6} = y = \frac{1}{2} \qquad \csc\frac{5\pi}{6} = \frac{1}{y} = 2$$

$$\cos\frac{5\pi}{6} = x = -\frac{\sqrt{3}}{2} \qquad \sec\frac{5\pi}{6} = \frac{1}{x} = -\frac{2\sqrt{3}}{3}$$

$$\tan\frac{5\pi}{6} = \frac{y}{x} = \frac{1/2}{-\sqrt{3}/2} = -\frac{\sqrt{3}}{3} \qquad \cot\frac{5\pi}{6} = \frac{x}{y} = -\sqrt{3}$$

1.2.23.

$t = \frac{4\pi}{3}$ corresponds to the point $(x, y) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$$\sin\frac{4\pi}{3} = y = -\frac{\sqrt{3}}{2} \qquad \csc\frac{4\pi}{3} = \frac{1}{y} = -\frac{2\sqrt{3}}{3}$$

$$\cos\frac{4\pi}{3} = x = -\frac{1}{2} \qquad \sec\frac{4\pi}{3} = \frac{1}{x} = -2$$

$$\tan\frac{4\pi}{3} = \frac{y}{x} = \sqrt{3} \qquad \cot\frac{4\pi}{3} = \frac{x}{y} = \frac{\sqrt{3}}{3}$$

1.2.24.

$t = \frac{7\pi}{4}$ corresponds to the point $(x, y) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

$$\sin\frac{7\pi}{4} = y = -\frac{\sqrt{2}}{2} \qquad \csc\frac{7\pi}{4} = \frac{1}{y} = -\sqrt{2}$$
$$\cos\frac{7\pi}{4} = x = \frac{\sqrt{2}}{2} \qquad \sec\frac{7\pi}{4} = \frac{1}{x} = \sqrt{2}$$
$$\tan\frac{7\pi}{4} = \frac{y}{x} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1 \qquad \cot\frac{7\pi}{4} = \frac{x}{y} = -1$$

1.2.25.

$t = -\frac{5\pi}{3}$ corresponds to the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

$$\sin\left(-\frac{5\pi}{3}\right) = y = \frac{\sqrt{3}}{2} \qquad \csc\left(-\frac{5\pi}{3}\right) = \frac{1}{y} = \frac{2}{\sqrt{3}}$$
$$\cos\left(-\frac{5\pi}{3}\right) = x = \frac{1}{2} \qquad \sec\left(-\frac{5\pi}{3}\right) = \frac{1}{x} = 2$$
$$\tan\left(-\frac{5\pi}{3}\right) = \frac{y}{x} = \sqrt{3} \qquad \cot\left(-\frac{5\pi}{3}\right) = \frac{x}{y} = \frac{\sqrt{3}}{3}$$

1.2.26.

$t = -\frac{3\pi}{2}$ corresponds to the point $(x, y) = (0, 1)$.

$$\sin\left(-\frac{3\pi}{2}\right) = y = 1 \qquad \csc\left(-\frac{3\pi}{2}\right) = \frac{1}{y} = 1$$
$$\cos\left(-\frac{3\pi}{2}\right) = x = 0 \qquad \sec\left(-\frac{3\pi}{2}\right) = \frac{1}{x} \text{ is undefined.}$$
$$\tan\left(-\frac{3\pi}{2}\right) = \frac{y}{x} \text{ is undefined.} \qquad \cot\left(-\frac{3\pi}{2}\right) = \frac{x}{y} = \frac{0}{1} = 0$$

1.2.27.

$t = -\frac{\pi}{2}$ corresponds to the point $(x, y) = (0, -1)$.

$$\sin\left(-\frac{\pi}{2}\right) = y = -1 \qquad \csc\left(-\frac{\pi}{2}\right) = \frac{1}{y} = -1$$
$$\cos\left(-\frac{\pi}{2}\right) = x = 0 \qquad \sec\left(-\frac{\pi}{2}\right) = \frac{1}{x} \text{ is undefined.}$$
$$\tan\left(-\frac{\pi}{2}\right) = \frac{y}{x} \text{ is undefined.} \qquad \cot\left(-\frac{\pi}{2}\right) = \frac{x}{y} = 0$$

1.2.28.

$t=-\pi$ corresponds to the point $(x, y)=(-1, 0)$.

$\sin(-\pi)=y=0 \qquad \csc(-\pi)=\dfrac{1}{y}$ is undefined.

$\cos(-\pi)=x=-1 \qquad \sec(-\pi)=\dfrac{1}{x}=-1$

$\tan(-\pi)=\dfrac{y}{x}=0 \qquad \cot(-\pi)=\dfrac{x}{y}$ is undefined.

1.2.29.

$\sin\ 4\pi=\sin\ 0=0$

1.2.30.

$\cos\ 3\pi=\cos\ \pi=-1$

1.2.31.

$$\cos\ \frac{7\pi}{3}=\cos\ \frac{\pi}{3}=\frac{1}{2}$$

1.2.32.

$$\sin\ \frac{9\pi}{4}=\sin\ \frac{\pi}{4}=\frac{\sqrt{2}}{2}$$

1.2.33.

$$\sin\ \frac{19\pi}{6}=\sin\ \frac{7\pi}{6}=-\frac{1}{2}$$

1.2.34.

$$\sin\left(-\frac{8\pi}{3}\right)=\sin\left(-\frac{2\pi}{3}\right)=-\sin\left(\frac{2\pi}{3}\right)=-\frac{\sqrt{3}}{2}$$

1.2.35.

$\sin\ t=\frac{1}{2}$

(a) $\sin(-t)=-\sin\ t=-\frac{1}{2}$

(b) $\csc(-t)=-\csc\ t=-2$

1.2.36.

$$\sin(-t) = \frac{3}{8}$$

(a) $\sin t = -\sin(-t) = -\frac{3}{8}$

(b) $\csc\ t = \frac{1}{\sin t} = -\frac{8}{3}$

1.2.37.

$$\cos(-t) = -\frac{1}{5}$$

(a) $\cos t = \cos(-t) = -\frac{1}{5}$

(b) $\sec(-t) = \frac{1}{\cos(-t)} = -5$

1.2.38.

$$\cos t = -\frac{3}{4}$$

(a) $\cos(-t) = \cos t = -\frac{3}{4}$

(b) $\sec(-t) = \sec t = \frac{1}{\cos\ t} = -\frac{4}{3}$

1.2.39.

$$\sin\ t = \frac{4}{5}$$

(a) $\sin(\pi - t) = \sin t = \frac{4}{5}$

(b) $\sin(t + \pi) = -\sin t = -\frac{4}{5}$

1.2.40.

$$\cos\ t = \frac{4}{5}$$

(a) $\cos(\pi - t) = -\cos t = -\frac{4}{5}$

(b) $\cos(t + \pi) = -\cos t = -\frac{4}{5}$

1.2.41.

$y(t) = 2\cos 6t$

(a) $y(0) = 2\cos 0 = 2$ feet

(b) $y\left(\frac{1}{4}\right) = 2\cos\frac{3}{2} \approx 0.14$ foot

1.2.42.

$y(t) = \frac{1}{2}e^{-t}\cos 6t$

(a)

t	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
y	0.5	0.0275	−0.3002	−0.0498	0.1766

(b) Using technology, you estimate that $y \approx 0$ when $t \approx 0.26$ and $t \approx 0.79$ seconds.

(c) As t increases, the displacement oscillates but decreases in amplitude.

1.2.43.

False. $\sin(-t) = -\sin t$ means the function is odd, not that the sine of a negative angle is a negative number.

For example: $\sin\left(-\frac{3\pi}{2}\right) = -\sin\left(\frac{3\pi}{2}\right) = -(-1) = 1.$

Even though the angle is negative, the sine value is positive.

1.2.44.

False. The real number 0 corresponds to the point $(1, 0)$ on the unit circle.

1.2.45.

True. $\tan\alpha = \tan(\alpha - 6\pi)$ because the period of the tangent function is π.

1.2.46.

True.

$-\frac{7\pi}{2}$ is coterminal with $\frac{\pi}{2}$.

$$\cos\left(-\frac{7\pi}{2}\right) = \cos\frac{\pi}{2} = 0$$

$$\cos\left(\pi + \frac{\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right) = 0$$

So, $\cos\left(-\frac{7\pi}{2}\right) = \cos\left(\pi + \frac{\pi}{2}\right)$.

1.2.47.

(a) The points have y-axis symmetry.

(b) $\sin t_1 = \sin(\pi - t_1)$ because they have the same y-value.

(c) $\cos(\pi - t_1) = -\cos t_1$ because the x-values have the opposite signs.

1.2.48.

Let $h(t) = f(t)g(t) = \sin t\ \tan t.$

$$\begin{aligned}\text{Then, } h(t) &= \sin(-t)\tan(-t)\\ &= (-\sin t)(-\tan t)\\ &= \sin t \tan t\\ &= h(t).\end{aligned}$$

So, $h(t)$ is even.

1.2.49.

$\sin(0.25) + \sin(0.75) \approx 0.2474 + 0.6816 = 0.9290$

$\sin 1 \approx 0.8415$

So, $\sin t_1 + \sin t_2 \neq \sin(t_1 + t_2)$.

1.2.50.

(a) Yes: the functions $\sin\ t$ and $\cos\ t$ are the y and x coordinates, respectively, of the point. Because these coordinates are nonzero, the functions $\csc t = \dfrac{1}{y}$, $\sec t = \dfrac{1}{x}$, $\tan t = \dfrac{y}{x}$, and $\cot t = \dfrac{x}{y}$ all exist as well.

(b) The functions $\sin t = y, \cos t = x,\ \csc t = \dfrac{1}{y}$, and $\sec t = \dfrac{1}{x}$ are all negative because (x, y) is in the third quadrant, which means that both x and y (and their reciprocals) are negative. The functions $\tan t = \dfrac{y}{x}$ and $\cot t = \dfrac{x}{y}$ are both positive because x and y are both negative, and the ratio of two negative numbers is positive.

1.2.51.

Center: $(17, 7)$, Radius: 6

$(x - 17)^2 + (y - 7)^2 = 6^2 = 36$

1.2.52.

Center: $(12, -12)$, Radius: 7

$(x-12)^2 + (y+12)^2 = 7^2 = 49$

1.2.53.

Endpoints of diameter: $(-4, -7), (-3, -5)$

Center is midpoint of diameter: $\left(\frac{-4-3}{2}, \frac{-7-5}{2}\right) = \left(\frac{-7}{2}, -6\right)$

Radius is one-half length of diameter:

$r = \frac{1}{2}\sqrt{(-4+3)^2 + (-7+5)^2} = \frac{1}{2}\sqrt{5}$

Circle: $\left(x + \frac{7}{2}\right)^2 + (y+6)^2 = \frac{5}{4}$

1.2.54.

Endpoints of diameter: $(-10, 20), (16, -3)$

Center is midpoint of diameter: $\left(\frac{-10+16}{2}, \frac{20-3}{2}\right) = \left(3, \frac{17}{2}\right)$

Radius is one-half length of diameter:

$r = \frac{1}{2}\sqrt{(16+10)^2 + (-3-20)^2} = \frac{1}{2}\sqrt{1205}$

Circle: $(x-3)^2 + \left(y - \frac{17}{2}\right)^2 = \frac{1205}{4}$

1.2.55.

$y^2 = 6 - x$

Let $y = 0$.

$0^2 = 6 - x$

$x = 6$

x-intercept: $(6, 0)$

Let $x = 0$.

$y^2 = 6 - (0)$

$y^2 = 6$

$y = \pm\sqrt{6}$

y-intercepts: $(0, \pm\sqrt{6})$

1.2.56.

$y = x^4 - 25$

Let $y = 0$.

$0 = x^4 - 25$

$0 = (x^2 - 5)(x^2 + 5)$

$x = \pm\sqrt{5}$

x-intercepts: $(\pm\sqrt{5}, 0)$

Let $x = 0$.

$y = (0)^4 - 25$

$y = -25$

y-intercept: $(0, -25)$

1.2.57.

$y = 4x^2 - 6x - 4 = 2(x - 2)(2x + 1)$

x-intercepts: $(2, 0), \left(-\frac{1}{2}, 0\right)$

y-intercept: $(0, -4)$

1.2.58.

$y^2 = 2|x - 4|$

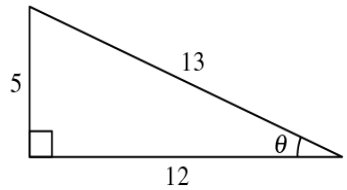
$y = 0 \Rightarrow 0 = 2|x - 4| \Rightarrow x = 4$

$x = 0 \Rightarrow y^2 = 8 \Rightarrow y = \pm 2\sqrt{2}$

y-intercepts: $(0, 2\sqrt{2}), (0, -2\sqrt{2})$

SECTION 1.3 RIGHT TRIANGLE TRIGONOMETRY

1.3.1.



(a) The side opposite θ has length 5.

(b) The side adjacent to θ has length 12.

(c) The hypotenuse has length 13.

1.3.2.

(a) $\sin \alpha = \dfrac{a}{c}$

(b) $\cos \beta = \dfrac{a}{c}$

(c) $\tan \beta = \dfrac{b}{a}$

(d) $\cot \alpha = \dfrac{b}{a}$

(e) $\sec \alpha = \dfrac{c}{b}$

(f) $\csc \beta = \dfrac{c}{b}$

1.3.3.

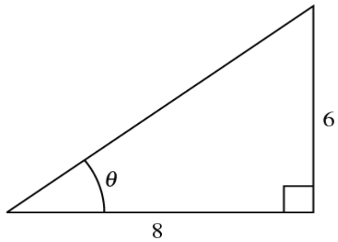
Complementary

1.3.4.

elevation; depression

1.3.5.

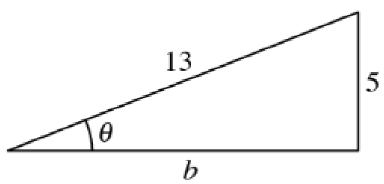
$$\text{hyp} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} = \frac{3}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{10}{6} = \frac{5}{3}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{8}{10} = \frac{4}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{10}{8} = \frac{5}{4}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{6}{8} = \frac{3}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{8}{6} = \frac{4}{3}$$

1.3.6.

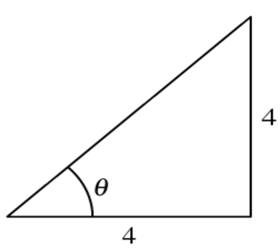
$$\text{adj} = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = 12$$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{13}{5}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{13}{12}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{5}{12} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{12}{5}$$

1.3.7.

$$\text{hyp} = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$$



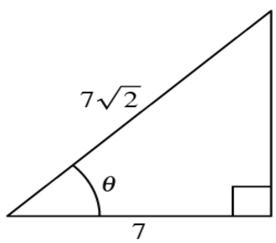
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{4\sqrt{2}}{4} = \sqrt{2}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{4\sqrt{2}}{4} = \sqrt{2}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{4} = 1 \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{4} = 1$$

1.3.8.

$$\text{opp} = \sqrt{\left(7\sqrt{2}\right)^2 - 7^2} = \sqrt{98-49} = \sqrt{49} = 7$$



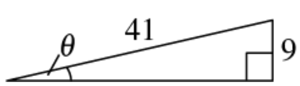
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{7}{7\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{7\sqrt{2}}{7} = \sqrt{2}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{7}{7\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{7\sqrt{2}}{7} = \sqrt{2}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{7}{7} = 1 \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{7}{7} = 1$$

1.3.9.

$$\text{adj} = \sqrt{41^2 - 9^2} = \sqrt{1681-81} = \sqrt{1600} = 40$$



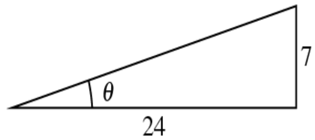
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{9}{41} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{41}{9}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{40}{41} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{41}{40}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{9}{40} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{40}{9}$$

1.3.10.

$$\text{hyp} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$$

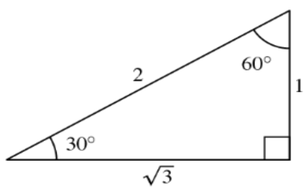


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{7}{25} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{25}{7}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{24}{25} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{25}{24}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{7}{24} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{24}{7}$$

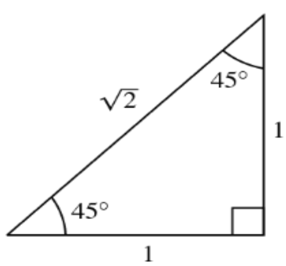
1.3.11.



$$30^\circ = 30^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{6} \text{ radian}$$

$$\tan 30^\circ = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

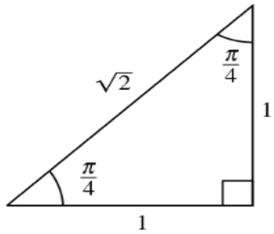
1.3.12.



$$45^\circ = 45^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{4} \text{ radian}$$

$$\cos 45^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

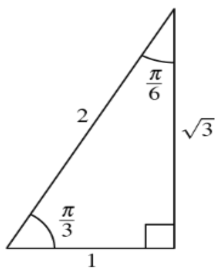
1.3.13.



$$\frac{\pi}{4} = \frac{\pi}{4}\left(\frac{180°}{\pi}\right) = 45°$$

$$\sec\frac{\pi}{4} = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

1.3.14.



$$\frac{\pi}{6} = \frac{\pi}{6}\left(\frac{180°}{\pi}\right) = 30°$$

$$\csc\ \frac{\pi}{6} = \frac{\text{hyp}}{\text{opp}} = 2$$

1.3.15.

(a) $\sin\ 20° \approx 0.3420$

(b) $\cos\ 70° \approx 0.3420$

1.3.16.

(a) $\tan 23.5° \approx 0.4348$

(b) $\cot 66.5° = \dfrac{1}{\tan\ 66.5°} \approx 0.4348$

1.3.17.

(a) $\sin 14.21° \approx 0.2455$

(b) $\csc 14.21° = \dfrac{1}{\sin 14.21°} \approx 4.0737$

1.3.18.

(a) $\cot 79.56° = \dfrac{1}{\tan\ 79.56°} \approx 0.1843$

(b) $\sec 79.56° = \dfrac{1}{\cos\ 79.56°} \approx 5.5186$

1.3.19.

(a) $\cos 4°\ 50'\ 15'' = \cos\left(4 + \dfrac{50}{60} + \dfrac{15}{3600}\right)^{\circ} \approx 0.9964$

(b) $\sec 4°\ 50'\ 15'' = \dfrac{1}{\cos\ 4°\ 50'\ 15''} \approx 1.0036$

1.3.20.

(a) $\sec 42°\ 12' = \sec 42.2° = \dfrac{1}{\cos\ 42.2°} \approx 1.3499$

(b) $\csc 48°\ 7' = \dfrac{1}{\sin\left(48 + \dfrac{7}{60}\right)^{\circ}} \approx 1.3432$

1.3.21.

(a) $\cot 17°\ 15' = \cot\left(17 + \dfrac{15}{60}\right)^{\circ} = \dfrac{1}{\tan 17.25°} \approx 3.2205$

(b) $\tan 17°\ 15' = \tan\left(17 + \dfrac{15}{60}\right)^{\circ} = \tan 17.25° \approx 0.3105$

1.3.22.

(a) $\sec 56°\ 8'\ 10'' = \sec\left(56 + \frac{8}{60} = \frac{10}{3600}\right)^{\circ} \approx 1.7946$

(b) $\cos 56°\ 8'\ 10'' = \cos\left(56 + \frac{8}{60} + \frac{10}{3600}\right)^{\circ} \approx 0.5572$

1.3.23.

$\sin 60° = \dfrac{\sqrt{3}}{2}$, $\cos 60° = \dfrac{1}{2}$

(a) $\sin 30° = \cos 60° = \dfrac{1}{2}$

(b) $\cos 30° = \sin 60° = \dfrac{\sqrt{3}}{2}$

(c) $\tan 60° = \dfrac{\sin\ 60°}{\cos\ 60°} = \sqrt{3}$

(d) $\cot 60° = \dfrac{\cos\ 60°}{\sin\ 60°} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

1.3.24.

$\sin 30° = \dfrac{1}{2}, \tan 30° = \dfrac{\sqrt{3}}{3}$

(a) $\csc 30° = \dfrac{1}{\sin 30°} = 2$

(b) $\cot 60° = \tan\left(90° - 60°\right) = \tan 30° = \dfrac{\sqrt{3}}{3}$

(c) $\cos 30° = \dfrac{\sin 30°}{\tan 30°} = \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{3}} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

(d) $\cot 30° = \dfrac{1}{\tan 30°} = \dfrac{3}{\sqrt{3}} = \dfrac{3\sqrt{3}}{3} = \sqrt{3}$

1.3.25.

$\cos\theta = \dfrac{1}{3}$

(a)
$$\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \sin^2\theta + \left(\frac{1}{3}\right)^2 &= 1 \\ \sin^2\theta &= \frac{8}{9} \\ \sin\theta &= \frac{2\sqrt{2}}{3} \end{aligned}$$

(b) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = 2\sqrt{2}$

(c) $\sec\theta = \dfrac{1}{\cos\theta} = 3$

(d) $\csc\left(90° - \theta\right) = \sec\theta = 3$

1.3.26.

$\sec\theta = 5$

(a) $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{5}$

(b) $1+\tan^2\theta = \sec^2\theta$

$1+\tan^2\theta = 5^2$

$\tan^2\theta = 24$

$\tan\theta = 2\sqrt{6}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{2\sqrt{6}} = \dfrac{\sqrt{6}}{12}$

(c) $\cot(90^\circ - \theta) = \tan\theta = 2\sqrt{6}$

(d) $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\dfrac{1}{5}\right)^2 = 1$

$\sin^2\theta = \dfrac{24}{25}$

$\sin\theta = \dfrac{2\sqrt{6}}{5}$

1.3.27.

$\cot\alpha = 3$

(a) $\tan\alpha = \dfrac{1}{\cot\alpha} = \dfrac{1}{3}$

(b) $\csc^2\alpha = 1+\cot^2\alpha$

$\csc^2\alpha = 1+3^2$

$\csc^2\alpha = 10$

$\csc\alpha = \sqrt{10}$

(c) $\cot(90^\circ - \alpha) = \tan\alpha = \dfrac{1}{3}$

(d) $\csc\alpha = \sqrt{10}$

$\sin\alpha = \dfrac{1}{\csc\alpha} = \dfrac{1}{\sqrt{10}} = \dfrac{\sqrt{10}}{10}$

1.3.28.

$$\cos\beta = \frac{\sqrt{7}}{4}$$

(a) $\sec\beta = \dfrac{1}{\cos\beta} = \dfrac{4\sqrt{7}}{7}$

(b) $\sin^2\beta + \cos^2\beta = 1$

$$\sin^2\beta + \left(\frac{\sqrt{7}}{4}\right)^2 = 1$$

$$\sin^2\beta = \frac{9}{16}$$

$$\sin\beta = \frac{3}{4}$$

(c) $\cot\beta = \dfrac{\cos\beta}{\sin\beta} = \dfrac{\frac{\sqrt{7}}{4}}{\frac{3}{4}} = \dfrac{\sqrt{7}}{3}$

(d) $\sin(90^\circ - \beta) = \cos\beta = \dfrac{\sqrt{7}}{4}$

1.3.29.

$$\tan\theta\cot\theta = \tan\theta\left(\frac{1}{\tan\theta}\right) = 1$$

1.3.30.

$$\cos\theta\sec\theta = \cos\theta\,\frac{1}{\cos\theta} = 1$$

1.3.31.

$$\tan\alpha\cos\alpha = \left(\frac{\sin\alpha}{\cos\alpha}\right)\cos\alpha = \sin\alpha$$

1.3.32.

$$\cot\alpha\sin\alpha = \frac{\cos\alpha}{\sin\alpha}\sin\alpha = \cos\alpha$$

1.3.33.

$$(1+\sin\theta)(1-\sin\theta) = 1-\sin^2\theta = \cos^2\theta$$

1.3.34.

$$\begin{aligned}(1+\cos\theta)(1-\cos\theta) &= 1-\cos^2\theta\\ &= (\sin^2\theta+\cos^2\theta)-\cos^2\theta\\ &= \sin^2\theta\end{aligned}$$

1.3.35.

$$\begin{aligned}(\sec\theta+\tan\theta)(\sec\theta-\tan\theta) &= \sec^2\theta-\tan^2\theta\\ &= (1+\tan^2\theta)-\tan^2\theta\\ &= 1\end{aligned}$$

1.3.36.

$$\begin{aligned}\sin^2\theta-\cos^2\theta &= \sin^2\theta-(1-\sin^2\theta)\\ &= \sin^2\theta-1+\sin^2\theta\\ &= 2\sin^2\theta-1\end{aligned}$$

1.3.37.

$$\begin{aligned}\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta} &= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\\ &= \frac{1}{\sin\theta\cos\theta}\\ &= \frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta}\\ &= \csc\theta\sec\theta\end{aligned}$$

1.3.38.

$$\begin{aligned}\frac{\tan\beta+\cot\beta}{\tan\beta} &= \frac{\tan\beta}{\tan\beta}+\frac{\cot\beta}{\tan\beta}\\ &= 1+\frac{\cot\beta}{\frac{1}{\cot\beta}}\\ &= 1+\cot^2\beta=\csc^2\beta\end{aligned}$$

1.3.39.

$$\cos 60^\circ=\frac{x}{18}$$

$$x=18\cos 60^\circ=18\left(\frac{1}{2}\right)=9$$

$$\sin 60^\circ=\frac{y}{18}$$

$$y=18\sin 60^\circ=18\frac{\sqrt{3}}{2}=9\sqrt{3}$$

1.3.40.

$$\tan 30^\circ = \frac{30}{x}$$
$$\frac{1}{\sqrt{3}} = \frac{30}{x}$$
$$x = 30\sqrt{3}$$
$$\sin 30^\circ = \frac{30}{r}$$
$$r = \frac{30}{\sin 30^\circ} = \frac{30}{\frac{1}{2}} = 60$$

1.3.41.

$$\tan 60^\circ = \frac{32}{x}$$
$$\sqrt{3} = \frac{32}{x}$$
$$\sqrt{3}x = 32$$
$$x = \frac{32}{\sqrt{3}} = \frac{32\sqrt{3}}{3}$$
$$\sin 60^\circ = \frac{32}{r}$$
$$r = \frac{32}{\sin 60^\circ}$$
$$r = \frac{32}{\frac{\sqrt{3}}{2}} = \frac{64\sqrt{3}}{3}$$

1.3.42.

$$\tan 45^\circ = \frac{20}{x}$$
$$x = \frac{20}{\tan 45^\circ}$$
$$x = \frac{20}{1} = 20$$
$$\sin 45^\circ = \frac{20}{r}$$
$$r = \frac{20}{\sin 45^\circ} = \frac{20}{\frac{\sqrt{2}}{2}} = 20\sqrt{2}$$

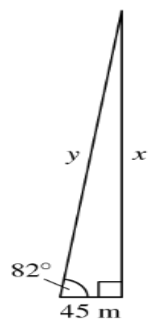
1.3.43.

(a) $\tan 82^\circ = \dfrac{x}{45}$

$x = 45 \tan 82^\circ$

Height of the building:

$123 + 45 \tan 82^\circ \approx 443.2$ meters



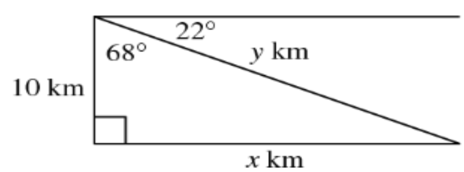
(b) Distance between friends:

$\cos 82^\circ = \dfrac{45}{y} \Rightarrow y = \dfrac{45}{\cos 82^\circ}$

≈ 323.34 meters

1.3.44.

(a)



(b) $\cos 68^\circ = \dfrac{10}{y}$

$\tan 68^\circ = \dfrac{x}{10}$

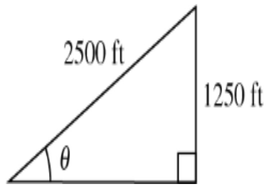
(c) $x = 10 \tan 68^\circ \approx 24.75$ km

(d) $y = \dfrac{10}{\cos 68^\circ} \approx 26.69$ km

1.3.45.

$\sin \theta = \dfrac{1250}{2500} = \dfrac{1}{2}$

$\theta = 30^\circ = \dfrac{\pi}{6}$



1.3.46.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 54° = \frac{w}{100}$$

$$w = 100 \tan 54° \approx 137.6 \text{ feet}$$

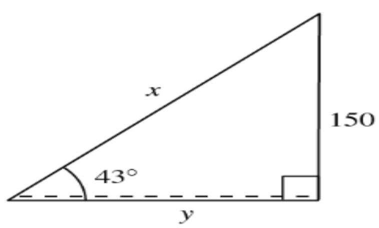
1.3.47.

(a) $\sin 43° = \dfrac{150}{x}$

$$x = \frac{150}{\sin 43°} \approx 219.9 \text{ ft}$$

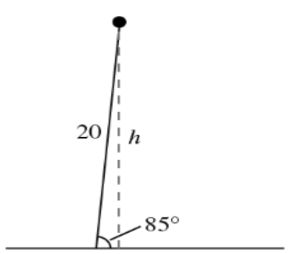
(b) $\tan 43° = \dfrac{150}{y}$

$$y = \frac{150}{\tan 43°} \approx 160.9 \text{ ft}$$



1.3.48.

(a)



(b) $\sin 85° = \dfrac{h}{20}$

$$h = 20 \sin 85° \approx 19.9 \text{ meters}$$

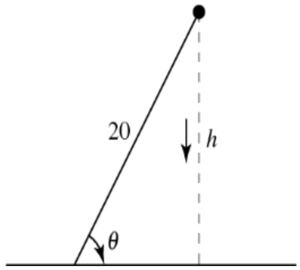
(c) The side of the triangle labeled h will become shorter.

(d)

Angle, θ	Height (in meters)
$\sin\theta$	19.7
$\cos\theta$	18.8
60°	17.3
50°	15.3
40°	12.9
30°	10.0
20°	6.8
10°	3.5

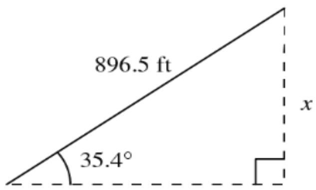
(e) The height of the balloon decreases.

As $\theta \to 0^\circ$, $h \to 0$.



1.3.49.

(a)



$$\sin 35.4^\circ = \frac{x}{896.5}$$
$$x = 896.5 \sin 35.4^\circ \approx 519.33 \text{ feet}$$

(b) Because the top of the incline is 1693.5 feet above sea level and the vertical rise of the inclined plane is 519.33 feet, the elevation of the lower end of the inclined plan is about $1693.5 - 519.33 = 1174.17$ feet.

1.3.50.

Ascent time:
$$d = rt$$
$$896.5 = 300t$$
$$3 \approx t$$

It takes about 3 minutes for the cars to get from the bottom to the top.

Vertical rate: $d = rt$

$$519.33 \approx r(3)$$
$$r \approx 173.11 \text{ ft/min}$$

1.3.51.

$\sin 60° \csc 60° = 1$

True.

$$\csc x = \frac{1}{\sin x} \Rightarrow \sin 60° \csc 60° = \sin 60°\left(\frac{1}{\sin 60°}\right)$$
$$= 1$$

1.3.52.

False.

Because $\sec 30° = \dfrac{1}{\cos 30°} = \dfrac{2\sqrt{3}}{3}$

and

$\sin 30° = \dfrac{1}{\csc 30°} = 2$, $\sec 30° \neq \csc 30°$.

1.3.53.

False. $\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} = \sqrt{2} \neq 1$

1.3.54.

True. $\cos 60° - \sin 30° = \dfrac{1}{2} - \dfrac{1}{2} = 0$

1.3.55.

False. $\dfrac{\sin 60°}{\sin 30°} = \dfrac{\cos 30°}{\sin 30°} = \cot 30° \approx 1.7321$

$\sin 2° \approx 0.0349$

1.3.56.

False.

$$\tan\left[(5°)^2\right] = \tan 25° \approx 0.466$$
$$\tan^2 5° = (\tan 5°)^2 \approx 0.008$$

1.3.57.

Yes. Given $\tan\theta, \sec\theta$ can be found from the identity

$1 + \tan^2\theta = \sec^2\theta.$

1.3.58.

(a) Side y is opposite $\theta.$

(b) Side y is adjacent to $90° - \theta.$

(c) Because side y is the side opposite angle θ and is the same side that is adjacent to the angle $90° - \theta, \sin\theta = \cos(90° - \theta).$

1.3.59.

$$\cos 60° = \frac{\text{adj}}{\text{hyp}} = \frac{1}{2}$$

1.3.60.

This is true because the corresponding sides of similar triangles are proportional.

1.3.61.

The point $(1, 3)$ lies in Quadrant I.

1.3.62.

The point $(9, -2)$ lies in Quadrant IV.

1.3.63.

The point $(-5, 4)$ lies in Quadrant II.

1.3.64.

The point $(-6, -7)$ lies in Quadrant III.

1.3.65.

$y = -x$ lies in Quadrants II and IV.

Sample points: $(-1, 1), (1, -1)$

1.3.66.

$y = \frac{1}{3}x$ lies in Quadrants I and III.

Sample points: $(3, 1), (-3, -1)$

1.3.67.

$2x - y = 0 \Leftrightarrow y = 2x$ lies in Quadrants I and III.

Sample points: $(1, 2), (-1, -2)$

1.3.68.

$4x + 3y = 0 \Leftrightarrow y = -\dfrac{4x}{3}$ lies in Quadrants II and IV.

Sample points: $(-3, 4), (3, -4)$

1.3.69.

(a) Coterminal angles for $\theta = 20°$

$20° + 360° = 380°$
$20° - 360° = -340°$

(b) Coterminal angles for $\theta = -30°$

$-30° + 360° = 330°$
$-30° - 360° = -390°$

1.3.70.

(a) Coterminal angles for $\theta = \dfrac{\pi}{12}$

$$\frac{\pi}{12} + 2\pi = \frac{25\pi}{12}$$
$$\frac{\pi}{12} - 2\pi = -\frac{23\pi}{12}$$

(b) Coterminal angles for $\theta = -\dfrac{\pi}{7}$

$$-\frac{\pi}{7} + 2\pi = \frac{13\pi}{7}$$
$$-\frac{\pi}{7} - 2\pi = -\frac{15\pi}{7}$$

1.3.71.

$$f(x) = \frac{1}{x^3 - x}$$
$$f(-x) = \frac{1}{(-x)^3 - (-x)} = \frac{1}{-x^3 + x} = \frac{-1}{x^3 - x}$$
$$= -f(x)$$

f is odd and symmetric with respect to the origin.

1.3.72.

$$f(x) = \frac{x^2}{4x^2 + 1}$$

$$f(-x) = \frac{(-x)^2}{4(-x)^2 + 1} = \frac{x^2}{4x^2 + 1} = f(x)$$

f is even and symmetric with respect to the y-axis.

1.3.73.

$$f(x) = 12x^2$$

$$f(-x) = 12(-x)^2 = 12x^2 = f(x)$$

f is even and symmetric with respect to the y-axis.

1.3.74.

$$f(x) = 5x + 4x^4$$

$$f(-x) = 5(-x) + 4(-x)^4 = -5x + 4x^4$$

f is neither even nor odd and has no symmetry.

SECTION 1.4 TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

1.4.1.

$\frac{y}{r}$

1.4.2.

csc θ

1.4.3.

$\frac{y}{x}$

1.4.4.

$\frac{r}{x}$

1.4.5.

cos θ

1.4.6.

cot θ

1.4.7.

Reference angle

1.1.8.

Sine and cosine are defined for all real numbers because the denominator $s = \sqrt{x^2 + y^2} \neq 0.$

1.4.9.

(a) $(x, y) = \left(-\sqrt{3}, -1\right)$

$r = \sqrt{3+1} = 2$

$$\sin\theta = \frac{y}{r} = -\frac{1}{2} \qquad \csc\theta = \frac{r}{y} = -2$$

$$\cos\theta = \frac{x}{r} = -\frac{\sqrt{3}}{2} \qquad \sec\theta = \frac{r}{x} = -\frac{2\sqrt{3}}{3}$$

$$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{3} \qquad \cot\theta = \frac{x}{y} = \sqrt{3}$$

(b) $(x, y) = (4, -1)$

$r = \sqrt{16+1} = \sqrt{17}$

$\sin\theta = \dfrac{y}{r} = -\dfrac{1}{\sqrt{17}} = -\dfrac{\sqrt{17}}{17}$ $\qquad$ $\csc\theta = \dfrac{r}{y} = -\sqrt{17}$

$\cos\theta = \dfrac{x}{r} = \dfrac{4}{\sqrt{17}} = \dfrac{4\sqrt{17}}{17}$ $\qquad$ $\sec\theta = \dfrac{r}{x} = \dfrac{\sqrt{17}}{4}$

$\tan\theta = \dfrac{y}{x} = -\dfrac{1}{4}$ $\qquad$ $\cot\theta = \dfrac{x}{y} = -4$

1.4.10.

(a) $x = 3,\ y = 1$

$r = \sqrt{3^2 + 1^2} = \sqrt{10}$

$\sin\theta = \dfrac{y}{r} = \dfrac{1}{\sqrt{10}} = \dfrac{\sqrt{10}}{10}$

$\cos\theta = \dfrac{x}{r} = \dfrac{3}{\sqrt{10}} = \dfrac{3\sqrt{10}}{10}$

$\tan\theta = \dfrac{y}{x} = \dfrac{1}{3}$

$\csc\theta = \dfrac{r}{y} = \dfrac{\sqrt{10}}{1} = \sqrt{10}$

$\sec\theta = \dfrac{r}{x} = \dfrac{\sqrt{10}}{3}$

$\cot\theta = \dfrac{x}{y} = \dfrac{3}{1} = 3$

(b) $(x, y) = \left(-2, \sqrt{21}\right)$

$r = \sqrt{(-2)^2 + \left(\sqrt{21}\right)^2} = \sqrt{4+21} = \sqrt{25} = 5$

$\sin\theta = \dfrac{y}{r} = \dfrac{\sqrt{21}}{5}$ $\qquad$ $\csc\theta = \dfrac{r}{y} = \dfrac{5}{\sqrt{21}} = \dfrac{5\sqrt{21}}{21}$

$\cos\theta = \dfrac{x}{r} = -\dfrac{2}{5}$ $\qquad$ $\sec\theta = \dfrac{r}{x} = -\dfrac{5}{2}$

$\tan\theta = \dfrac{y}{x} = -\dfrac{\sqrt{21}}{2}$ $\qquad$ $\cot\theta = \dfrac{x}{y} = -\dfrac{2\sqrt{21}}{21}$

1.4.11.

$$(x, y) = (5, 12)$$
$$r = \sqrt{25 + 144} = 13$$

$$\sin\theta = \frac{y}{r} = \frac{12}{13} \qquad \csc\theta = \frac{r}{y} = \frac{13}{12}$$
$$\cos\theta = \frac{x}{r} = \frac{5}{13} \qquad \sec\theta = \frac{r}{x} = \frac{13}{5}$$
$$\tan\theta = \frac{y}{x} = \frac{12}{5} \qquad \cot\theta = \frac{x}{y} = \frac{5}{12}$$

1.4.12.

$$x = 8,\ y = 15$$
$$r = \sqrt{8^2 + 15^2} = 17$$

$$\sin\theta = \frac{y}{r} = \frac{15}{17} \qquad \csc\theta = \frac{r}{y} = \frac{17}{15}$$
$$\cos\theta = \frac{x}{r} = \frac{8}{17} \qquad \sec\theta = \frac{r}{x} = \frac{17}{8}$$
$$\tan\theta = \frac{y}{x} = \frac{15}{8} \qquad \cot\theta = \frac{x}{y} = \frac{8}{15}$$

1.4.13.

$$x = -5,\ y = -2$$
$$r = \sqrt{(-5)^2 + (-2)^2} = \sqrt{29}$$

$$\sin\theta = \frac{y}{r} = \frac{-2}{\sqrt{29}} = -\frac{2\sqrt{29}}{29}$$
$$\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{29}} = -\frac{5\sqrt{29}}{29}$$
$$\tan\theta = \frac{y}{x} = \frac{-2}{-5} = \frac{2}{5}$$
$$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{-2} = -\frac{\sqrt{29}}{2}$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{29}}{-5} = -\frac{\sqrt{29}}{5}$$
$$\cot\theta = \frac{x}{y} = \frac{-5}{-2} = \frac{5}{2}$$

1.4.14.

$$(x, y) = (-4, 10)$$
$$r = \sqrt{16 + 100} = 2\sqrt{29}$$
$$\sin\theta = \frac{y}{r} = \frac{5\sqrt{29}}{29}$$
$$\cos\theta = \frac{x}{r} = -\frac{2\sqrt{29}}{29}$$
$$\tan\theta = \frac{y}{x} = -\frac{5}{2}$$
$$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{5}$$
$$\sec\theta = \frac{r}{x} = -\frac{\sqrt{29}}{2}$$
$$\cot\theta = \frac{x}{y} = -\frac{2}{5}$$

1.4.15.

$$(x, y) = (-5.4, 7.2)$$
$$r = \sqrt{29.16 + 51.84} = 9$$
$$\sin\theta = \frac{y}{r} = \frac{7.2}{9} = \frac{4}{5} \qquad \csc\theta = \frac{r}{y} = \frac{9}{7.2} = \frac{5}{4}$$
$$\cos\theta = \frac{x}{r} = -\frac{5.4}{9} = -\frac{3}{5} \qquad \sec\theta = \frac{r}{x} = -\frac{9}{5.4} = -\frac{5}{3}$$
$$\tan\theta = \frac{y}{x} = -\frac{7.2}{5.4} = -\frac{4}{3} \qquad \tan\theta = \frac{x}{y} = -\frac{5.4}{7.2} = -\frac{3}{4}$$

1.4.16.

$$x = 3\frac{1}{2} = \frac{7}{2},\ y = -2\sqrt{15}$$
$$r = \sqrt{\left(\frac{7}{2}\right)^2 + \left(-2\sqrt{15}\right)^2} = \sqrt{\frac{49}{4} + 60} = \sqrt{\frac{289}{4}} = \frac{17}{2}$$

$$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{15}}{\frac{7}{2}} = -\frac{4\sqrt{15}}{7} \qquad \csc\theta = \frac{r}{y} = -\frac{7}{4\sqrt{15}} = -\frac{7\sqrt{15}}{60}$$

$$\cos\theta = \frac{x}{r} = \frac{\frac{7}{2}}{\frac{17}{2}} = \frac{7}{17} \qquad \sec\theta = \frac{r}{x} = \frac{17}{7}$$

$$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{15}}{\frac{7}{2}} = -\frac{4\sqrt{15}}{7} \qquad \cot\theta = \frac{x}{y} = \frac{7}{-4\sqrt{15}} = -\frac{7\sqrt{15}}{60}$$

1.4.17.

$\sin\theta > 0 \Rightarrow \theta$ lies in Quadrant I or in Quadrant II.

$\cos\theta > 0 \Rightarrow \theta$ lies in Quadrant I or in Quadrant IV.

$\sin\theta > 0$ and $\cos\theta > 0 \Rightarrow \theta$ lies in Quadrant I.

1.4.18.

$\sin\theta < 0 \Rightarrow \theta$ lies in Quadrant III or in Quadrant IV.

$\cos\theta < 0 \Rightarrow \theta$ lies in Quadrant II or in Quadrant III.

$\sin\theta < 0$ and $\cos\theta < 0 \Rightarrow \theta$ lies in Quadrant III.

1.4.19.

$\csc\theta > 0 \Rightarrow \theta$ lies in Quadrant I or in Quadrant II.

$\tan\theta < 0 \Rightarrow \theta$ lies in Quadrant II or in Quadrant III.

$\csc\theta > 0$ and $\tan\theta < 0 \Rightarrow \theta$ lies in Quadrant II.

1.4.20.

$\sec\theta > 0 \Rightarrow \theta$ lies in Quadrant I or in Quadrant IV.

$\cot\theta < 0 \Rightarrow \theta$ lies in Quadrant II or in Quadrant IV.

$\sec\theta > 0$ and $\cot\theta < 0 \Rightarrow \theta$ lies in Quadrant IV.

1.4.21.

$\tan\theta > 0$ and $\sin\theta > 0 \Rightarrow \theta$ is in Quadrant I $\Rightarrow x > 0$ and $y > 0$.

$$\tan\theta = \frac{y}{x} = \frac{15}{8} \Rightarrow r = 17$$

$$\sin\theta = \frac{y}{r} = \frac{15}{7} \qquad \sec\theta = \frac{r}{x} = \frac{17}{8}$$

$$\cos\theta = \frac{x}{r} = \frac{8}{17} \qquad \cot\theta = \frac{x}{y} = \frac{8}{15}$$

$$\csc\theta = \frac{r}{y} = \frac{17}{15}$$

1.4.22.

$\cos\theta > 0$ and $\tan\theta < 0 \Rightarrow \theta$ is in Quadrant IV $\Rightarrow x > 0$ and $y < 0$.

$$\cos\theta = \frac{x}{r} = \frac{8}{17} \Rightarrow y = -15$$

$$\sin\theta = \frac{y}{r} = \frac{-15}{17} = -\frac{15}{17} \qquad \sec\theta = \frac{17}{8}$$

$$\tan\theta = \frac{y}{x} = \frac{-15}{8} = -\frac{15}{8} \qquad \cot\theta = -\frac{8}{15}$$

$$\csc\theta = -\frac{17}{15}$$

1.4.23.

$\sin\theta = 0.6 = \dfrac{3}{5}$ and θ in Quadrant II

$$x^2 + y^2 = r^2$$

$$\sin\theta = \frac{3}{5}, \qquad x = -\sqrt{r^2 - y^2}$$

$$x = -\sqrt{5^2 - 3^2} = -4$$

$$\cos\theta = \frac{x}{r} = -\frac{4}{5} \qquad \sec\theta = \frac{r}{x} = -\frac{5}{4}$$

$$\tan\theta = \frac{y}{x} = -\frac{3}{4} \qquad \cot\theta = \frac{x}{y} = -\frac{4}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{3}$$

1.4.24.

$$\cos\theta = \frac{x}{r} = -0.8 = \frac{-4}{5} \Rightarrow y = -3,\ \theta \text{ in Quadrant III}$$

$\sin\theta = \frac{y}{r} = -\frac{3}{5}$ $\qquad$ $\sec\theta = -\frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{3}{4}$ $\qquad$ $\cot\theta = \frac{4}{3}$

$\csc\theta = -\frac{5}{3}$

1.4.25.

$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2} + \pi n$

$\csc\theta = 1 \Rightarrow \theta = \frac{\pi}{2} + 2\pi n$

$y = 1,\ x = 0,\ r = 1$

$\sin\theta = \frac{y}{r} = 1$

$\tan\theta = \frac{y}{x}$ is undefined.

$\sec\theta$ is undefined.

$\cot\theta = 0$

1.4.26.

$\sin\theta = 0 \Rightarrow \theta = 0 + \pi n$

$\sec\theta = -1 \Rightarrow \theta = \pi + 2\pi n$

$y = 0,\ \ x = -1,\ \ r = 1$

$\cos\theta = \frac{x}{r} = -1$.

$\tan\theta = \frac{y}{x} = 0$

$\csc\theta$ is undefined.

$\cot\theta$ is undefined.

1.4.27.

$\cot\theta$ is undefined, $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2} \Rightarrow y = 0 \Rightarrow \theta = \pi$

$\sin\theta = 0$ $\qquad$ $\csc\theta$ is undefined.

$\cos\theta = -1$ $\qquad$ $\sec\theta = -1$

$\tan\theta = 0$ $\qquad$ $\cot\theta$ is undefined.

1.4.28.

$\tan\theta$ is undefined $\Rightarrow \theta = n\pi + \dfrac{\pi}{2}$

$\pi \le \theta \le 2\pi \Rightarrow \theta = \dfrac{3\pi}{2},\ x = 0,\ y = -r$

$\sin\theta = \dfrac{y}{r} = \dfrac{-r}{r} = -1$ $\qquad \csc\theta = \dfrac{r}{y} = -1$

$\cos\theta = \dfrac{x}{r} = \dfrac{0}{r} = 0$ $\qquad$ $\sec\theta$ is undefined.

$\tan\theta$ is undefined. $\qquad \cot\theta = \dfrac{x}{y} = \dfrac{0}{y} = 0$

1.4.29.

To find a point on the terminal side of θ, use any point on the line $y = -x$ that lies in Quadrant II. $(-1, 1)$ is one such point.

$x = -1,\ y = 1,\ r = \sqrt{2}$

$\sin\theta = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ $\qquad \csc\theta = \sqrt{2}$

$\cos\theta = -\dfrac{1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$ $\qquad \sec\theta = -\sqrt{2}$

$\tan\theta = -1$ $\qquad \cot\theta = -1$

1.4.30.

Let $x > 0$.

$\left(-x, -\dfrac{1}{3}x\right)$, Quadrant III

$$r = \sqrt{x^2 + \frac{1}{9}x^2} = \frac{\sqrt{10}x}{3}$$

$\sin\theta = \dfrac{y}{r} = \dfrac{\left(-\frac{1}{3}\right)x}{\frac{\sqrt{10}x}{3}} = -\dfrac{\sqrt{10}}{10}$ $\qquad \csc\theta = \dfrac{r}{y} = \dfrac{\frac{\sqrt{10}x}{3}}{\left(-\frac{1}{3}\right)x} = -\sqrt{10}$

$\cos\theta = \dfrac{x}{r} = \dfrac{-x}{\frac{\sqrt{10}x}{3}} = -\dfrac{3\sqrt{10}}{10}$ $\qquad \sec\theta = \dfrac{r}{x} = \dfrac{\frac{\sqrt{10}x}{3}}{-x} = -\dfrac{\sqrt{10}}{3}$

$\tan\theta = \dfrac{y}{x} = \dfrac{\left(-\frac{1}{3}\right)x}{-x} = \dfrac{1}{3}$ $\qquad \cot\theta = \dfrac{x}{y} = \dfrac{-x}{\left(-\frac{1}{3}\right)x} = 3$

1.4.31.

To find a point on the terminal side of θ, use any point on the line $y = 2x$ that lies in Quadrant I. (1, 2) is one such point.

$$x = 1,\ y = 2,\ y = \sqrt{5}$$

$$\sin\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \qquad \csc\theta = \frac{\sqrt{5}}{2} = \frac{\sqrt{5}}{2}$$

$$\cos\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} \qquad \sec\theta = \frac{\sqrt{5}}{1} = \sqrt{5}$$

$$\tan\theta = \frac{2}{1} = 2 \qquad \cot\theta = \frac{1}{2}$$

1.4.32.

Let $x > 0$.

$$4x + 3y = 0 \Rightarrow y = -\frac{4}{3}x$$

$\left(x, -\frac{4}{3}x\right)$, Quadrant IV

$$r = \sqrt{x^2 + \frac{16}{9}x^2} = \frac{5}{3}x$$

$$\sin\theta = \frac{y}{r} = \frac{\left(-\frac{4}{3}\right)x}{\frac{5}{3}x} = -\frac{4}{5} \qquad \csc\theta = -\frac{5}{4}$$

$$\cos\theta = \frac{x}{r} = \frac{x}{\frac{5}{3}x} = \frac{3}{5} \qquad \sec\theta = \frac{5}{3}$$

$$\tan\theta = \frac{y}{x} = \frac{\left(-\frac{4}{3}\right)x}{x} = -\frac{4}{3} \qquad \cot\theta = -\frac{3}{4}$$

1.4.33.

$$(x, y) = (-1, 0),\ r = 1$$

$$\sin\pi = \frac{y}{r} = \frac{0}{1} = 0$$

1.4.34.

$$(x, y) = (0, -1), r = 1$$
$$\csc \frac{3\pi}{2} = \frac{1}{-1} = -1$$

1.4.35.

$$(x, y) = (0, -1), r = 1$$
$$\sec \frac{3\pi}{2} = \frac{r}{x} = \frac{1}{0} \Rightarrow \text{undefined}$$

1.4.36.

$$(x, y) = (-1, 0), r = 1$$
$$\sec \pi = \frac{r}{x} = \frac{1}{-1} = -1$$

1.4.37.

$$(x, y) = (0, 1), r = 1$$
$$\sin \frac{\pi}{2} = \frac{y}{r} = \frac{1}{1} = 1$$

1.4.38.

$$(x, y) = (-1, 0), r = 1$$

$$\cot \pi = \frac{-1}{0} \text{ undefined.}$$

1.4.39.

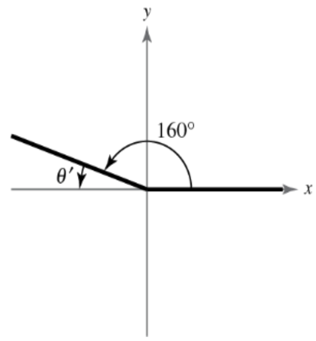
$$(x, y) = (0, 1)$$
$$\cot \frac{9\pi}{2} = \frac{x}{y} = \frac{0}{1} = 0$$

1.4.40.

$$(x, y) = (0, -1)$$
$$\tan\left(-\frac{\pi}{2}\right) = \frac{y}{x} = \frac{-1}{0} \text{ is undefined.}$$

1.4.41.

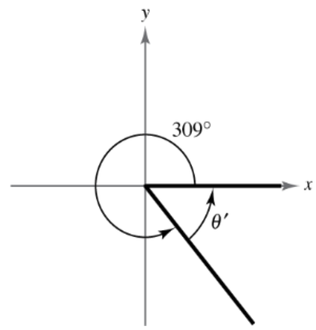
$$\theta = 160°$$
$$\theta' = 180° - 160° = 20°$$



1.4.42.

$\theta = 309°$

$\theta' = 360° - 309° = 51°$

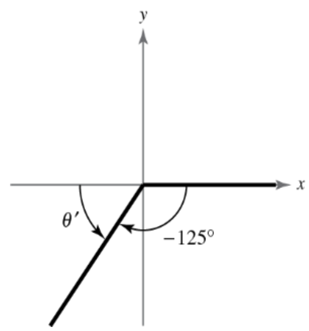


1.4.43.

$\theta = -125°$

$360° - 125° = 235° \quad (\text{coterminal angle})$

$\theta' = 235° - 180° = 55°$

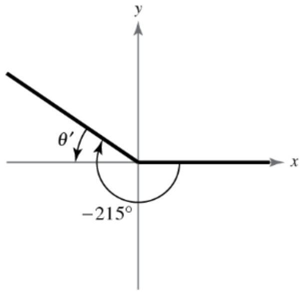


1.4.44.

$\theta = -215°$

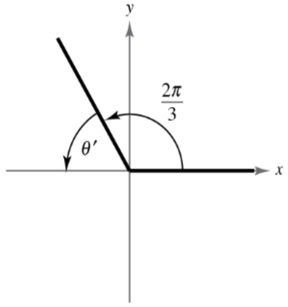
$360° - 215° = 145° \quad (\text{coterminal angle})$

$\theta' = 180° - 145° = 35°$



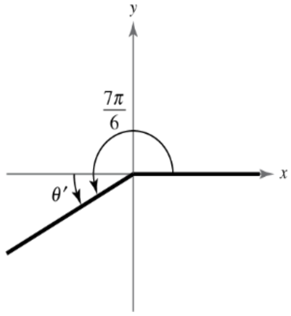
1.4.45.

$$\theta = \frac{2\pi}{3}$$
$$\theta' = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$



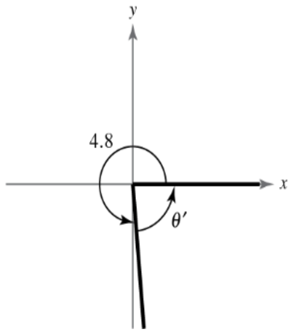
1.4.46.

$$\theta = \frac{7\pi}{6}$$
$$\theta' = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$$



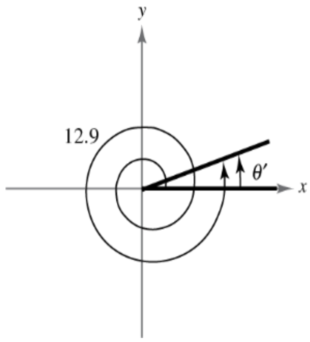
1.4.47.

$$\theta = 4.8$$
$$\theta' = 2\pi - 4.8 \approx 1.48$$



1.4.48.

$$\theta = 12.9$$
$$\theta' = 12.9 - 4\pi \approx 0.3336$$



1.4.49.

$\theta = 225°, \theta' = 45°,$ Quadrant III

$$\sin 225° = -\sin 45° = -\frac{\sqrt{2}}{2}$$
$$\cos 225° = -\cos 45° = -\frac{\sqrt{2}}{2}$$
$$\tan 225° = \tan 45° = 1$$

1.4.50.

$\theta = 300°, \theta' = 360° - 300° = 60°$ in Quadrant IV.

$$\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}$$
$$\cos 300° = \cos 60° = \frac{1}{2}$$
$$\tan 300° = -\tan 60° = -\sqrt{3}$$

1.4.51.

$\theta = 750°, \theta' = 30°,$ Quadrant I

$$\sin 750° = \sin 30° = \frac{1}{2}$$

$$\cos 750° = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\tan 750° = \tan 30° = \frac{\sqrt{3}}{3}$$

1.4.52.

$\theta = 675°, \theta' = 45°,$ Quadrant IV

$$\sin 675° = -\sin 45° = -\frac{\sqrt{2}}{2}$$

$$\cos 675° = \cos 45° = \frac{\sqrt{2}}{2}$$

$$\tan 675° = -\tan 45° = -1$$

1.4.53.

$\theta = -120°, \theta' = 60°,$ Quadrant III

$$\sin(-120°) = -\sin 60° = -\frac{\sqrt{3}}{2}$$

$$\cos(-120°) = -\cos 60° = -\frac{1}{2}$$

$$\tan(-120°) = \tan 60° = \sqrt{3}$$

1.4.54.

$\theta = -570°, \theta' = 30°$ in Quadrant II.

$$\sin(-570°) = \sin 30° = \frac{1}{2}$$

$$\cos(-570°) = -\cos 30° = -\frac{\sqrt{3}}{2}$$

$$\tan(-570°) = -\tan 30° = -\frac{\sqrt{3}}{3}$$

1.4.55.

$\theta = \frac{2\pi}{3}, \theta' = \frac{\pi}{3}$ in Quadrant II

$$\sin \frac{2\pi}{3} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$
$$\cos \frac{2\pi}{3} = -\cos \frac{\pi}{3} = -\frac{1}{2}$$
$$\tan \frac{2\pi}{3} = -\tan \frac{\pi}{3} = -\sqrt{3}$$

1.4.56.

$\theta = \frac{3\pi}{4}, \theta' = \frac{\pi}{4}$ in Quadrant II

$$\sin \frac{3\pi}{4} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\cos \frac{3\pi}{4} = -\cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$
$$\tan \frac{3\pi}{4} = -\tan \frac{\pi}{4} = -1$$

1.4.57.

$\theta = \frac{11\pi}{4}, \theta' = \frac{\pi}{4}$, Quadrant II

$$\sin \frac{11\pi}{4} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\cos \frac{11\pi}{4} = -\cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$
$$\tan \frac{11\pi}{4} = -\tan \frac{\pi}{4} = -1$$

1.4.58.

$\theta = \frac{13\pi}{6}, \theta' = \frac{\pi}{6}$ in Quadrant I

$$\sin \frac{13\pi}{6} = \sin \frac{\pi}{6} = \frac{1}{2}$$
$$\cos \frac{13\pi}{6} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$
$$\tan \frac{13\pi}{6} = \tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

1.4.59.

$$\theta = -\frac{\pi}{6}, \theta' = \frac{\pi}{6} \text{ in Quadrant IV}$$

$$\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$$

$$\cos\left(-\frac{\pi}{6}\right) = -\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\tan\left(-\frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

1.4.60.

$$\theta = -\frac{23\pi}{4}, \theta' = \frac{\pi}{4} \text{ in Quadrant I}$$

$$\sin\left(-\frac{23\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos\left(-\frac{23\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\tan\left(-\frac{23\pi}{4}\right) = \tan\frac{\pi}{4} = 1$$

1.4.61.

$$\sin\theta = -\frac{3}{5}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos^2\theta = 1 - \left(-\frac{3}{5}\right)^2$$

$$\cos^2\theta = 1 - \frac{9}{25}$$

$$\cos^2\theta = \frac{16}{25}$$

$\cos\theta > 0$ in Quadrant IV.

$$\cos\theta = \frac{4}{5}$$

1.4.62.

$$\cot\theta = -3$$
$$1+\cot^2\theta = \csc^2\theta$$
$$1+(-3)^2 = \csc^2\theta$$
$$10 = \csc^2\theta$$

$\csc\theta > 0$ in Quadrant II.

$$\csc\theta = \sqrt{10}$$

1.4.63.

$$\tan\theta = \frac{3}{2}$$
$$\sec^2\theta = 1+\tan^2\theta$$
$$\sec^2\theta = 1+\left(\frac{3}{2}\right)^2$$
$$\sec^2\theta = 1+\frac{9}{4}$$
$$\sec^2\theta = \frac{13}{4}$$

$\sec\theta < 0$ in Quadrant III.

$$\sec\theta = -\frac{\sqrt{13}}{2}$$

1.4.64.

$$\csc\theta = -2$$
$$1+\cot^2\theta = \csc^2\theta$$
$$\cot^2\theta = \csc^2\theta - 1$$
$$\cot^2\theta = (-2)^2 - 1$$
$$\cot^2\theta = 3$$

$\cot\theta < 0$ in Quadrant IV.

$$\cot\theta = -\sqrt{3}$$

1.4.65.

$$\cos\theta = \frac{5}{8}$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta + \left(\frac{5}{8}\right)^2 = 1$$
$$\sin^2\theta = 1 - \frac{25}{64}$$
$$\sin^2\theta = \frac{39}{64}$$

$\sin\theta > 0$ in Quadrant I.

$$\sin\theta = \frac{\sqrt{39}}{8}$$
$$\csc\theta = \frac{1}{\sin\theta}$$
$$\csc\theta = \frac{8\sqrt{39}}{39}$$

1.4.66.

$$\sec\theta = -\frac{9}{4}$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$\tan^2\theta = \sec^2\theta - 1$$
$$\tan^2\theta = \left(-\frac{9}{4}\right)^2 - 1$$
$$\tan^2\theta = \frac{65}{16}$$

$\tan\theta > 0$ in Quadrant III.

$$\tan\theta = \frac{\sqrt{65}}{4}$$
$$\cot\theta = \frac{1}{\tan\theta}$$
$$\cot\theta = \frac{4\sqrt{65}}{65}$$

1.4.67.

(a) $\sin\theta = \dfrac{1}{2} \Rightarrow$ reference angle is 30° or $\dfrac{\pi}{6}$ and θ is in Quadrant I or Quadrant II.

Values in degrees: $30^\circ,\ 150^\circ$

Values in radian: $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$

(b) $\sin\theta = \dfrac{1}{2} \Rightarrow$ reference angle is 30° or $\dfrac{\pi}{6}$ and θ is in Quadrant III or Quadrant IV.

Values in degrees: $210^\circ, 330^\circ$

Values in radians: $\dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

1.4.68.

(a) $\cos\theta = \dfrac{\sqrt{2}}{2} \Rightarrow$ reference angle is 45° or $\dfrac{\pi}{4}$ and θ is in Quadrant I or IV.

Values in degrees: $45^\circ, 315^\circ$

Values in radians: $\dfrac{\pi}{4}, \dfrac{7\pi}{4}$

(b) $\cos\theta = -\dfrac{\sqrt{2}}{2} \Rightarrow$ reference angle is 45° or $\dfrac{\pi}{4}$ and θ is in Quadrant II or III.

Values in degrees: $135^\circ, 225^\circ$

Values in radians: $\dfrac{3\pi}{4}, \dfrac{5\pi}{4}$

1.4.69.

(a) $\tan\theta = 1 \Rightarrow$ reference angle is 45° or $\dfrac{\pi}{4}$ and θ is in Quadrant I or Quadrant III.

Values in degrees: $45^\circ, 225^\circ$

Values in radians: $\dfrac{\pi}{4}, \dfrac{5\pi}{4}$

(b) $\cot\theta = -\sqrt{3} \Rightarrow$ reference angle is 30° or $\dfrac{\pi}{6}$ and θ is in Quadrant II or Quadrant IV.

Values in degrees: $150^\circ, 330^\circ$

Values in radians: $\frac{5\pi}{6}, \frac{11\pi}{6}$

1.4.70.

(a) $\cot\theta = 0 \Rightarrow \tan\theta$ is undefined$\Rightarrow$no reference angle, so the solutions are quadrants angles. Values in degrees: $90°, 270°$

Values in radians: $\frac{\pi}{2}, \frac{3\pi}{2}$

(b) $\sec\theta = -\sqrt{2} \Rightarrow \cos\theta = -\frac{\sqrt{2}}{2}$ reference angle is $45°$ or $\frac{\pi}{4}$ and θ is in Quadrant II or Quadrant III.

Values in degrees: $135°, 225°$

Values in radians: $\frac{3\pi}{4}, \frac{5\pi}{4}$

1.4.71.

(a) Using technology, the equations are as follows.

$B = 18.534\sin(0.569t - 2.609) + 59.703$

$F = 33.567\sin(0.530t - 2.109) + 40.564$

(b) February $(t = 2)$: $B = 18.534\sin[(0.569)(2) - 2.609] + 59.703 \approx 41.3°\text{F}$

$F = 33.567\sin[(0.530)(2) - 2.109] + 40.564 \approx 11.5°\text{F}$

April $(t = 4)$: $B = 18.534\sin[(0.569)(4) - 2.609] + 59.703 \approx 53.6°\text{F}$

$F = 33.567\sin[(0.530)(4) - 2.109] + 40.564 \approx 40.9°\text{F}$

May $(t = 5)$: $B = 18.534\sin[(0.569)(5) - 2.609] + 59.703 \approx 64.0°\text{F}$

$F = 33.567\sin[(0.530)(5) - 2.109] + 40.564 \approx 57.9°\text{F}$

July $(t = 7)$: $B = 18.534\sin[(0.569)(7) - 2.609] + 59.703 \approx 77.9°\text{F}$

$F = 33.567\sin[(0.530)(7) - 2.109] + 40.564 \approx 74.1°\text{F}$

September $(t = 9)$: $B = 18.534\sin[(0.569)(8) - 2.609] + 59.703 \approx 70.6°\text{F}$

$F = 33.567\sin[(0.530)(9) - 2.109] + 40.564 \approx 56.1°\text{F}$

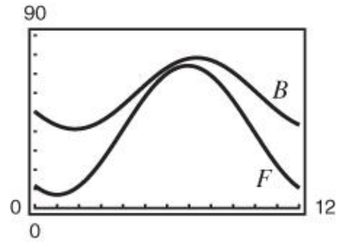
October $(t = 10)$: $B = 18.534\sin[(0.569)(10) - 2.609] + 59.703 \approx 60.8°\text{F}$

$F = 33.567\sin[(0.530)(10) - 2.109] + 40.564 \approx 38.9°\text{F}$

December $(t = 12)$: $B = 18.534\sin[(0.569)(12) - 2.609] + 59.703 \approx 43.4°\text{F}$

$F = 33.567\sin[(0.530)(12) - 2.109] + 40.564 \approx 10.5°\text{F}$

(c)



Answers will vary.

1.4.72.

$$S = 23.1 + 0.442t + 4.3\cos\frac{\pi t}{6}$$

(a) $t = 2;\ S = 23.1 + 0.442(2) + 4.3\cos\frac{\pi(2)}{6}$

$= 26{,}134$ snowboards

(b) $t = 14;\ S = 23.1 + 0.442(14) + 4.3\cos\frac{\pi(14)}{6}$

$\approx 31{,}438$ snowboards

(c) $t = 6;\ S = 23.1 + 0.442(6) + 4.3\cos\frac{\pi(6)}{6}$

$\approx 21{,}452$ snowboards

(d) $t = 18;\ S = 23.1 + 0.442(18) + 4.3\cos\frac{\pi(18)}{6}$

$\approx 26{,}756$ snowboards

1.4.73.

$$\sin\theta = \frac{6}{d} \Rightarrow d = \frac{6}{\sin\theta}$$

(a) $\theta = 30°$

$$d = \frac{6}{\sin 30°}$$

$$= \frac{6}{1/2} = 12 \text{ miles}$$

(b) $\theta = 90°$

$$d = \frac{6}{\sin 90°} = \frac{6}{1} = 6 \text{ miles}$$

(c) $\theta = 120°$

$$d = \frac{6}{\sin 120°}$$

$$= \frac{6}{\sqrt{3}/2} \approx 6.9 \text{ miles}$$

1.4.74.

$y(t) = 2 \cos 6t$

(a) $y(0) = 2 \cos 0 = 2$ centimeters

(b) $y\left(\frac{1}{4}\right) = 2 \cos\left(\frac{3}{2}\right) \simeq 0.14$ centimeter

(c) $y\left(\frac{1}{2}\right) = 2 \cos 3 \simeq -1.98$ centimeters

1.4.75.

False. In each of the four quadrants, the sign of the secant function and the cosine function will be the same because they are reciprocals of each other.

1.4.76.

False. The reference angle is always acute by definition. For θ in Quadrant II, $\theta' = 180° - \theta$. For θ in Quadrant III, $\theta' = \theta - 180°$. For θ in Quadrant IV, $\theta' = 360° - \theta$.

1.4.77.

Answers will vary.

1.4.78.

As θ increases from $0°$ to $90°$, x decreases from 12 cm to 0 cm and y increases from 0 cm to 12 cm. Therefore, $\sin \theta = \frac{y}{12}$ increases from 0 to 1, $\cos \theta = \frac{y}{12}$ decreases from 1 to 0, and $\tan \theta = \frac{y}{x}$ increases without bound (and is undefined at $\theta = 90°$).

1.4.79.

$g(x) = \sqrt{x+12} + 9$

(a) Parent function: $f(x) = \sqrt{x}$

(b) g is a reflection in the x-axis, a left shift of 12 units, and an upward shift of 9 units of the graph of f.

1.4.80.

$g(x) = 15(x+2)^3 - 4$

(a) Parent function: $f(x) = x^3$

(b) g is a vertical stretch, a left shift of 2 units, and a downward shift of 4 units of the graph of f.

1.4.81.

$(-4, 5)$ is a relative maximum.

1.4.82.

$(1, -3)$ is a relative minimum.

1.4.83.

$$y = x^2 + x - 132$$
$$0 = x^2 + x - 132$$
$$0 = (x+12)(x-11)$$
$$x = -12, 11$$
$$x\text{-intercepts: } (-12, 0), (11, 0)$$

1.4.84.

$$y = 2x^7 - x^5 - x^3$$
$$0 = x^3(2x^4 - x^2 - 1)$$
$$0 = x^3(2x^2 + 1)(x^2 - 1)$$
$$x = 0, \quad \pm 1$$
$$x\text{-intercepts: } (0, 0), (1, 0), (-1, 0)$$

SECTION 1.5 GRAPHS OF SINE AND CONSINE FUNCTIONS

1.5.1.

cycle

1.5.2.

amplitude

1.5.3.

The constant *d* of $y = \sin x + d$ is a vertical shift of *d* units.

1.5.4.

$$y = 3\cos\left(2x - \frac{\pi}{4}\right)$$

$$2x - \frac{\pi}{4} = 0$$
$$2x = \frac{\pi}{4}$$
$$x = \frac{\pi}{8}$$
$$2x - \frac{\pi}{4} = 2\pi$$
$$2x = \frac{9\pi}{4}$$
$$x = \frac{9\pi}{8}$$

The interval $\left[\frac{\pi}{8}, \frac{9\pi}{8}\right]$ corresponds to one cycle of the graph of

$y = 3\cos\left(2x - \frac{\pi}{4}\right)$.

1.5.5.

$$y = 2\sin 5x$$

Period: $\frac{2\pi}{5}$

Amplitude: $|2| = 2$

1.5.6.

$y = 3 \cos 2x$

Period: $\frac{2\pi}{2} = \pi$

Amplitude: $|3| = 3$

1.5.7.

$y = \frac{3}{4} \cos \frac{\pi x}{2}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{\pi/2} = 4$

Amplitude: $\left|\frac{3}{4}\right| = \frac{3}{4}$

1.5.8.

$y = -5 \sin \frac{\pi x}{3}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{\pi/3} = 6$

Amplitude: $|-5| = 5$

1.5.9.

$y = -\frac{1}{2} \sin \frac{5x}{4}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{5/4} = \frac{8\pi}{5}$

Amplitude: $\left|-\frac{1}{2}\right| = \frac{1}{2}$

1.5.10.

$y = \frac{1}{4} \sin \frac{x}{6}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{1/6} = 12\pi$

Amplitude: $\left|\frac{1}{4}\right| = \frac{1}{4}$

1.5.11.

$$y = -\frac{5}{3}\cos\frac{\pi x}{12}$$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{\pi/12} = 24$

Amplitude: $\left|-\dfrac{5}{3}\right| = \dfrac{5}{3}$

1.5.12.

$$y = -\frac{2}{5}\cos 10\pi x$$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{10\pi} = \dfrac{1}{5}$

Amplitude: $\left|-\dfrac{2}{5}\right| = \dfrac{2}{5}$

1.5.13.

$f(x) = \cos x$

$g(x) = \cos 5x$

The period of g is one-fifth the period of f.

1.5.14.

$f(x) = \sin x$

$g(x) = 2\sin x$

The amplitude of g is twice the amplitude of f.

1.5.15.

$f(x) = \cos 2x$

$g(x) = -\cos 2x$

g is a reflection of the graph of f in the x-axis.

1.5.16.

$f(x) = \sin 3x$

$g(x) = \sin(-3x)$

g is a reflection of the graph of f in the y-axis.

1.5.17.

$f(x) = \sin x$
$g(x) = \sin(x - \pi)$

g is a horizontal shift to the right π units of the graph of *f*.

1.5.18.

$f(x) = \cos x$
$g(x) = \cos(x + \pi)$

g is a horizontal shift to the left π units of the graph of *f*.

1.5.19.

$f(x) = \sin 2x$
$f(x) = 3 + \sin 2x$

g is a vertical shift three units upward of the graph of *f*.

1.5.20.

$f(x) = \cos 4x$
$g(x) = -2 + \cos 4x$

g is a vertical shift two units downward of the graph of *f*.

1.5.21.

The graph of *g* has twice the amplitude as the graph of *f*. The period is the same.

1.5.22.

The period of *g* is one-third the period of *f*.

1.5.23.

The graph of *g* is a horizontal shift π units to the right of the graph of *f*.

1.5.24.

The graph of *g* is a vertical shift two units upward of the graph of *f*.

1.5.25.

$f(x) = \sin x$

Period: $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

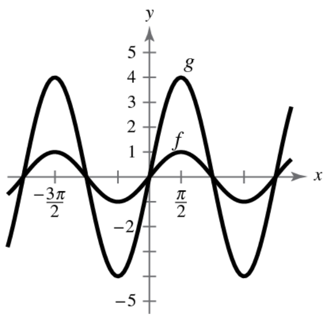
Amplitude: 1

Symmetry: origin

Key points: Intercept	Maximum	Intercept	Minimum	Intercept
$(0, 0)$	$\left(\frac{\pi}{2}, 1\right)$	$(\pi, 0)$	$\left(\frac{3\pi}{2}, -1\right)$	$(2\pi, 0)$

Because $g(x) = 4 \sin x = 4f(x)$, the graph of $g(x)$ is the graph of $f(x)$, but stretched vertically by a factor of 4.
The amplitude of $g(x)$ is four times the amplitude of $f(x)$.

Generate key points for the graph $g(x)$ by multiplying the x-coordinate of each key point of $f(x)$ by 4.



1.5.26.

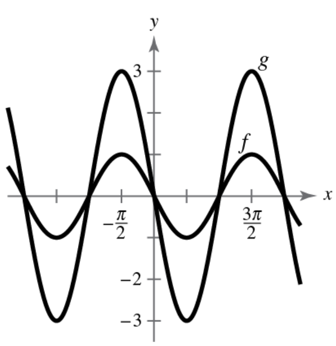
$f(x) = -\sin x$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{1} = 2\pi$

Amplitude: 1
Symmetry: origin

Key points: Intercept	Minimum	Intercept	Maximum	Intercept
$(0, 0)$	$\left(\frac{\pi}{2}, -1\right)$	$(\pi, 0)$	$\left(\frac{3\pi}{2}, 1\right)$	$(2\pi, 0)$

Because $g(x) = -3 \sin x = 3f(x)$, generate key points for the graph of $g(x)$ by multiplying the y-coordinate of each key point of $f(x)$ by 3.



1.5.27.

$f(x) = \cos\ x$

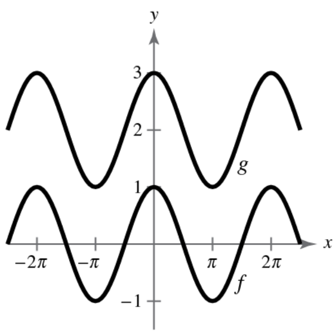
Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{1} = 2\pi$

Amplitude: $|1| = 1$

Symmetry: y-axis

Key points: Maximum	Intercept	Minimum	Intercept	Maximum
$(0, 1)$	$\left(\dfrac{\pi}{2}, 0\right)$	$(\pi, -1)$	$\left(\dfrac{3\pi}{2}, 0\right)$	$(2\pi, 1)$

Because $g(x) = 2 + \cos\ x = f(x) + 2$, the graph of $g(x)$ is the graph of $f(x)$, but translated upward by two units. Generate key points of $g(x)$ by adding 2 to the y-coordinate of each key point of $f(x)$.



1.5.28.

$f(x) = \cos x$

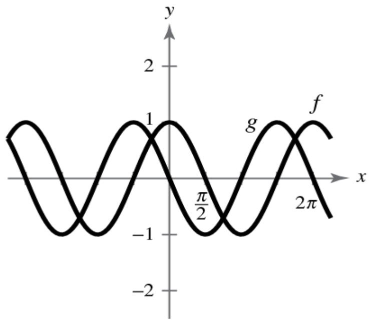
Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{1} = 2\pi$

Amplitude: 1

Symmetry: y-axis

Key points: Maximum	Intercept	Minimum	Intercept	Maximum
$(0, 1)$	$\left(\dfrac{\pi}{2}, 0\right)$	$(\pi, -1)$	$\left(\dfrac{3\pi}{2}, 0\right)$	$(2\pi, 1)$

Because $g(x) = \cos\left(x + \dfrac{\pi}{2}\right) = f\left(x + \dfrac{\pi}{2}\right)$, the graph of $g(x)$ is the graph of $f(x)$, but with a phase shift (horizontal translation) of $-\dfrac{\pi}{2}$. Generate key points for the graph of $g(x)$ by shifting each key point of $f(x)$ $\dfrac{\pi}{2}$ units to the left.



1.5.29.

$f(x) = -\cos x$

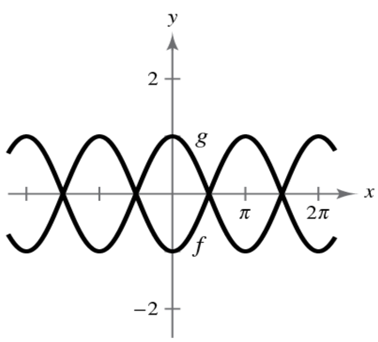
Period: $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

Amplitude: 1

Symmetry: y-axis

Key points:	Minimum	Intercept	Maximum	Intercept	Minimum
	$(0, -1)$	$\left(\frac{\pi}{2}, 0\right)$	$(\pi, 1)$	$\left(\frac{3\pi}{2}, 0\right)$	$(2\pi, -1)$

Because $g(x) = -\cos(x - \pi) = f(x - \pi)$, the graph of $g(x)$ is the graph of $f(x)$, but with a phase shift (horizontal translation) of π. Generate key points for the graph of $g(x)$ by shifting each key point of $f(x)$ π units to the right.



1.5.30.

$f(x) = \sin x$

Period: $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

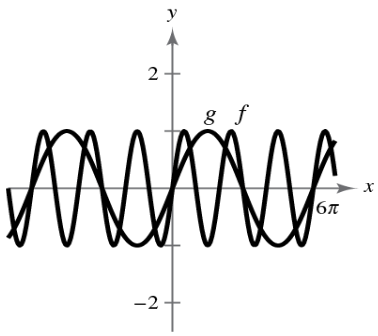
Amplitude: 1

Symmetry: origin

Key points:	Intercept	Maximum	Intercept	Minimum	Intercept
	$(0, 0)$	$\left(\frac{\pi}{2}, 1\right)$	$(\pi, 0)$	$\left(\frac{3\pi}{2}, -1\right)$	$(2\pi, 0)$

Because $g(x) = \sin\left(\frac{x}{3}\right) = f\left(\frac{x}{3}\right)$, the graph of $g(x)$ is the graph of $f(x)$, but stretched horizontally by a factor of 3.

Generate key points for the graph of $g(x)$ by multiplying the x-coordinate of each key point of $f(x)$ by 3.



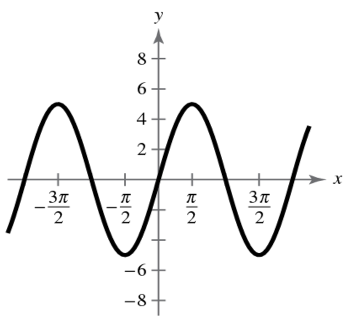
1.5.31.

$y = 5 \sin x$

Period: 2π

Amplitude: 5

Key points: $(0, 0)$, $\left(\frac{\pi}{2}, 5\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, -5\right)$, $(2\pi, 0)$



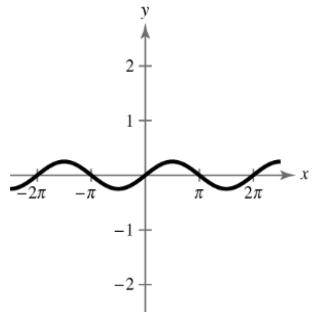
1.5.32.

$y = \frac{1}{4} \sin x$

Period: 2π

Amplitude: $\frac{1}{4}$

Key points: $(0, 0)$, $\left(\frac{\pi}{2}, \frac{1}{4}\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, -\frac{1}{4}\right)$, $(2\pi, 0)$



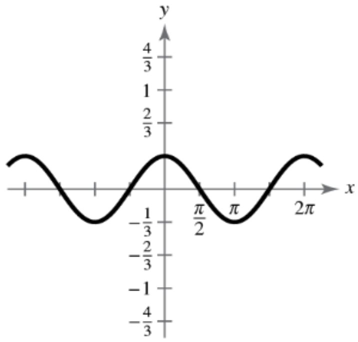
1.5.33.

$y = \frac{1}{3}\cos x$

Period: 2π

Amplitude: $\frac{1}{3}$

Key points: $\left(0, \frac{1}{3}\right)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\pi, -\frac{1}{3}\right)$, $\left(\frac{3\pi}{2}, 0\right)$, $\left(2\pi, \frac{1}{3}\right)$



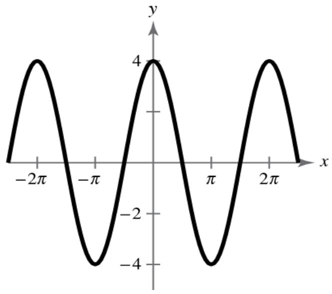
1.5.34.

$y = 4\cos x$

Period: 2π

Amplitude: 4

Key points: $(0, 4)$, $\left(\frac{\pi}{2}, 0\right)$, $(\pi, -4)$, $\left(\frac{3\pi}{2}, 0\right)$, $(2\pi, 4)$



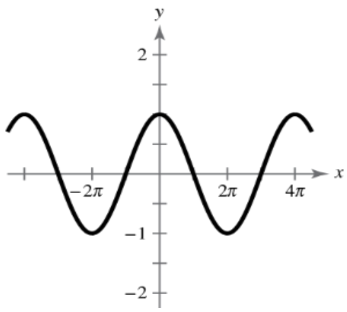
1.5.35.

$$y = \cos\frac{x}{2}$$

Period $\dfrac{2\pi}{1/2} = 4\pi$

Amplitude: 1

Key points: $(0, 1), (\pi, 0), (2\pi, -1), (3\pi, 0), (4\pi, 1)$



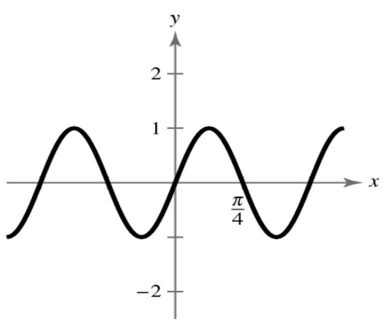
1.5.36.

$$y = \sin 4x$$

Period: $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$

Amplitude: 1

Key points: $(0, 0), \left(\dfrac{\pi}{8}, 1\right), \left(\dfrac{\pi}{4}, 0\right), \left(\dfrac{3\pi}{8}, -1\right), \left(\dfrac{\pi}{2}, 0\right)$



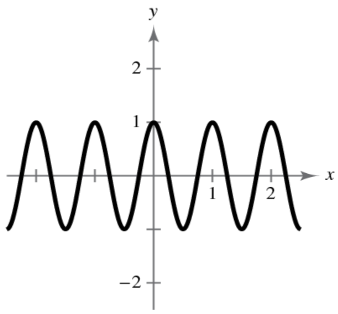
1.5.37.

$y = \cos 2\pi x$

Period: $\dfrac{2\pi}{2\pi} = 1$

Amplitude: 1

Key points: $(0, 1), \left(\dfrac{1}{4}, 0\right), \left(\dfrac{1}{2}, -1\right), \left(\dfrac{3}{4}, 0\right)$



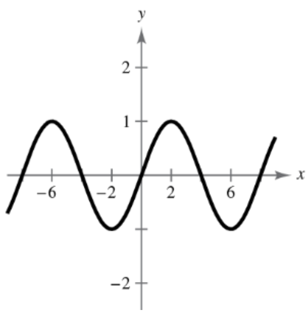
1.5.38.

$y = \sin \dfrac{\pi x}{4}$

Period: $\dfrac{2\pi}{\pi/4} = 8$

Amplitude: 1

Key points: $(0, 0), (2, 1), (4, 0), (6, -1), (8, 0)$



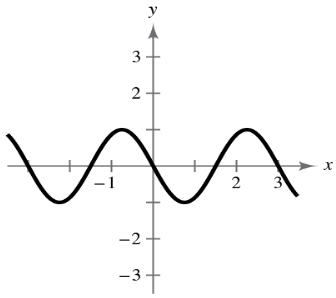
1.5.39.

$y = -\sin \dfrac{2\pi x}{3}$

Period: $\dfrac{2\pi}{2\pi/3} = 3$

Amplitude: 1

Key points: $(0, 0), \left(\frac{3}{4}, -1\right), \left(\frac{3}{2}, 0\right), \left(\frac{9}{4}, 1\right), (3, 0)$



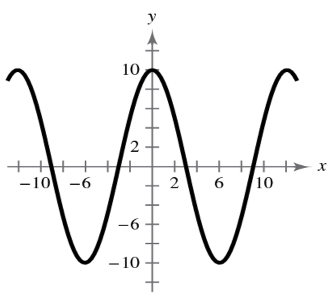
1.5.40.

$$y = 10 \cos \frac{\pi x}{6}$$

Period: $\dfrac{2\pi}{\pi/6} = 12$

Amplitude: 10

Key points: $(0, 10), (3, 0), (6, -10), (9, 0), (12, 10)$



1.5.41.

$$y = 3 \sin(x + \pi) - 3$$

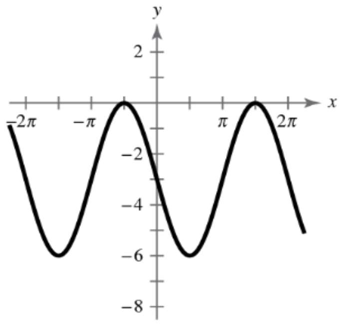
Period: 2π

Amplitude: 3

Shift: Set $x + \pi = 0$ and $x + \pi = 2\pi$

$x = -\pi$ $\quad$ $x = \pi$

Key points: $(-\pi, -3), \left(-\frac{\pi}{2}, 0\right), (0, -3), \left(\frac{\pi}{2}, -6\right), (\pi, -3)$



1.5.42.

$$y = -3\sin(6x + \pi)$$

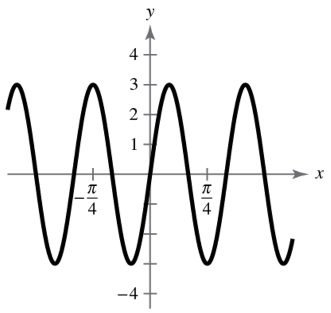
Period: $\dfrac{2\pi}{6} = \dfrac{\pi}{3}$

Amplitude: 3

Shift: Set $6x + \pi = 0$ and $6x + \pi = 2\pi$

$$x = -\frac{\pi}{6} \qquad x = \frac{\pi}{6}$$

Key points: $\left(-\dfrac{\pi}{6}, 0\right), \left(-\dfrac{\pi}{12}, -3\right), (0, 0), \left(\dfrac{\pi}{12}, 3\right), \left(\dfrac{\pi}{6}, 0\right)$

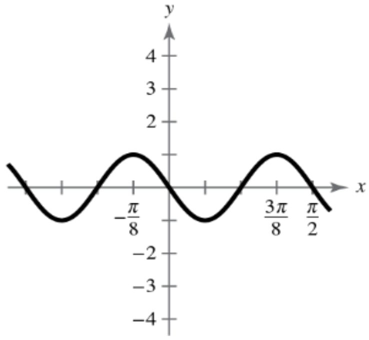


1.5.43.

$$g(x) = \sin(4x - \pi)$$

(a) $g(x)$ is obtained by a horizontal shrink and a phase shift of $\dfrac{\pi}{4}$. One cycle of $g(x)$ corresponds to the interval $\left[\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right]$.

(b)



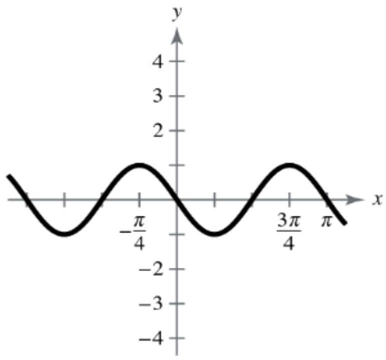
(c) $g(x) = f(4x - \pi)$ where $f(x) = \sin x$

1.5.44.

$g(x) = \sin(2x + \pi)$

(a) $g(x)$ is obtained by a horizontal shrink and a phase shift of $-\dfrac{\pi}{2}$. One cycle of $g(x)$ corresponds to the interval $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

(b)



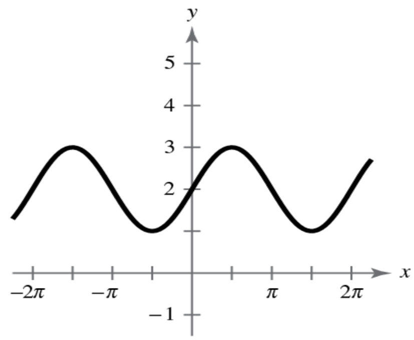
(c) $g(x) = f(2x + \pi)$ where $f(x) = \sin x$

1.5.45.

$g(x) = \cos\left(x - \dfrac{\pi}{2}\right) + 2$

(a) $g(x)$ is obtained by shifting $f(x)$ two units upward and a phase shift of $\dfrac{\pi}{2}$. One cycle of $g(x)$ corresponds to the interval $[\pi, 3\pi]$.

(b)



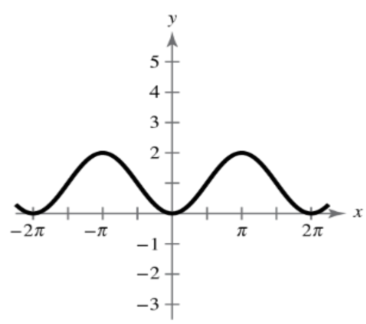
(c) $g(x) = f\left(x - \frac{\pi}{2}\right) + 2$ where $f(x) = \cos x$

1.5.46.

$g(x) = 1 + \cos(x + \pi)$

(a) $g(x)$ is obtained by shifting $f(x)$ one unit upward and a phase shift of $-\pi$. One cycle of $g(x)$ corresponds to the interval $[-\pi, \pi]$.

(b)



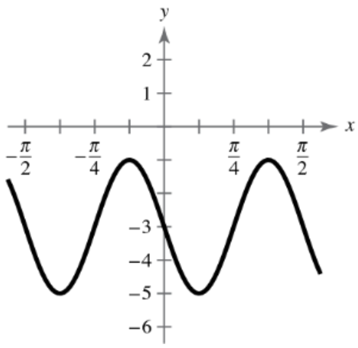
(c) $g(x) = f(x + \pi) + 1$ where $f(x) = \cos x$

1.5.47.

$g(x) = 2\sin(4x - \pi) - 3$

(a) $g(x)$ is obtained by a vertical stretch, a horizontal shrink, a phase shift of $\frac{\pi}{4}$, and shifting $f(x)$ three units downward. One cycle of $g(x)$ corresponds to the interval $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.

(b)



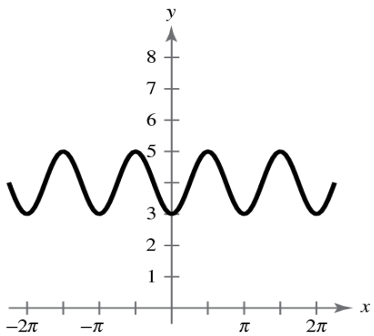
(c) $g(x) = 2f(4x - \pi) - 3$ where $f(x) = \sin\ x$

1.5.48.

$$g(x) = 4 - \sin\left(2x + \frac{\pi}{2}\right)$$

(a) $g(x)$ is obtained by a horizontal shrink, a phase shift of $-\frac{\pi}{4}$, reflecting $f(x)$ over the x-axis and shifting $f(x)$ four units upward. One cycle of $g(x)$ corresponds to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

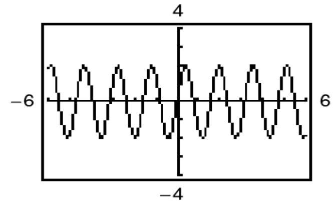
(b)



(c) $g(x) = 4 - f(2x + \pi)$ where $f(x) = \sin x$

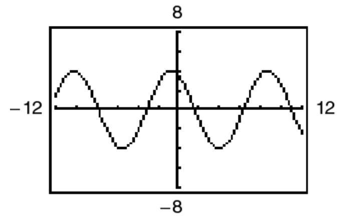
1.5.49.

$$y = -2\ \sin(4x + \pi)$$



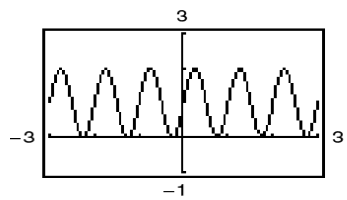
1.5.50.

$$y = -4\sin\left(\frac{2}{3}x = \frac{\pi}{3}\right)$$



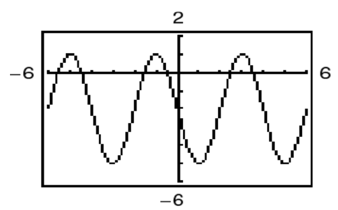
1.5.51.

$$y = \cos\left(2\pi x - \frac{\pi}{2}\right) + 1$$



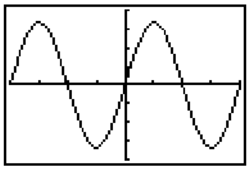
1.5.52.

$$y = 3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) - 2$$



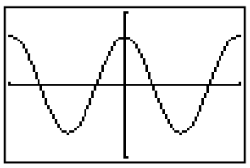
1.5.53.

$$y = -0.1\sin\left(\frac{\pi x}{10} + \pi\right)$$



1.5.54.

$$y = \frac{1}{100}\cos 120\pi t$$



1.5.55.

$f(x) = a\cos x + d$

Amplitude: $\frac{1}{2}\left[3 - (-1)\right] = 2 \Rightarrow a = 2$

$3 = 2\cos 0 + d$

$d = 3 - 2 = 1$

$a = 2, \ \ d = 1$

1.5.56.

$f(x) = a\cos x + d$

Amplitude: $\dfrac{1 - (-3)}{2} = 2 \Rightarrow a = 2$

$1 = 2\cos 0 + d$

$d = 1 - 2 = -1$

$a = 2, \ \ d = -1$

1.5.57.

$f(x) = a \ \cos \ x + d$

Amplitude: $\frac{1}{2}\left[8 - 0\right] = 4$

Reflected in the x-axis: $a = -4$

$0 = -4\cos 0 + d$

$d = 4$

$a = -4, d = 4$

1.5.58.

$f(x) = a \cos x + d$

Amplitude: $\dfrac{-2-(-4)}{2} = 1$

Reflected in the x-axis: $a = -1$

$-4 = -1 \cos 0 + d$

$d = -3$

$a = -1, d = -3$

1.5.59.

$y = a \sin(bx - c)$

Amplitude: $|a| = |3|$

Since the graph is reflected in the x-axis, we have $a = -3$.

Period: $\dfrac{2\pi}{b} = \pi \Rightarrow b = 2$

Phase shift: $c = 0$

$a = -3, b = 2, c = 0$

1.5.60.

$y = a \sin(bx - c)$

Amplitude: $2 \Rightarrow a = 2$

Period: 4π

$\dfrac{2\pi}{b} = 4\pi \Rightarrow b = \dfrac{1}{2}$

Phase shift: $c = 0$

$a = 2,\ b = \dfrac{1}{2},\ c = 0$

1.5.61.

$y = a \sin(bx - c)$

Amplitude: $a = 2$

Period: $2\pi \Rightarrow b = 1$

Phase shift: $bx - c = 0$ when $x = -\dfrac{\pi}{4}$

$(1)\left(-\dfrac{\pi}{4}\right) - c = 0 \Rightarrow c = -\dfrac{\pi}{4}$

$a = 2, b = 1, c = -\frac{\pi}{4}$

1.5.62.

$y = a \sin(bx - c)$

Amplitude: $2 \Rightarrow a = 2$

Period: 4

$\frac{2\pi}{b} = 4 \Rightarrow b = \frac{\pi}{2}$

Phase shift: $\frac{c}{b} = 1 \Rightarrow \frac{c}{\pi/2} = 1 \Rightarrow c = \frac{\pi}{2}$

$a = 2, b = \frac{\pi}{2}, c = \frac{\pi}{2}$

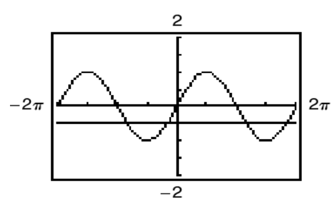
1.5.63.

$y_1 = \sin x$

$y_2 = -\frac{1}{2}$

In the interval $[-2\pi, 2\pi]$,

$y_1 = y_2$ when $x = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$.



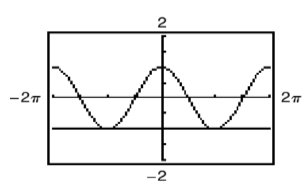
1.5.64.

$y_1 = \cos x$

$y_2 = -1$

In the interval $[-2\pi, 2\pi]$,

$y_1 = y_2$ when $x = \pi, -\pi$.



Answers for 65–68 are sample answers.

1.5.65.

$$f(x) = 2\sin(2x - \pi) + 1$$

1.5.66.

$$f(x) = 3\sin\left(\frac{1}{2}x + \frac{\pi}{8}\right) - 1$$

1.5.67.

$$f(x) = \cos(2x + 2\pi) - \tfrac{3}{2}$$

1.5.68.

$$f(x) = 3\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right) + 2$$

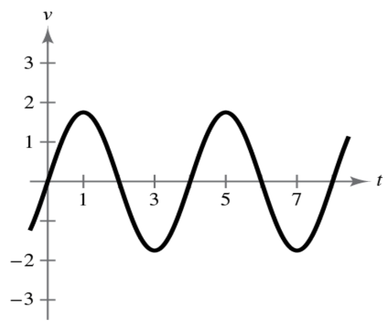
1.5.69.

$$v = 1.75\sin\frac{\pi t}{2}$$

(a) Period $= \dfrac{2\pi}{\pi/2} = 4$ seconds

(b) $\dfrac{1\text{ cycle}}{4\text{ seconds}} \cdot \dfrac{60\text{ seconds}}{1\text{ minute}} = 15$ cycles per minute

(c)



1.5.70.

$$y = 0.85 \sin \frac{\pi t}{3}$$

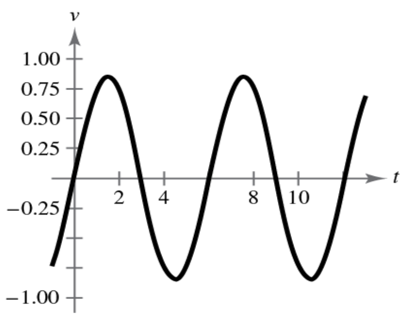
(a) Time for one cycle $= \dfrac{2\pi}{\pi/3} = 6$ seconds

(b) Cycles per min $= \dfrac{60}{6} = 10$ cycles per minute

(c) Amplitude: 0.85; Period: 6

Key points:

$(0, 0), \left(\dfrac{3}{2}, 0.85\right), (3, 0), \left(\dfrac{9}{2}, -0.85\right), (6, 0)$



1.5.71.

$$P = 100 - 20 \cos \frac{5\pi t}{3}$$

(a) Period: $\dfrac{2\pi}{(5\pi)/3} = \dfrac{6}{5}$ seconds

(b) $\dfrac{1 \text{ heartbeat}}{6/5 \text{ seconds}} \cdot \dfrac{60 \text{ seconds}}{1 \text{ minute}} = 50$ heartbeats per minute

1.5.72.

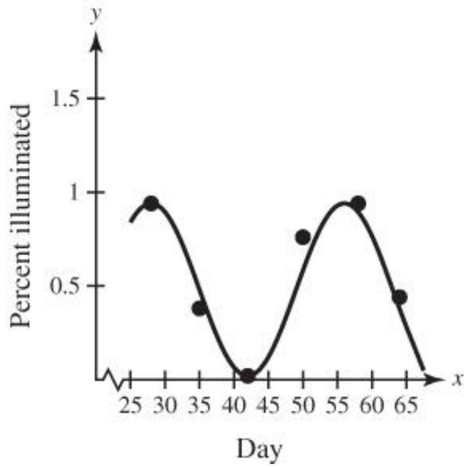
$$y = 0.001 \sin 880\pi t$$

(a) Period: $\dfrac{2\pi}{880\pi} = \dfrac{1}{440}$ second

(b) $f = \dfrac{1}{p} = 440$ cycles per second

1.5.73.

(a) and (c)



The model fits the data well.

(b) Amplitude: $\alpha = \frac{1}{2}(\text{max.percent} - \text{min.percent})$

$= \frac{1}{2}(0.94 - 0.02) = 0.46$

Period: $p = 2(\text{day of min.percent} - \text{day of max.percent})$

$= 2(42 - 28) = 28$

$b = \frac{2\pi}{p} = \frac{2\pi}{28} = \frac{\pi}{14}$

The maximum percent occurs 27 days after January 1, so the left endpoint of one cycle is $c/b = 28$. So, the value of c is

$$\frac{c}{b} = 28 \rightarrow \frac{c}{\pi/14} = 28 \rightarrow c = 28\left(\frac{\pi}{14}\right) \rightarrow c = 2\pi.$$

The average percent illuminated is $\frac{1}{2}(0.94 + 0.02) = 0.48,$ so it follows that $d = 0.48.$ A model for the percent of the moon's face illuminated is $y = 0.46\cos\left(\frac{\pi}{14}x - 2\pi\right) + 0.48.$

(d) The period is $p = 28$ days.

(e) Because March 27, 2024, is the 87th day of the year, $x = 87.$ So, the percent of the moon's face illuminated is $y = 0.46\cos\left(\frac{\pi}{14}(87) - 2\pi\right) + 0.48 \approx 84, \text{or } 84\%.$

1.5.74.

(a) Amplitude: $\alpha = \frac{1}{2}(\text{max. temp.} - \text{min. temp.})$

$= \frac{1}{2}(77.7 - 15.7) = 31$

Period: $p = 2(\text{month of max. temp.}) - (\text{month of min. temp.})$

$= 2(7 - 1) = 12$

$b = \frac{2\pi}{p} = \frac{2\pi}{12} = \frac{\pi}{6}$

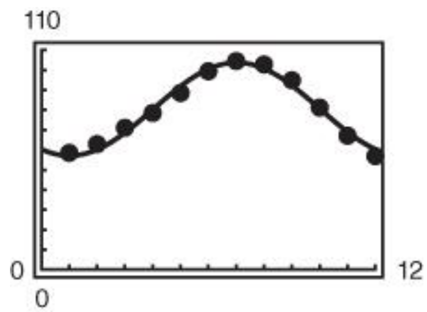
Because the maximum temperature occurs in the seventh month,

$\frac{c}{b} = 7 \Rightarrow c = \frac{7\pi}{6}$.

The average temperature is $\frac{1}{2}(77.7 + 15.7) = 46.7°$, so $d = 46.7$.

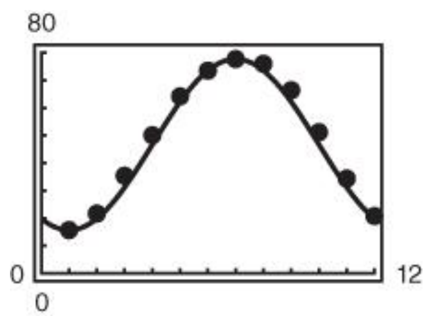
So, $I(t) = 46.7 + 31.0\cos\left(\frac{\pi t}{6} - \frac{7\pi}{6}\right)$.

(b)



The model fits the data well.

(c)



The model fits the data well.

(d) Las Vegas: $80.5°$ F; International Falls: $46.7°$ F; The constant term gives the annual average normal daily maximum temperature.

(e) The period of each model is 12 months. This is what you would expect because the time period is 1 year, or 12 months.

(f) The amplitude determines the variability. The greater the amplitude, the greater the variability in temperature. So, International Falls has the greater variability in temperature (31.0 > 23.3).

1.5.75.

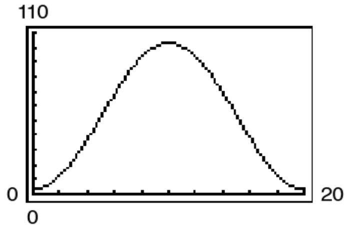
(a) $\text{Period} = \dfrac{2\pi}{\left(\dfrac{\pi}{10}\right)} = 20 \text{ seconds}$

The wheel takes 20 seconds to revolve once.

(b) Amplitude: 50 feet

The radius of the wheel is 50 feet.

(c)



1.5.76.

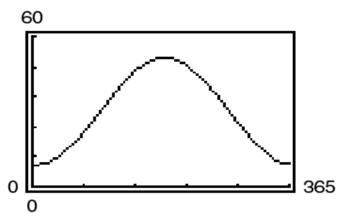
$$C = 30.3 + 21.6 \sin\left(\frac{2\pi t}{365} + 10.9\right)$$

(a) $\text{Period} = \dfrac{2\pi}{\dfrac{2\pi}{365}} = 365$

Yes, this is what is expected because there are 365 days in a year.

(b) 30.3 gallons; the average daily fuel consumption is given by the amount of the vertical shift (from 0) which is given by the constant 30.3.

(c)



The consumption exceeds 40 gallons per day when $124 < t < 252$.

1.5.77.

False. $y = \frac{1}{2}\cos 2x$ has an amplitude that is half that of $y = \cos x$. For $y = a \cos bx$, the amplitude is $|a|$.

1.5.78.

True. $y = -\cos x$ is a reflection of $y = \sin\left(x + \frac{\pi}{2}\right)$ in the x-axis. So,

$$-\cos x = -\sin\left(x + \frac{\pi}{2}\right).$$

1.5.79.

The key points are for the graph of $y = \cos x$. The key points for the graph of $y = \sin x$ on the interval $[0, 2\pi]$ are $(0, 0)$, $\left(\frac{\pi}{2}, 1\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, -1\right)$, and $(2\pi, 0)$.

1.5.80.

(a) For $c \neq 0$, the value of c shifts the graph of $f(x) = \sin x$ c units to the left when $c < 0$ and c units to the right when $c > 0$.

(b) An intercept of the graph $y = -\cos\left(x + \frac{\pi}{4}\right)$ occurs at $\left(\frac{\pi}{4}, 0\right)$. Because the graph of $y = \sin\left(x - \frac{\pi}{4}\right)$ also has an intercept at $\left(\frac{\pi}{4}, 0\right)$, the green graph is equivalent to $y = -\cos\left(x + \frac{\pi}{4}\right)$.

1.5.81.

$y = x^2 + 4$

(a)

x	0	1	2	3	4
y	4	5	8	13	20

(b) $y = (-x)^2 + 4 \Rightarrow y = x^2 + 4$

$\Rightarrow y$-axis symmetry

$-y = x^2 + 4 \Rightarrow y = -x^2 - 4$

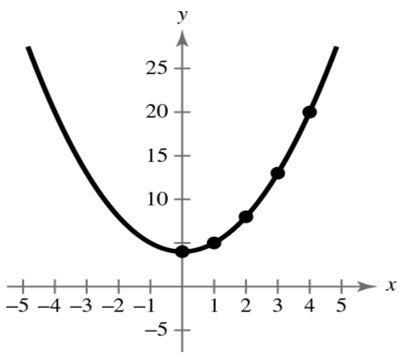
$\Rightarrow$ No x-axis symmetry

$-y = (-x)^2 + 4 \Rightarrow -y = x^2 + 4$

$\Rightarrow y = -x^2 - 4$

$\Rightarrow$ No origin symmetry

(c)



1.5.82.

$y^2 = 1 - x$

(a)

x	1	0	−3	−8	−15
y	0	1	2	3	4

(b) $y^2 = 1 - (-x) \Rightarrow y^2 = 1 + x$

$\Rightarrow$ No y-axis symmetry

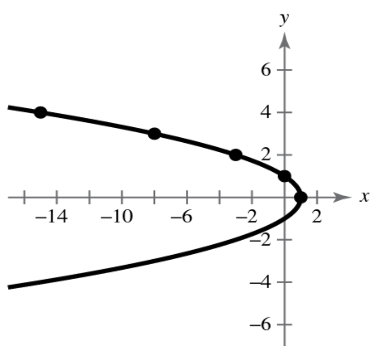
$(-y)^2 = 1 - x \Rightarrow y^2 = 1 - x$

$\Rightarrow$ x-axis symmetry

$(-y)^2 = 1 - (-x) \Rightarrow y^2 = 1 + x$

$\Rightarrow$ No origin symmetry

(c)



1.5.83.

$y = x^3 - x$

(a)

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
y	0	$-\frac{3}{8}$	0	$\frac{15}{8}$	6

(b) $y = (-x)^3 - (-x) \Rightarrow y = -x^3 + x$

$\Rightarrow$ No y-axis symmetry

$-y = x^3 - x \Rightarrow y = -x^3 + x$

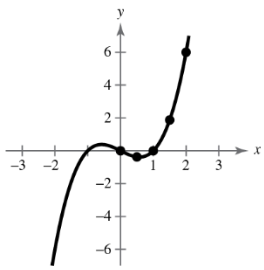
$\Rightarrow$ No x-axis symmetry

$-y = (-x)^3 - (-x) \Rightarrow -y = -x^3 + x$

$\Rightarrow y = x^3 - x$

$\Rightarrow$ origin symmetry

(c)



1.5.84.

$y = 2x^{-3} = \dfrac{2}{x^3}$

(a)

x	-2	$\frac{-3}{2}$	-1	$\frac{-1}{2}$	$\frac{-1}{4}$
y	$-\frac{1}{4}$	$\frac{-16}{27}$	-2	-16	-128

(b) $y = \dfrac{2}{(-x)^3} \Rightarrow y = -\dfrac{2}{x^3}$

$\Rightarrow$ No y-axis symmetry

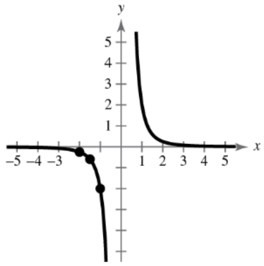
$-y = \dfrac{2}{x^3} \Rightarrow y = -\dfrac{2}{x^3}$

$\Rightarrow$ No x-axis symmetry

$-y = \dfrac{2}{(-x)^3} \Rightarrow -y = -\dfrac{2}{x^3} \Rightarrow y = \dfrac{2}{x^3}$

$\Rightarrow$ origin symmetry

(c)



Section 1.6 Graphs of Other Trigonometric Functions

1.6.1.

odd; origin

1.6.2.

reciprocal

1.6.3.

damping

1.6.4.

$x \neq n\pi$

1.6.5.

The range of both $y = \csc x$ and $y = \sec x$ is $(-\infty, -1] \cup [1, \infty)$.

1.6.6.

The functions $y = \sin x$, $y = \cos x$, $y = \csc x$, and $y = \sec x$ have a period of 2π.

1.6.7.

$y = \tan \dfrac{x}{2}$

Period: $\dfrac{\pi}{b} = \dfrac{\pi}{1/2} = 2\pi$

Asymptotes: $x = -\pi$, $x = \pi$

Matches graph (c).

1.6.8.

$y = \dfrac{1}{2}\cot \pi x$

Period: $\dfrac{\pi}{\pi} = 1$

Matches graph (a).

1.6.9.

$y = -\csc x$

Period: 2π

Matches graph (d).

1.6.10.

$y = -2 \sec \dfrac{\pi x}{2}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{\pi/2} = 4$

Asymptotes: $x = -1,\ x = 1$

Reflected in x-axis
Matches graph (b).

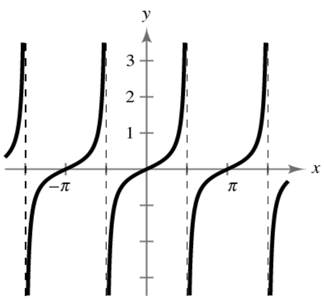
1.6.11.

$y = \dfrac{1}{3} \tan x$

Period: π

Two consecutive asymptotes: $x = -\dfrac{\pi}{2}$ and $x = \dfrac{\pi}{2}$

x	$-\dfrac{\pi}{4}$	0	$\dfrac{\pi}{4}$
y	$-\dfrac{1}{3}$	0	$\dfrac{1}{3}$



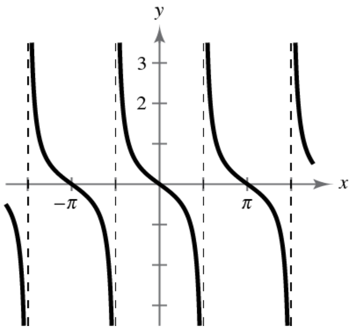
1.6.12.

$y = -\dfrac{1}{2} \tan x$

Period: π

Two consecutive asymptotes: $x = -\dfrac{\pi}{2},\ x = \dfrac{\pi}{2}$

x	$-\dfrac{\pi}{4}$	0	$\dfrac{\pi}{4}$
y	$\dfrac{1}{2}$	0	$-\dfrac{1}{2}$



1.6.13.

$y = \tan\dfrac{\pi x}{4}$

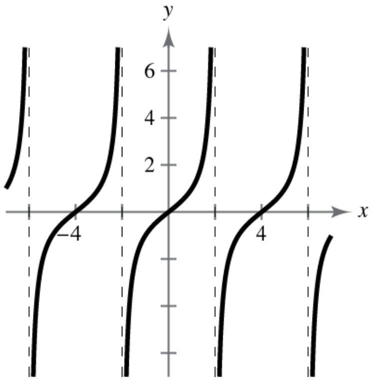
Period: $\dfrac{\pi}{\pi/4} = 4$

Two consecutive asymptotes:

$\dfrac{\pi x}{4} = -\dfrac{\pi}{2} \Rightarrow x = -2$

$\dfrac{\pi x}{4} = \dfrac{\pi}{2} \Rightarrow x = 2$

x	–1	0	1
y	–1	0	1



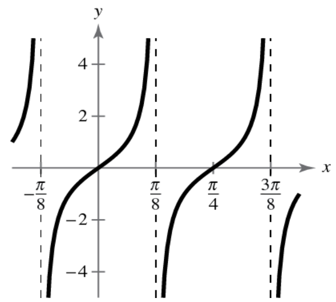
1.6.14.

$y = \tan 4x$

Period: $\dfrac{\pi}{4}$

Two consecutive asymptotes: $x = -\dfrac{\pi}{8},\ x = \dfrac{\pi}{8}$

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	0	0	0



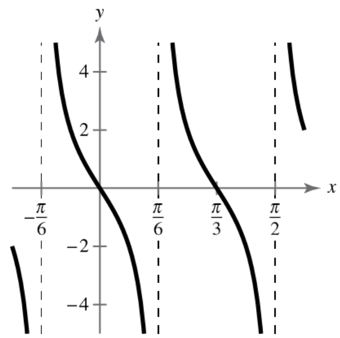
1.6.15.

$y = -2\tan 3x$

Period: $\dfrac{\pi}{3}$

Two consecutive asymptotes: $x = -\dfrac{\pi}{6}, x = \dfrac{\pi}{6}$

x	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$
y	0	0	0



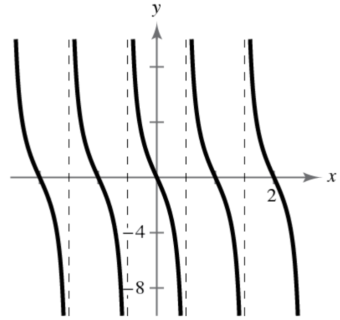
1.6.16.

$y = -3\tan \pi x$

Period: $\dfrac{\pi}{\pi} = 1$

Two consecutive asymptotes: $x = -\dfrac{1}{2}, x = \dfrac{1}{2}$

x	$-\frac{1}{4}$	0	$\frac{1}{4}$
y	3	0	-3



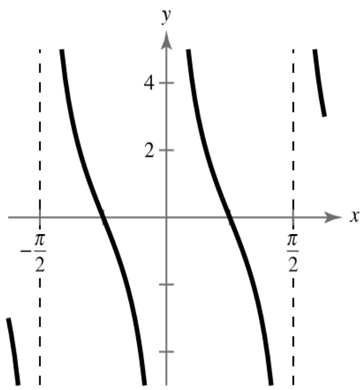
1.6.17.

$y = 3\cot 2x$

Period: $\frac{\pi}{2}$

Two consecutive asymptotes: $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$

x	$-\frac{\pi}{6}$	$-\frac{\pi}{8}$	$\frac{\pi}{8}$	$\frac{\pi}{6}$
y	$-3\sqrt{3}$	-3	3	$3\sqrt{3}$



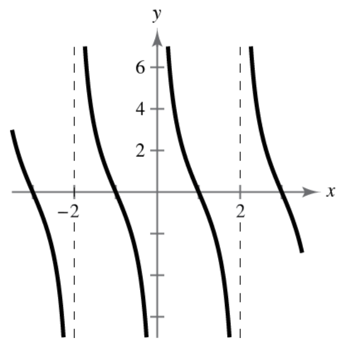
1.6.18.

$y = 3\cot\frac{\pi x}{2}$

Period: $\frac{\pi}{\pi/2} = 2$

Two consecutive asymptotes: $x = 0$, $x = 2$

x	$\frac{1}{2}$	1	$\frac{3}{2}$
y	3	0	–3



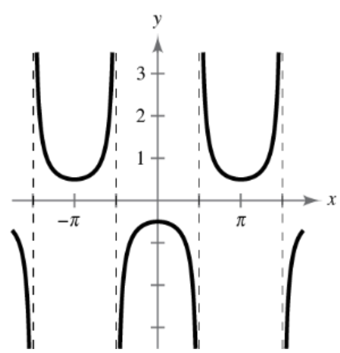
1.6.19.

$$y = -\frac{1}{2}\sec x$$

Period: 2π

Two consecutive asymptotes: $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$

x	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$
y	–1	$-\frac{1}{2}$	–1



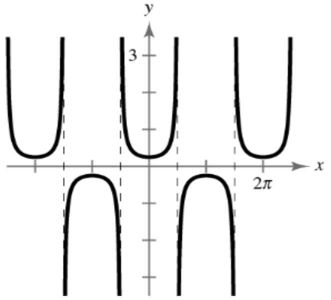
1.6.20.

$$y = \frac{1}{4}\sec x$$

Period: 2π

Two consecutive asymptotes: $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$

x	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$
y	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$



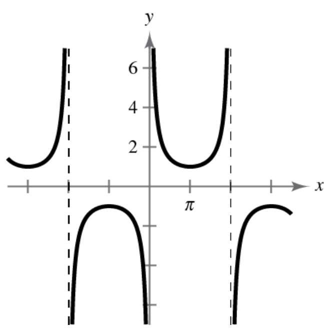
1.6.21.

$$y = \csc\frac{x}{2}$$

Period: $\dfrac{2\pi}{1/2} = 4\pi$

Two consecutive asymptotes: $x = 0,\ x = 2\pi$

x	$\frac{\pi}{3}$	π	$\frac{5\pi}{3}$
y	2	1	2



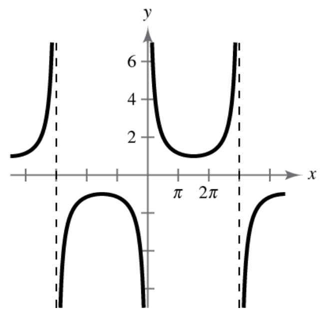
1.6.22.

$$y = \csc\frac{x}{3}$$

Period: $\dfrac{2\pi}{1/3} = 6\pi$

Two consecutive asymptotes: $x = 0,\ x = 3\pi$

x	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	$\frac{5\pi}{2}$
y	2	1	2



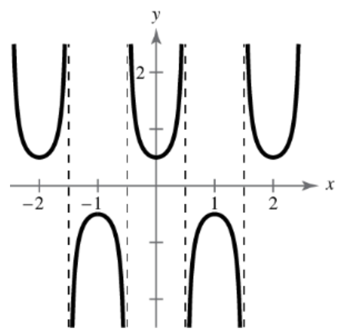
1.6.23.

$y = \frac{1}{2}\sec \pi x$

Period: 2

Two consecutive asymptotes: $x = -\frac{1}{2}, x = \frac{1}{2}$

x	–1	0	1
y	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$



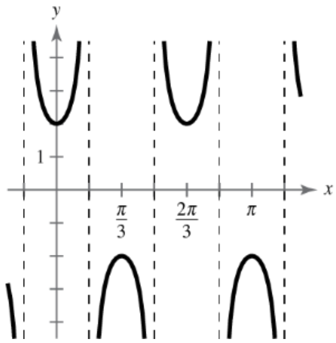
1.6.24.

$y = 2\sec 3x$

Period: $\frac{2\pi}{3}$

Two consecutive asymptotes: $x = -\frac{\pi}{6}, x = \frac{\pi}{6}$

x	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$
y	–2	2	–2



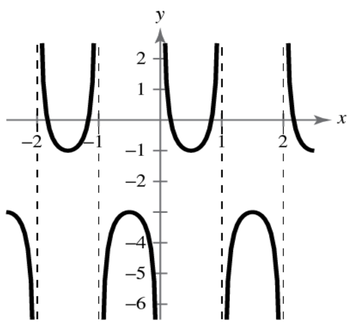
1.6.25.

$y = \csc \pi x - 2$

Period: $\dfrac{2\pi}{\pi} = 2$

Two consecutive asymptotes: $x = 0$, $x = 1$

x	0.5	1.5	2.5
y	–1	–3	–1



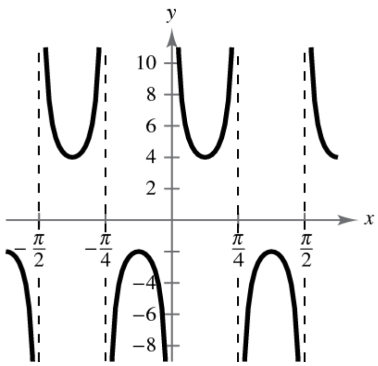
1.6.26.

$y = 3\ \csc 4x + 1$

Period: $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$

Two consecutive asymptotes: $x = 0$, $x = \dfrac{\pi}{4}$

x	$\frac{\pi}{8}$	$\frac{3\pi}{8}$	$\frac{5\pi}{8}$
y	4	–2	4



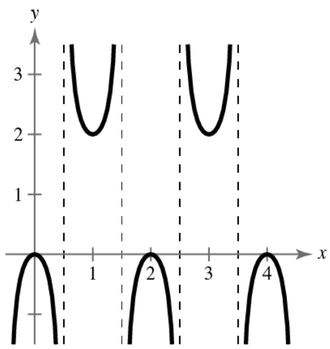
1.6.27.

$y = -\sec \pi x + 1$

Period: $\dfrac{2\pi}{\pi} = 2$

Two consecutive asymptotes: $x = -\dfrac{1}{2}, x = \dfrac{1}{2}$

x	$-\dfrac{1}{3}$	0	$\dfrac{1}{3}$
y	–1	0	1



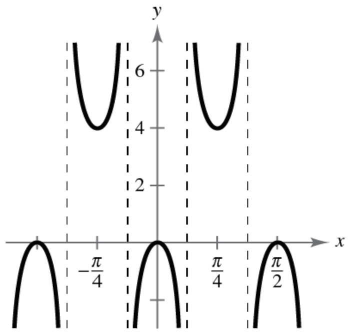
1.6.28.

$y = -2 \sec 4x + 2$

Period: $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$

Two consecutive asymptotes: $x = -\dfrac{\pi}{8}, x = \dfrac{\pi}{8}$

x	$-\dfrac{\pi}{12}$	0	$\dfrac{\pi}{12}$
y	–2	0	–2



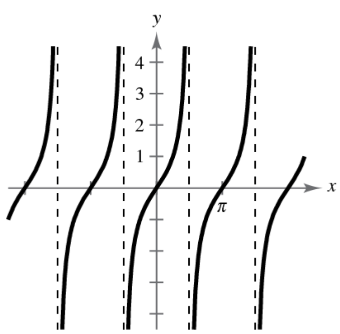
1.6.29.

$$y = \tan(x + \pi)$$

Period: π

Two consecutive asymptotes: $x = -\frac{\pi}{2}, x = \frac{\pi}{2}$

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	–1	0	1



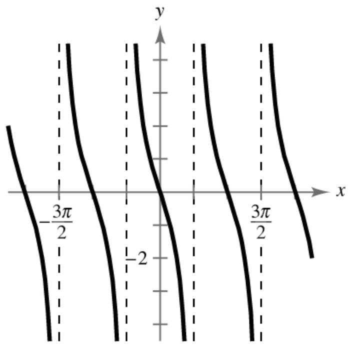
1.6.30.

$$y = 2\cot\left(x + \frac{\pi}{2}\right)$$

Period: π

Two consecutive asymptotes: $x = -\frac{\pi}{2}, x = \frac{\pi}{2}$

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	2	0	–2



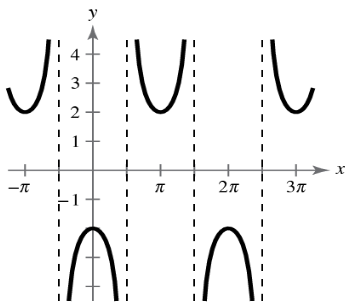
1.6.31.

$y = 2\ \sec(x + \pi)$

Period: 2π

Two consecutive asymptotes: $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$

x	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$
y	-4	-2	-4



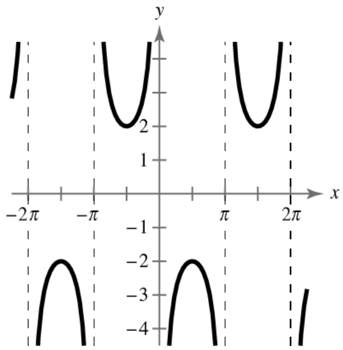
1.6.32.

$y = 2\csc(x - \pi)$

Period: 2π

Two consecutive asymptotes: $x = -\pi$, $x = \pi$

x	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{3\pi}{2}$
y	2	-2	-2



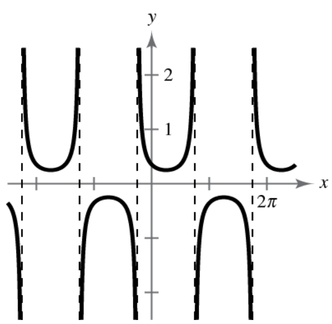
1.6.33.

$$y = \frac{1}{4}\csc\left(x + \frac{\pi}{4}\right)$$

Period: 2π

Two consecutive asymptotes: $x = -\frac{\pi}{4},\ x = \frac{3\pi}{4}$

x	$-\frac{\pi}{12}$	$\frac{\pi}{4}$	$\frac{7\pi}{12}$
y	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$



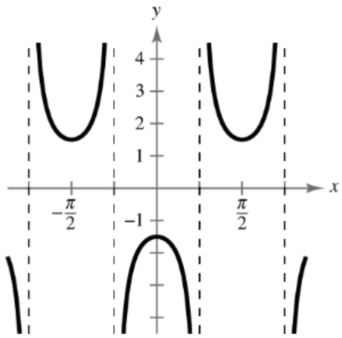
1.6.34.

$$y = \frac{3}{2}\sec(2x - \pi)$$

Period: $\frac{2\pi}{2} = \pi$

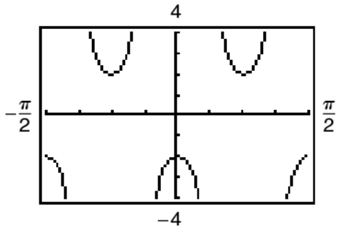
Two consecutive asymptotes: $\frac{\pi}{4}, \frac{3\pi}{4}$

x	0	$\frac{\pi}{2}$	π
y	-1.5	1.5	-1.5



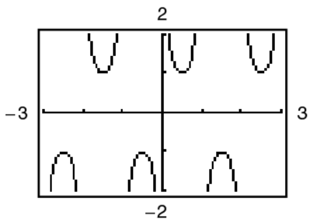
1.6.35.

$$y = -2\sec 4x = \frac{-2}{\cos 4x}$$



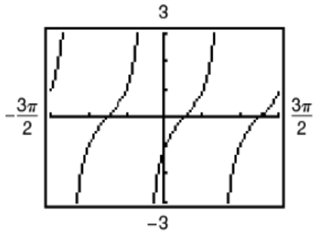
1.6.36.

$$y = \csc \pi x$$



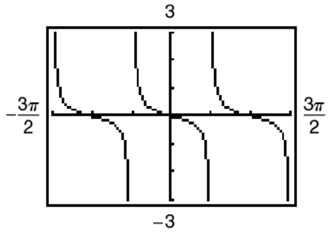
1.6.37.

$$y = \tan\left(x - \frac{\pi}{4}\right)$$



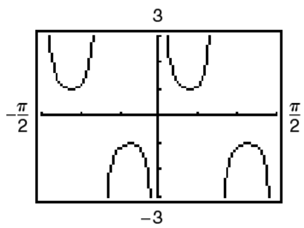
1.6.38.

$$y = \frac{1}{4}\cot\left(x - \frac{\pi}{2}\right)$$
$$= \frac{1}{4\tan\left(x - \frac{\pi}{2}\right)}$$



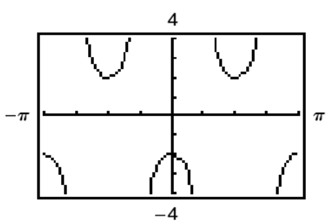
1.6.39.

$$y = -\csc(4x - \pi)$$
$$y = \frac{-1}{\sin(4x - \pi)}$$



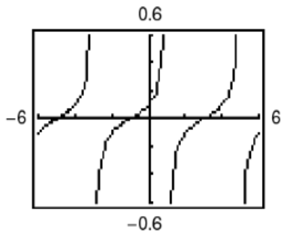
1.6.40.

$$y = 2\sec(2x - \pi)$$
$$y = \frac{2}{\cos(2x - \pi)}$$



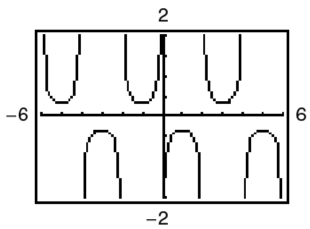
1.6.41.

$$y = 0.1\tan\left(\frac{\pi x}{4} + \frac{\pi}{4}\right)$$



1.6.42.

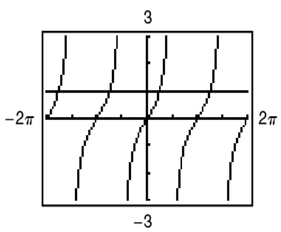
$$y = \frac{1}{3}\sec\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) \Rightarrow y = \frac{1}{3\ \cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right)}$$



1.6.43.

$$\tan x = 1$$

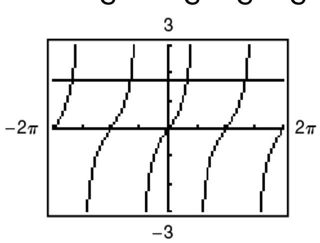
$$x = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$$



1.6.44.

$$\tan x = \sqrt{3}$$

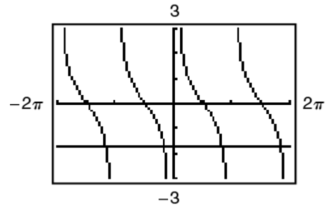
$$x = -\frac{5\pi}{3}, -\frac{2\pi}{3}, \frac{\pi}{3}, \frac{4\pi}{3}$$



1.6.45.

$$\cot x = -\sqrt{3}$$

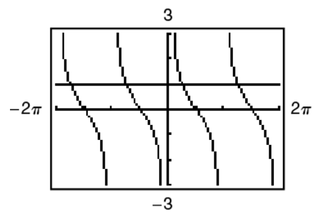
$$x = -\frac{7\pi}{6}, -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$$



1.6.46.

$$\cot x = 1$$

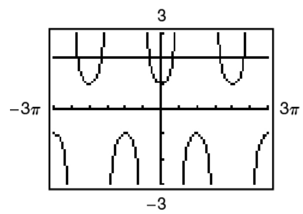
$$x = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$$



1.6.47.

$$\sec x = 2$$

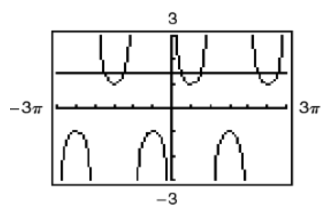
$$x = -\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$$



1.6.48.

$$\csc x = \sqrt{2}$$

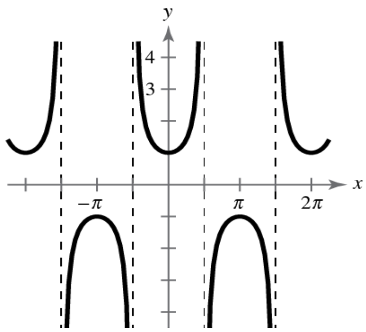
$$x = -\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$$



1.6.49.

$$\begin{aligned} f(x) &= \sec x = \frac{1}{\cos x} \\ f(-x) &= \sec(-x) \\ &= \frac{1}{\cos(-x)} \\ &= \frac{1}{\cos x} \\ &= f(x) \end{aligned}$$

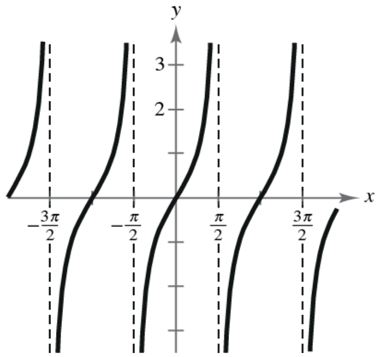
So, $f(x) = \sec x$ is an even function and the graph has y-axis symmetry.



1.6.50.

$$\begin{aligned} f(x) &= \tan x \\ \tan(-x) &= -\tan x \end{aligned}$$

So, $f(x) = \tan x$ is an odd function and the graph is symmetric about the origin.



1.6.51.

The function $f(x) = \csc 2x = \dfrac{1}{\sin 2x}$ has origin symmetry. Thus, the function is odd.

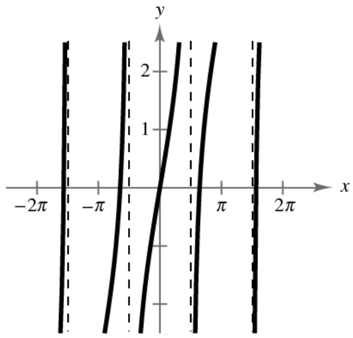
1.6.52.

$$f(x) = \cot 2x$$
$$f(-x) = \cot(-2x)$$
$$= \frac{\cos(-2x)}{\sin(-2x)} = -\frac{\cos(2x)}{\sin(2x)} = -f(x)$$

Thus, the function is odd.

1.6.53.

$$f(x) = x + \tan x$$

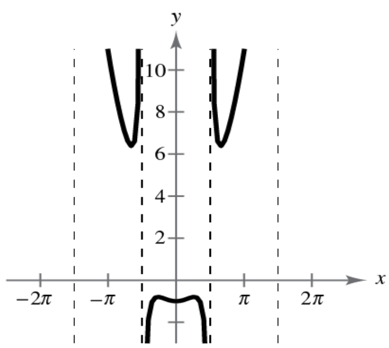


$$f(-x) = (-x) + \tan(-x)$$
$$= -x - \tan x$$
$$= -(x + \tan x)$$
$$= -f(x)$$

So, $f(x) = x + \tan x$ is an odd function and the graph has origin symmetry.

1.6.54.

$$f(x) = x^2 - \sec x = x^2 - \frac{1}{\cos x}$$

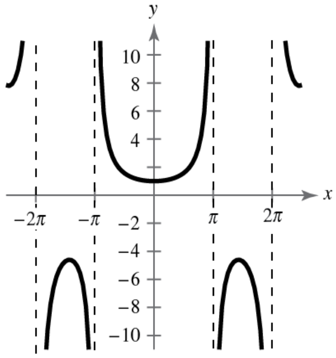


$$\begin{aligned} f(-x) &= (-x)^2 - \sec(-x) \\ &= x^2 - \frac{1}{\cos(-x)} \\ &= x^2 - \frac{1}{\cos x} \\ &= x^2 - \sec x \\ &= f(x). \end{aligned}$$

So, $f(x) = x^2 - \sec x$ is an even function and the graph has y-axis symmetry.

1.6.55.

$$g(x) = x \csc x = \frac{x}{\sin x}$$

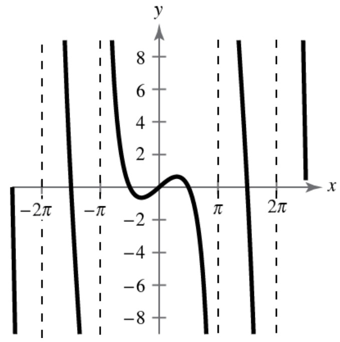


$$\begin{aligned} g(-x) &= (-x)\csc(-x) \\ &= \frac{-x}{\sin(-x)} \\ &= \frac{-x}{-\sin x} \\ &= \frac{x}{\sin x} \\ &= x \csc x \\ &= g(x) \end{aligned}$$

So, $g(x) = x \csc x$ is an even function and the graph has y-axis symmetry.

1.6.56.

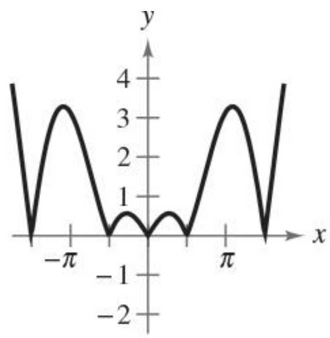
$$g(x) = x^2 \cot x = \frac{x^2}{\tan x}$$



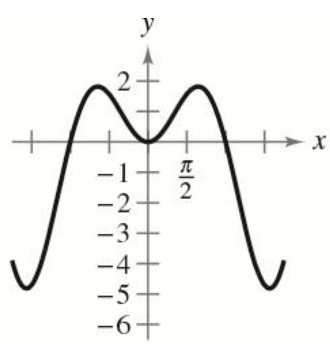
$$\begin{aligned} g(-x) &= (-x)^2 \cot(-x) \\ &= \frac{x^2}{\tan(-x)} \\ &= -\frac{x^2}{\tan x} \\ &= -x^2 \cot x \\ &= -g(x) \end{aligned}$$

So, $g(x) = x^2 \cot x$ is an odd function and the graph has origin symmetry.

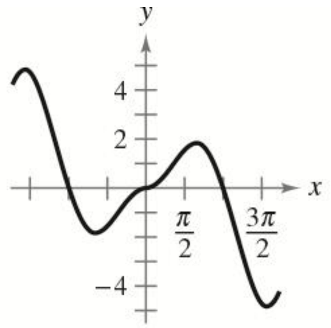
1.6.57.



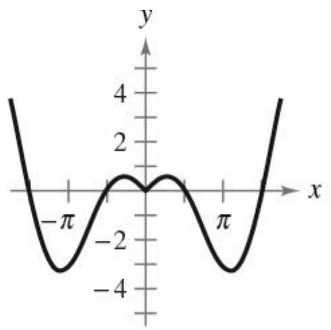
1.6.58.



1.6.59.



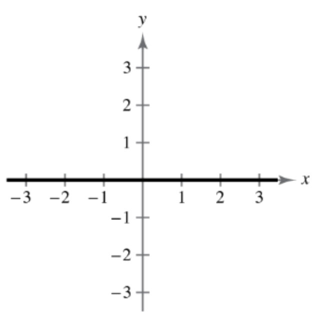
1.6.60.



1.6.61.

$$f(x) = \sin x + \cos\left(x + \frac{\pi}{2}\right)$$
$$g(x) = 0$$
$$f(x) = g(x)$$

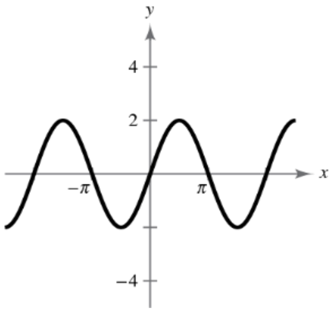
The functions are equal.



1.6.62.

$$f(x) = \sin x - \cos\left(x + \frac{\pi}{2}\right)$$
$$g(x) = 2\sin x$$
$$f(x) = g(x)$$

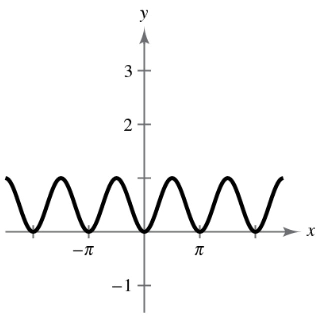
The functions are equal.



1.6.63.

$$f(x) = \sin^2 x$$
$$g(x) = \tfrac{1}{2}(1 - \cos 2x)$$
$$f(x) = g(x)$$

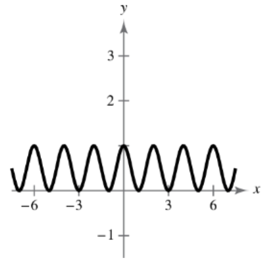
The functions are equal.



1.6.64.

$$f(x) = \cos^2 \frac{\pi x}{2}$$
$$g(x) = \frac{1}{2}(1 + \cos \pi x)$$
$$f(x) = g(x)$$

The functions are equal.

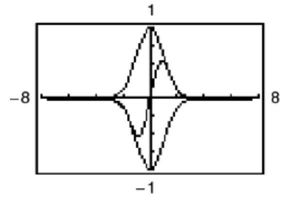


1.6.65.

$$g(x) = e^{-x^2/2} \sin x$$

Damping factor: $e^{-x^2/2}$

As $x \to \infty, g(x) \to 0.$

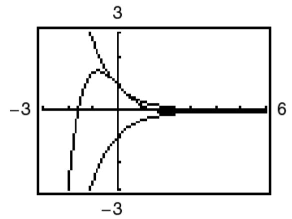


1.6.66.

$$f(x) = e^{-x} \cos x$$

Damping factor: e^{-x}

As $x \to \infty, f(x) \to 0.$

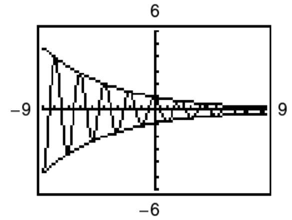


1.6.67.

$$f(x) = 2^{-x/4} \cos \pi x$$

Damping factor: $y = 2^{-x/4}$

As $x \to \infty, f(x) \to 0.$

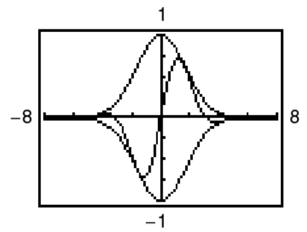


1.6.68.

$h(x) = 2^{-x^2/4} \sin x$

Damping factor: $2^{-x^2/4}$

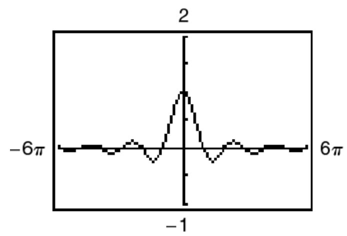
As $x \to \infty, h(x) \to 0.$



1.6.69.

$g(x) = \dfrac{\sin x}{x}$

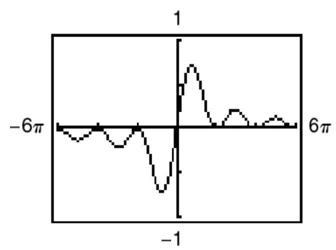
As $x \to 0, g(x) \to 1.$



1.6.70.

$f(x) = \dfrac{1 - \cos x}{x}$

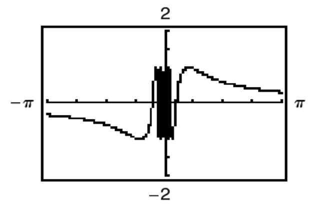
As $x \to 0, f(x) \to 0.$



1.6.71.

$f(x) = \sin \dfrac{1}{x}$

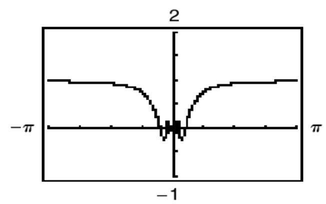
As $x \to 0, f(x)$ oscillates between −1 and 1.



1.6.72.

$h(x) = x \sin\dfrac{1}{x}$

As $x \to 0$, $h(x) \to 0$.



1.6.73.

(a) Period of $\cos\dfrac{\pi t}{6} = \dfrac{2\pi}{\pi/6} = 12$

Period of $\sin\dfrac{\pi t}{6} = \dfrac{2\pi}{\pi/6} = 12$

The period of $H(t)$ is 12 months.

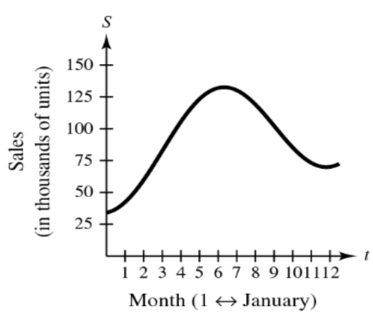
The period of $L(t)$ is 12 months.

(b) From the graph, it appears that the greatest difference between high and low temperatures occurs in the summer. The smallest difference occurs in the winter.

(c) The highest high and low temperatures appear to occur about half of a month after the time when the sun is northernmost in the sky.

1.6.74.

(a)



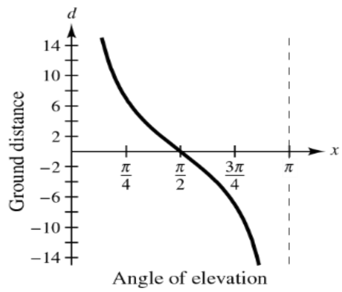
(b) $S(6) = 74 + 3(6) - 40\cos\left(\dfrac{6\pi}{6}\right)$

$= 74 + 18 - 40(-1) = 132 \Rightarrow 132{,}000$ units

1.6.75.

$\tan\ x = \dfrac{7}{d}$

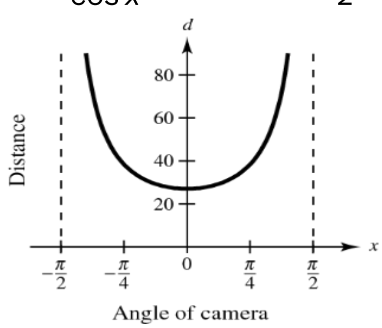
$d = \dfrac{7}{\tan x} = 7\ \cot x$



1.6.76.

$\cos x = \dfrac{27}{d}$

$d = \dfrac{27}{\cos x} = 27\ \sec x, -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$



1.6.77.

True. Because

$y = \csc x = \dfrac{1}{\sin x}$, for a given value of x, the y-coordinate of csc x is the reciprocal of the y-coordinate of sin x.

1.6.78.

True.

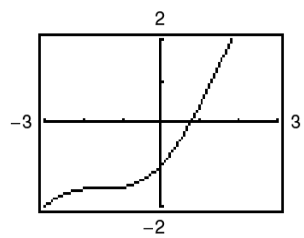
$y = \sec x = \dfrac{1}{\cos x}$

If the reciprocal of $y = \sin x$ is translated $\pi/2$ units to the left, then

$$y = \frac{1}{\sin\left(x + \frac{\pi}{2}\right)} = \frac{1}{\cos x} = \sec x.$$

1.6.79.

$f(x) = x - \cos x$

(a) 

The zero between 0 and 1 occurs at $x \approx 0.7391$.

(b) $x_n = \cos(x_{n-1})$

$x_0 = 1$

$x_1 = \cos 1 \approx 0.5403$

$x_2 = \cos 0.5403 \approx 0.8576$

$x_3 = \cos 0.8576 \approx 0.6543$

$x_4 = \cos 0.6543 \approx 0.7935$

$x_5 = \cos 0.7935 \approx 0.7014$

$x_6 = \cos 0.7014 \approx 0.7640$

$x_7 = \cos 0.7640 \approx 0.7221$

$x_8 = \cos 0.7221 \approx 0.7504$

$x_9 = \cos 0.7504 \approx 0.7314$

$\vdots$

This sequence appears to be approaching the zero of $f: x \approx 0.7391$.

1.6.80.

(a) (i) $f(x) = \tan 2x$

The period of the graph is $\frac{\pi}{2}$ and the graph is increasing on $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$.

(b) (ii) $f(x) = \csc 4x$

The period of the graph $\frac{\pi}{2}$ and the graph has vertical asymptotes at multiples of $\frac{\pi}{4}$.

1.6.81.

$f(x)=\sqrt[3]{x},\ g(x)=x^2$

(a) $(f\circ g)(x)=f(x^2)=(x^2)^{1/3}=x^{2/3}$

(b) $(g\circ f)(x)=g(x^{1/3})=(x^{1/3})^2=x^{2/3}$

Domains of $f, g, f\circ g,\ g\circ f$: all real numbers x

1.6.82.

$f(x)=x+4,\ g(x)=\dfrac{1}{x}$

(a) $(f\circ g)(x)=f\left(\dfrac{1}{x}\right)=\dfrac{1}{x}+4$

(b) $(g\circ f)(x)=g(x+4)=\dfrac{1}{x+4}$

Domain of f: all real numbers x

Domains of g and $f\circ g$: all real numbers x such that $x\neq 0$

Domain of $g\circ f$: all real numbers x such that $x\neq -4$

1.6.83.

$f(x)=x^4,\ g(x)=\sqrt{x-1}$

(a) $(f\circ g)(x)=f\sqrt{x-1}=(x-1)^2,\ x\geq 1$

(b) $(g\circ f)(x)=g(x^4)=\sqrt{x^4-1}$

Domain of f: all real numbers x

Domains of g and $f\circ g$: all real numbers x such that $x\geq 1$

Domain of $g\circ f$: all real numbers x such that $x\geq 1$ or $x\leq -1$

1.6.84.

$f(x)=\dfrac{x}{x^2-4},\ g(x)=2x$

(a) $(f\circ g)(x)=f(2x)=\dfrac{2x}{(2x)^2-4}=\dfrac{2x}{4x^2-4}$

$=\dfrac{x}{2x^2-2}$

(b) $(g\circ f)(x)=g\left(\dfrac{x}{x^2-4}\right)=\dfrac{2x}{x^2-4}$

Domains of f and $g\circ f$: all real numbers x such that $x\neq\pm 2$

Domain of g: all real numbers x

Domain of $f \circ g$: all real numbers x such that $x \neq \pm 1$

1.6.85.

$f(x) = 9 - x^2$

f is not one-to-one $\left(f(1) = f(-1) = 8\right)$.

No inverse function

1.6.86.

$g(x) = 4x + 1$ is one-to-one, so it has an inverse function.

$$y = 4x + 1$$
$$x = 4y + 1$$
$$y = \frac{1}{4}(x - 1)$$
$$g^{-1}(x) = \frac{1}{4}(x - 1)$$

1.6.87.

$h(x) = \dfrac{1}{x^3}$ is one-to-one, so it has an inverse function.

$$y = \frac{1}{x^3}$$
$$x = \frac{1}{y^3}$$
$$y^3 = \frac{1}{x}$$
$$y = \frac{1}{\sqrt[3]{x}}$$
$$h^{-1}(x) = \frac{1}{\sqrt[3]{x}}$$

1.6.88.

$h(x) = |x - 3|$

h is not one-to-one $\left(h(0) = h(6) = 3\right)$.

No inverse function

1.6.89.

$$\begin{aligned} f(x) &= \sqrt{3x-6},\ x \ge 2 \\ y &= \sqrt{3x-6} \\ x &= \sqrt{3y-6} \\ x^2 &= 3y-6 \\ 3y &= x^2+6 \\ y &= \frac{x^2+6}{3} \\ f^{-1}(x) &= \frac{x^2+6}{3},\ x \ge 0 \end{aligned}$$

1.6.90.

$$\begin{aligned} f(x) &= (x-5)^2,\ x \le 5 \\ y &= (x-5)^2 \\ x &= (y-5)^2 \\ -\sqrt{x} &= y-5 \\ y &= 5-\sqrt{x} \\ f^{-1}(x) &= 5-\sqrt{x} \end{aligned}$$

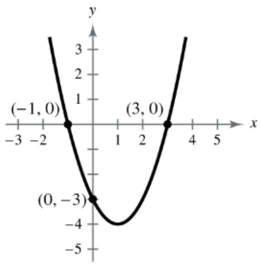
1.6.91.

$y = x^2 - 2x - 3 = (x-3)(x+1)$

$y = (-x)^2 - 2(-x) - 3 \Rightarrow y = x^2 + 2x - 3 \Rightarrow$ No y-axis symmetry

$-y = x^2 - 2x - 3 \Rightarrow y = -x^2 + 2x + 3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^2 - 2(-x) - 3 \Rightarrow -y = x^2 + 2x - 3 = y = -x^2 - 2x + 3 \Rightarrow$ No origin symmetry



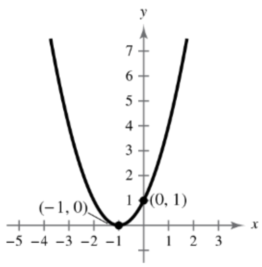
x-intercepts: $(-1,\quad 0)(3,\quad 0)$

y-intercept: $(0,\quad -3)$

1.6.92.

$$y = x^2 + 2x + 1 = (x+1)^2$$

$y = (-x)^2 + 2(-x) + 1 \Rightarrow y = x^2 - 2x + 1 \Rightarrow$ No y-axis symmetry
$-y = x^2 + 2x + 1 \Rightarrow y = -x^2 - 2x - 1 \Rightarrow$ No x-axis symmetry
$-y = (-x)^2 + 2(-x) + 1 \Rightarrow -y = x^2 - 2x + 1 \Rightarrow y = -x^2 + 2x - 1 \Rightarrow$ No origin symmetry

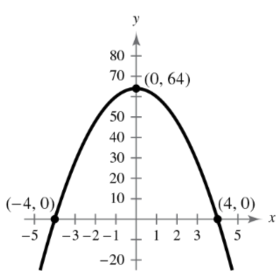


x-intercept: $(-1,\quad 0)$
y-intercept: $(0,\quad 1)$

1.6.93.

$$y = 64 - 4x^2$$

$y = 64 - 4(-x)^2 \Rightarrow y = 64 - 4x^2 \Rightarrow$ y-axis symmetry
$-y = 64 - 4x^2 \Rightarrow y = -64 + 4x^2 \Rightarrow$ No x-axis symmetry
$-y = 64 - 4(-x)^2 \Rightarrow -y = 64 - 4x^2 \Rightarrow y = -64 + 4x^2 \Rightarrow$ No origin symmetry



x-intercepts: $(\pm 4,\quad 0)$
y-intercept: $(0,\quad 64)$

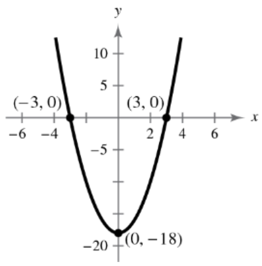
1.6.94.

$y = 2x^2 - 18$

$y = 2(-x)^2 - 18 \Rightarrow y = 2x^2 - 18 \Rightarrow y$-axis symmetry

$-y = 2x^2 - 18 \Rightarrow y = -2x^2 + 18 \Rightarrow$ No x-axis symmetry

$-y = 2(-x)^2 - 18 \Rightarrow -y = 2x^2 - 18 \Rightarrow y = -2x^2 + 18 \Rightarrow$ No origin symmetry



x-intercepts: $(\pm 3, \quad 0)$

y-intercept: $(0, \quad -18)$

Section 1.7 Inverse Trigonometric Functions

	Function	*Alternative Notation*	*Domain*	*Range*
1.7.1.	$y = \arcsin x$	$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
1.7.2.	$y = \arccos x$	$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
1.7.3.	$y = \arctan x$	$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

1.7.4.

inverse

1.7.5.

The inverse cosecant function can be denoted as $\csc^{-1} x$ or arccsc x.

1.7.6.

No. $\arccos x \ne \dfrac{1}{\cos x}$, $\arccos x = \cos^{-1} x$.

1.7.7.

$y = \arcsin \frac{1}{2} \Rightarrow \sin y = \frac{1}{2}$ for

$-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = \frac{\pi}{6}$

1.7.8.

$y = \arcsin\ 0 \Rightarrow \sin\ y = 0$ for $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = 0$

1.7.9.

$y = \arccos 0 \Rightarrow \cos\ y = 0$ for $0 \le y \le \pi \Rightarrow y = \frac{\pi}{2}$

1.7.10.

$y = \arccos \frac{1}{2} \Rightarrow \cos\ y = \frac{1}{2}$ for $0 \le y \le \pi \Rightarrow y = \frac{\pi}{3}$

1.7.11.

$$y = \arctan\frac{\sqrt{3}}{3} \Rightarrow \tan y = \frac{\sqrt{3}}{3} \text{ for } -\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = \frac{\pi}{6}$$

1.7.12.

$$y = \arctan(1) \Rightarrow \tan y = 1 \text{ for } -\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = \frac{\pi}{4}$$

1.7.13.

It is not possible to evaluate arcsin 3. The domain of the inverse sine function is $[-1, 1]$.

1.7.14.

It is not possible to evaluate arcsin $\sqrt{3}$. The domain of the inverse sine function is $[-1, 1]$.

1.7.15.

$$y = \arctan\left(-\sqrt{3}\right) \Rightarrow \tan y = -\sqrt{3} \text{ for } -\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = -\frac{\pi}{3}$$

1.7.16.

It is not possible to evaluate $\cos^{-1}(-2)$. The domain of the inverse cosine function is $[-1, 1]$.

1.7.17.

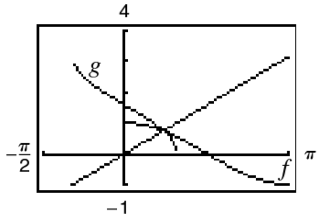
$$y = \arccos\left(-\frac{1}{2}\right) \Rightarrow \cos y = -\frac{1}{2} \text{ for } 0 \le y \le \pi \Rightarrow y = \frac{2\pi}{3}$$

1.7.18.

$$y = \arcsin\left(\frac{\sqrt{2}}{2}\right) \Rightarrow \sin y = \frac{\sqrt{2}}{2} \text{ for } -\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = \frac{\pi}{4}$$

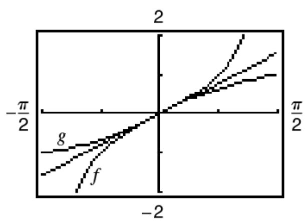
1.7.19.

$$f(x) = \cos x$$
$$g(x) = \text{arc}\cos x$$
$$y = x$$



1.7.20.

$$f(x) = \tan x$$
$$g(x) = \arctan x$$
$$y = x$$



1.7.21.

The graph of g is a transformation of the graph of f.

$g(x) = \arcsin(x+2) = f(x+2)$

The graph of g can be obtained by shifting the graph of f two units to the left.

1.7.22.

The graph of g is a transformation of the graph of f.

$g(x) = \arccos(x-1) = f(x-1)$

The graph of g can be obtained by shifting the graph of f one unit to the right.

1.7.23.

The graph of g is a transformation of the graph of f.

$g(x) = -\arctan x = -f(x)$

The graph of g can be obtained by reflecting the graph of f in the x-axis.

1.7.24.

The graph of g is a transformation of the graph of f.

$g(x) = \arcsin(x-3) = f(x-3)$

The graph of g can be obtained by shifting the graph of f three units to the right.

1.7.25.

$$\tan\theta = \frac{x}{4}$$
$$\theta = \arctan\frac{x}{4}$$

1.7.26.

$$\cos\theta = \frac{4}{x}$$
$$\theta = \arccos\frac{4}{x}$$

1.7.27.

$$\sin\theta = \frac{x+2}{5}$$
$$\theta = \arcsin\left(\frac{x+2}{5}\right)$$

1.7.28.

$$\cos\theta = \frac{x+3}{2x}$$
$$\theta = \arccos\frac{x+3}{2x}$$

1.7.29.

$$\sin(\arcsin 0.3) = 0.3$$

1.7.30.

It is not possible to evaluate $\cos\left[\arccos\left(-\sqrt{3}\right)\right]$. The domain of the inverse cosine function is $[-1, 1]$.

1.7.31.

$$\arcsin\left[\sin\left(\frac{9\pi}{4}\right)\right] = \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}.$$

Note: $\frac{9\pi}{4}$ is not in the range of the arcsin function.

1.7.32.

$$\arccos\left(\cos\frac{-3\pi}{2}\right) = \arccos 0 = \frac{\pi}{2}$$

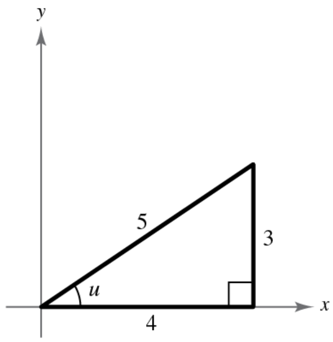
Note: $\frac{-3\pi}{2}$ is not in the range of the arccosine function.

1.7.33.

Let $u = \arctan\frac{3}{4}$.

$\tan u = \frac{3}{4}, 0 < u < \frac{\pi}{2},$

$\sin\left(\arctan\frac{3}{4}\right) = \sin u = \frac{3}{5}$

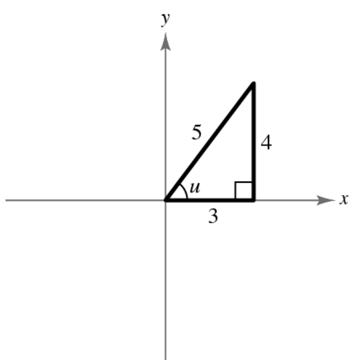


1.7.34.

Let $u = \arcsin\frac{4}{5}$,

$\sin\ u = \frac{4}{5}, 0 < u < \frac{\pi}{2},$

$\cos\left(\arcsin\frac{4}{5}\right) = \cos u = \frac{3}{5}.$

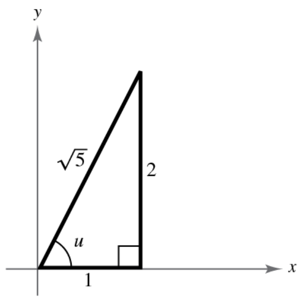


1.7.35.

Let $u = \tan^{-1} 2$,

$\tan u = 2 = \frac{2}{1}, 0 < u < \frac{\pi}{2}$,

$\cos\left(\tan^{-1} 2\right) = \cos u = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$.

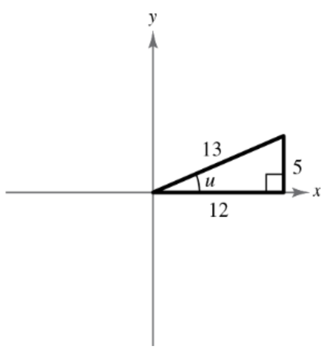


1.7.36.

It is not possible to evaluate $\sin\left(\cos^{-1}\sqrt{5}\right)$. The domain of the inverse cosine function is $\left[-1, 1\right]$.

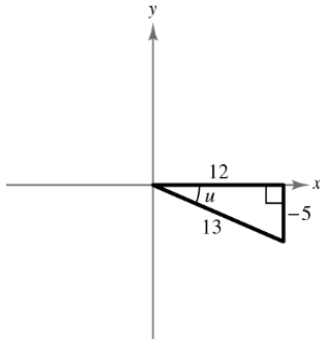
1.7.37.

Let $u = \arcsin\frac{5}{13}$, $\sin u = \frac{5}{13}, 0 < u < \frac{\pi}{2}$, $\sec\left(\arcsin\frac{5}{13}\right) = \sec u = \frac{13}{12}$.



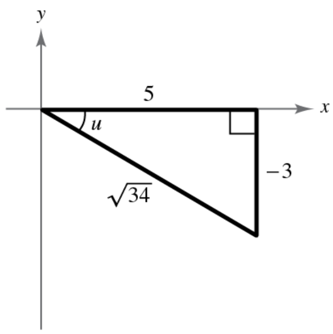
1.7.38.

Let $u = \arctan\left(-\frac{5}{12}\right)$, $\tan u = -\frac{5}{12}, -\frac{\pi}{2} < u < 0$, $\csc\left[\arctan\left(-\frac{5}{12}\right)\right] = \csc u = -\frac{13}{5}$.



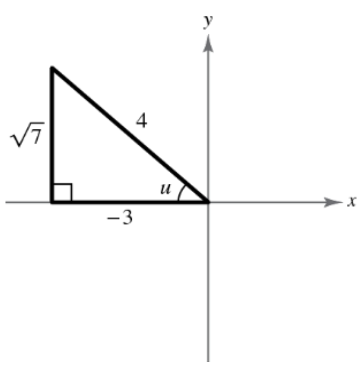
1.7.39.

Let $u = \arctan\left(-\dfrac{3}{5}\right)$, $\tan u = -\dfrac{3}{5}$, $-\dfrac{\pi}{2} < y < 0$, $\cot\left[\arctan\left(-\dfrac{3}{5}\right)\right] = \cot\ u = -\dfrac{5}{3}$.



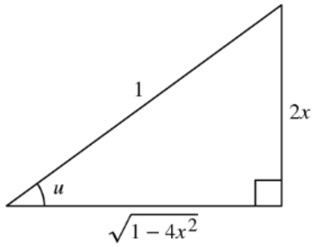
1.7.40.

Let $u = \arccos\left(-\dfrac{3}{4}\right)$. $\cos\ u = -\dfrac{3}{4}$, $\dfrac{\pi}{2} < u < \pi$, $\sec\left[\arccos\left(-\dfrac{3}{4}\right)\right] = \sec\ u = -\dfrac{4}{3}$



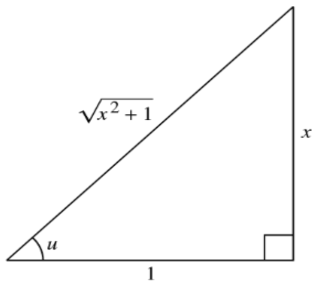
1.7.41.

Let $u = \arcsin(2x)$. $\sin\ u = 2x = \dfrac{2x}{1}$, $\cos(\arcsin 2x) = \cos\ u = \sqrt{1-4x^2}$



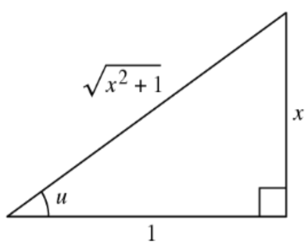
1.7.42.

Let $u = \arctan x$, $\tan u = x = \dfrac{x}{1}$, $\sin(\arctan x) = \sin u = \dfrac{x}{\sqrt{x^2+1}}$



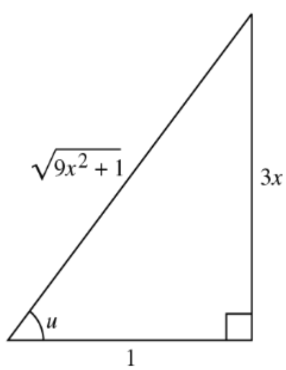
1.7.43.

Let $u = \arctan x$. $\tan u = x = \dfrac{x}{1}$, $\cot(\arctan x) = \cot u = \dfrac{1}{x}$



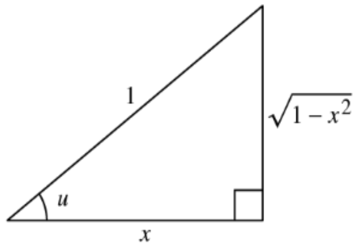
1.7.44.

Let $u = \arctan 3x$. $\tan u = 3x = \dfrac{3x}{1}$, $\sec(\arctan 3x) = \sec u = \sqrt{9x^2+1}$



1.7.45.

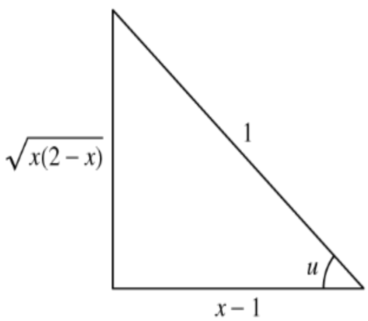
Let $u = \arccos x$. $\cos u = x = \dfrac{x}{1}$, $\sin(\arccos x) = \sin u = \sqrt{1-x^2}$



1.7.46.

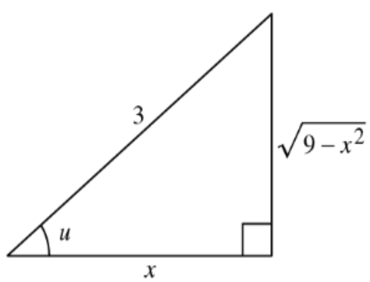
Let

$u = \arccos(x-1)$. $\cos u = x-1 = \dfrac{x-1}{1}$, $\csc[\arccos(x-1)] = \csc u = \dfrac{1}{\sqrt{x(2-x)}}$



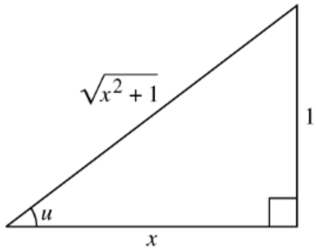
1.7.47.

Let $u = \arccos\left(\dfrac{x}{3}\right)$. $\cos u = \dfrac{x}{3}$, $\tan\left(\arccos\dfrac{x}{3}\right) = \tan u = \dfrac{\sqrt{9-x^2}}{x}$



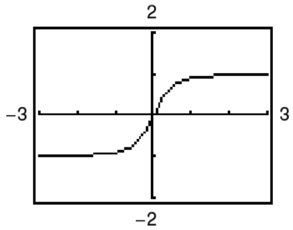
1.7.48.

Let $u = \arctan\dfrac{1}{x}$. $\tan u = \dfrac{1}{x}$, $\cot\left(\arctan\dfrac{1}{x}\right) = \cot u = x$



1.7.49.

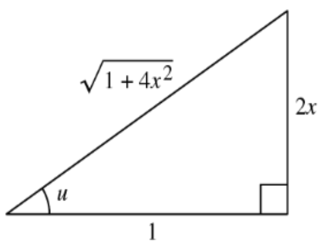
$$f(x) = \sin(\arctan 2x),\ g(x) = \frac{2x}{\sqrt{1+4x^2}}$$



They are equal. Let $u = \arctan 2x$, $\tan u = 2x = \dfrac{2x}{1}$, and $\sin u = \dfrac{2x}{\sqrt{1+4x^2}}$.

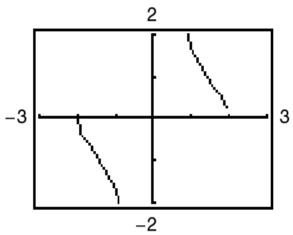
$$g(x) = \frac{2x}{\sqrt{1+4x^2}} = f(x)$$

The graph has horizontal asymptotes at $y = \pm 1$.



1.7.50.

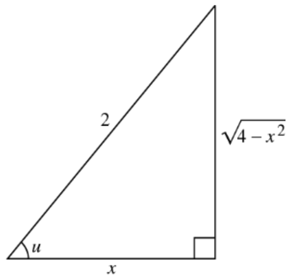
$$f(x) = \tan\left(\arccos \frac{x}{2}\right),\ g(x) = \frac{\sqrt{4-x^2}}{x}$$



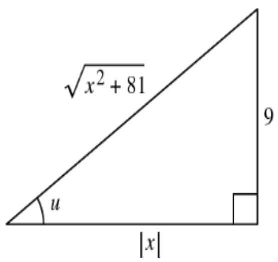
These are equal because: Let $u = \arccos\frac{x}{2}$.

$$f(x) = \tan\left(\arccos\frac{x}{2}\right) = \tan u = \frac{\sqrt{4-x^2}}{x} = g(x)$$

The graph has a horizontal asymptote at $x = 0$.



1.7.51.

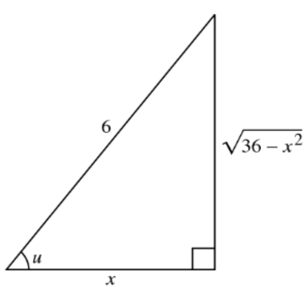


Let $u = \arctan\frac{9}{x}$.

$$\tan u = \frac{9}{x} \text{ and } \sin u = \frac{9}{\sqrt{x^2+81}},\ x > 0$$

$$\text{So, } \arctan\frac{9}{x} = \arcsin\frac{9}{\sqrt{x^2+81}},\ x > 0.$$

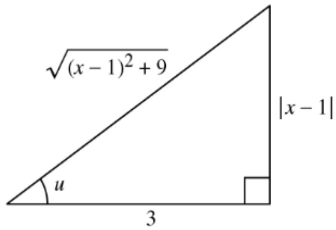
1.7.52.



Let $u = \arcsin\frac{\sqrt{36-x^2}}{6}$.

$$\sin u = \frac{\sqrt{36-x^2}}{6},\ \arcsin\frac{\sqrt{36-x^2}}{6} = \arccos\frac{x}{6}$$

1.7.53.

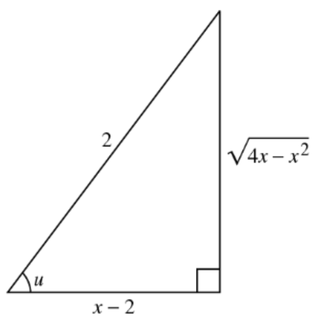


Let $u = \arccos \dfrac{3}{\sqrt{x^2 - 2x + 10}}$.

Then, $\cos u = \dfrac{3}{\sqrt{x^2 - 2x + 10}} = \dfrac{3}{\sqrt{(x-1)^2 + 9}}$ and $\sin u = \dfrac{|x-1|}{\sqrt{(x-1)^2 + 9}}$.

So, $u = \arcsin \dfrac{|x-1|}{\sqrt{x^2 - 2x + 10}}$.

1.7.54.



Let $u = \arccos \dfrac{x-2}{2}$, $2 < x < 4$

$\cos u = \dfrac{x-2}{2}$, $\arccos \dfrac{x-2}{2} = \arctan \dfrac{\sqrt{4x - x^2}}{x-2}$, $2 < x < 4$

1.7.55.

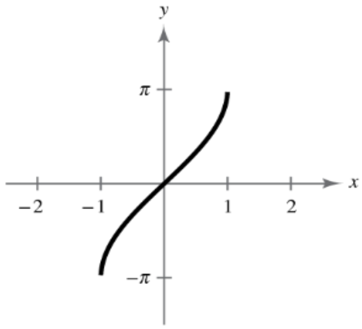
$g(x) = 2 \arcsin x$

Domain: $-1 \le x \le 1$

Range: $-\pi \le y \le \pi$

This is the graph of

$f(x) = \arcsin(x)$ with a vertical stretch.



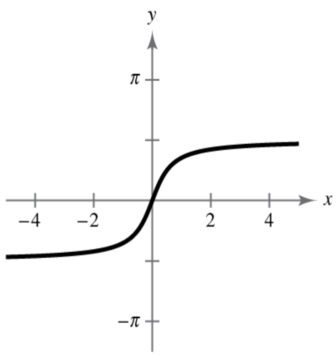
1.7.56.

$f(x) = \arctan 2x$

Domain: all real numbers

Range: $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

This is the graph of $g(x) = \arctan(x)$ with a horizontal shrink.

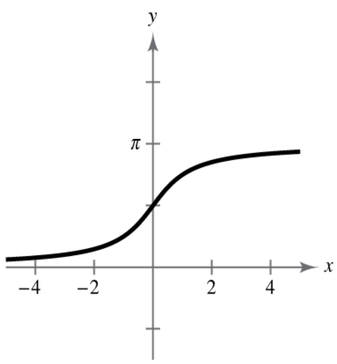


1.7.57.

$f(x) = \dfrac{\pi}{2} + \arctan x$

Domain: all real numbers
Range: $0 < y \le \pi$

This is the graph of $y = \arctan x$ shifted upward $\pi/2$ units.



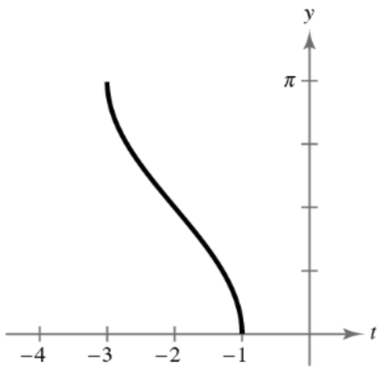
1.7.58.

$g(t) = \arccos(t+2)$

Domain: $-3 \le t \le -1$

Range: $0 \le y \le \pi$

This is the graph of $y = \arccos t$ shifted two units to the left.



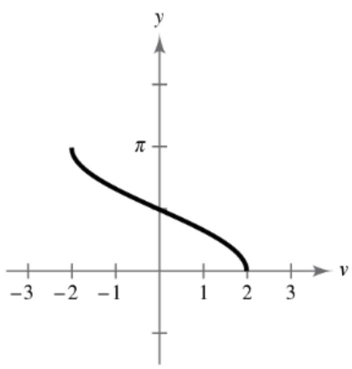
1.7.59.

$h(v) = \arccos \dfrac{v}{2}$

Domain: $-2 \le v \le 2$

Range: $0 \le y \le \pi$

This is the graph of $h(v) = \arccos v$ with a horizontal stretch.



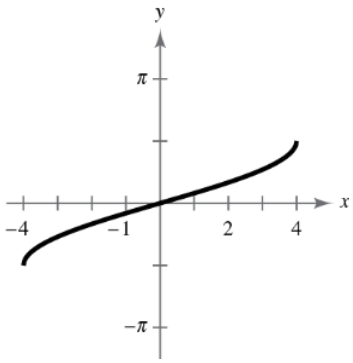
1.7.60.

$f(x) = \arcsin \dfrac{x}{4}$

Domain: $-4 \le x \le 4$

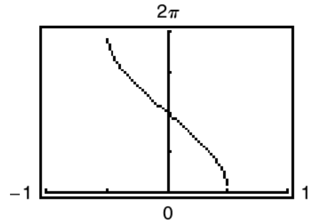
Range: $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$

This is the $f(x) = \arcsin x$ with a horizontal stretch.



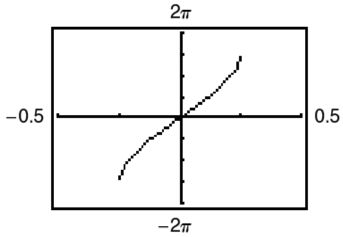
1.7.61.

$$f(x) = 2\arccos(2x)$$



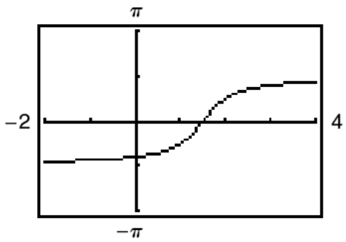
1.7.62.

$$f(x) = \pi \arcsin(4x)$$



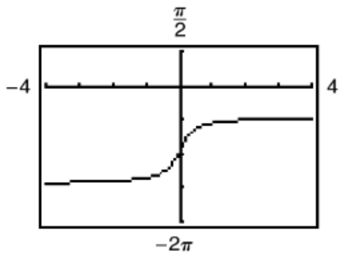
1.7.63.

$$f(x) = \arctan(2x - 3)$$



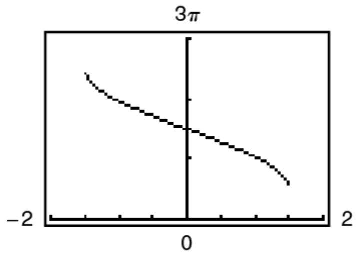
1.7.64.

$$f(x) = -3 + \arctan(\pi x)$$



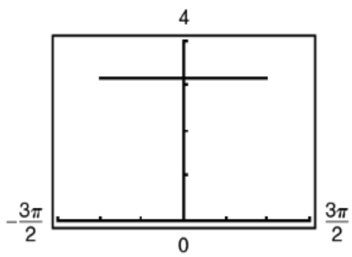
1.7.65.

$$f(x) = \pi - \sin^{-1}\frac{2x}{3} + \cos^{-1}\frac{2x}{3}$$



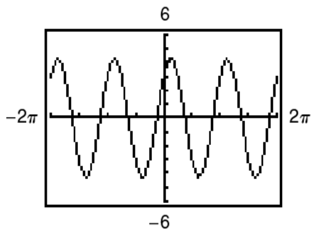
1.7.66.

$$f(x) = \frac{\pi}{2} + \cos^{-1}\frac{x}{\pi} + \sin^{-1}\frac{x}{\pi}$$



1.7.67.

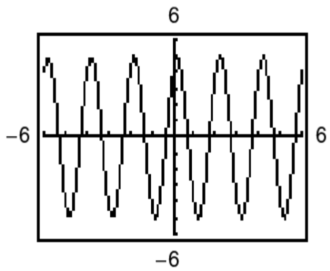
$$\begin{aligned} f(t) = 3\cos 2t + 3\sin 2t &= \sqrt{3^2 + 3^2}\sin\left(2t + \arctan\frac{3}{3}\right) \\ &= 3\sqrt{2}\sin(2t + \arctan 1) \\ &= 3\sqrt{2}\sin\left(2t + \frac{\pi}{4}\right) \end{aligned}$$



The graph implies that the identity is true.

1.7.68.

$$
\begin{aligned}
f(t) &= 4\cos \pi t + 3\sin \pi t \\
&= \sqrt{4^2+3^2}\sin\left(\pi t + \arctan\frac{4}{3}\right) \\
&= 5\sin\left(\pi t + \arctan\frac{4}{3}\right)
\end{aligned}
$$



The graph implies that $A\cos \omega t + B\sin \omega t = \sqrt{A^2+B^2}\sin\left(\omega t + \arctan\frac{A}{B}\right)$ is true.

1.7.69.

$$\frac{\pi}{2}$$

1.7.70.

0

1.7.71.

$$\frac{\pi}{2}$$

1.7.72.

$$-\frac{\pi}{2}$$

1.7.73.

(a) $\sin\theta = \frac{5}{s}$

$\theta = \arcsin\frac{5}{s}$

(b) $s = 40: \theta = \arcsin\frac{5}{40} \approx 0.13$

$s = 20: \theta = \arcsin\frac{5}{20} \approx 0.25$

1.7.74.

(a) $\tan\theta = \frac{s}{750}$

$\theta = \arctan\frac{s}{750}$

(b) When $s = 300, \theta = \arctan\frac{300}{750} \approx 0.38 \approx 21.8°$.

When $s = 1200, \theta = \arctan\frac{1200}{750} \approx 1.01 \approx 58.0°$.

1.7.75.

(a) $\sin\theta = \frac{6}{x}$

$\theta = \arcsin\frac{6}{x}$

(b) $x = 12$ miles

$\theta = \arcsin\frac{6}{12} \approx 30.0°$

$x = 7$ miles

$\theta = \arcsin\frac{6}{7} \approx 59.0°$

1.7.76.

(a) $\tan\theta = \frac{x}{20}$

$\theta = \arctan\frac{x}{20}$

(b) $x = 5: \theta = \arctan\frac{5}{20} \approx 14.0°$

$x = 12: \theta = \arctan\frac{12}{20} \approx 31.0°$

1.7.77.

False. The domain of arctan x is all real x, but the domain of both arcsin x and arccos x are $-1 \le x \le 1$.

$$\arctan(-1) = -\frac{\pi}{4} \ne \frac{\arcsin(-1)}{\arcsin(-1)} = \frac{\left(-\frac{\pi}{2}\right)}{\pi} = -\frac{1}{2}$$

1.7.78.

False. $\sin^{-1} x \ne \dfrac{1}{\sin x}$

The function $\sin^{-1} x$ is equivalent to $\arcsin x$, which is the inverse sine function. The expression, $\dfrac{1}{\sin x}$ is the reciprocal of the sine function and is equivalent to $\csc x$.

1.7.79.

False.

$\dfrac{5\pi}{6}$ is not in the range of $\arcsin(x)$.

$$\arcsin\frac{1}{2} = \frac{\pi}{6}$$

1.7.80.

True. $-\dfrac{\pi}{4}$ is in the range of the arctangent function.

$$\tan\left(-\frac{\pi}{4}\right) = -1 \Leftrightarrow \arctan(-1) = -\frac{\pi}{4}.$$

1.7.81.

The range of the arcsine function is $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$. $\dfrac{5\pi}{4}$ is not in the range $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$ of the arcsine function. However, the reference angle for $\dfrac{5\pi}{4}$ is $\dfrac{5\pi}{4} - \pi = \dfrac{\pi}{4}$. Because sine is negative in Quadrants III and IV, you have

$$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}.$$

1.7.82.

(a) Because the graph of $y = \arcsin x$ lies below the graph of $y = \arccos x$, when x is greater than or equal to −1 and less than $\sqrt{2}/2$, $\arcsin x < \arccos x$ for $-1 \le x < \sqrt{2}/2$.

(b) The graphs intersect at the point $\left(\sqrt{2}/2, \pi/4\right)$, so $\arcsin x = \arccos x$ when

$$x = \frac{\sqrt{2}}{2}.$$

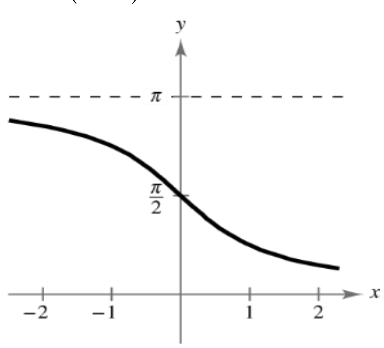
(c) Because the graph of $y = \arcsin x$ lies above the graph of $y = \arccos x$ when x is greater than $\frac{\sqrt{2}}{2}$ and less than or equal to 1, $\arcsin x > \arccos x$ for $\sqrt{2}/2 < x \le 1.$

1.7.83.

$y = \operatorname{arccot} x$ if and only if $\cot y = x$.

Domain: $(-\infty, \infty)$

Range: $(0, \pi)$



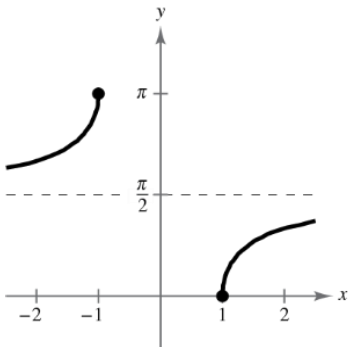
1.7.84.

$y = \operatorname{arcsec} x$ if and only if $\sec y = x$ where

$x \le -1 \cup x \ge 1$ and $0 \le y < \frac{\pi}{2}$ and $\frac{\pi}{2} < y \le \pi.$

Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

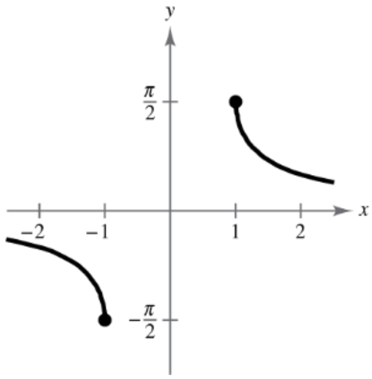


1.7.85.

$y = \operatorname{arccsc} x$ if and only if $\csc y = x$.

Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $\left[-\dfrac{\pi}{2}, 0\right) \cup \left(0, \dfrac{\pi}{2}\right]$



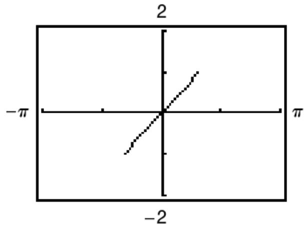
1.7.86.

Answers will vary.

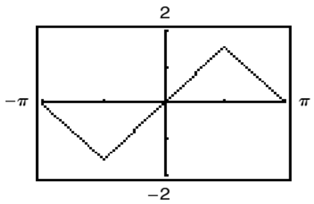
1.7.87.

$f(x) = \sin(x), f^{-1}(x) = \arcsin(x)$

(a) $f \circ f^{-1} = \sin(\arcsin x)$



$f^{-1} \circ f = \arcsin(\sin x)$



(b) The graphs coincide with the graph of $y = x$ only for certain values of x.

$f \circ f^{-1} = x$ over its entire domain, $-1 \le x \le 1$.

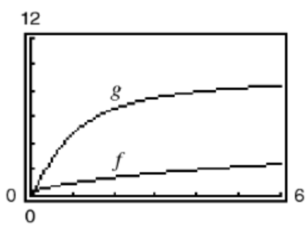
$f^{-1} \circ f = x$ over the region $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$, corresponding to the region where $\sin x$ is one-to-one and has an inverse.

1.7.88.

$f(x) = \sqrt{x}$

$g(x) = 6 \arctan x$

As x increases to infinity, g approaches 3π, but f has no maximum. Using technology, you find $a \approx 87.54$.



1.7.89.

Area $= \arctan b - \arctan a$

(a) $a = 0, b = 1$

Area $= \arctan 1 - \arctan 0 = \dfrac{\pi}{4} - 0 = \dfrac{\pi}{4}$

(b) $a = -1, b = 1$

$$\text{Area} = \arctan 1 - \arctan(-1)$$
$$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

(c) $a = 0, b = 3$

$$\text{Area} = \arctan 3 - \arctan 0$$
$$\approx 1.25 - 0 = 1.25$$

(d) $a = -1, b = 3$

$$\text{Area} = \arctan 3 - \arctan(-1)$$
$$\approx 1.25 - \left(-\frac{\pi}{4}\right) \approx 2.03$$

1.7.90.

(a) Let $y = \arcsin(-x)$. Then,

$$\begin{aligned} \sin y &= -x \\ -\sin y &= x \\ \sin(-y) &= x \\ -y &= \arcsin x \\ y &= -\arcsin x. \end{aligned}$$

So, $\arcsin(-x) = -\arcsin x$.

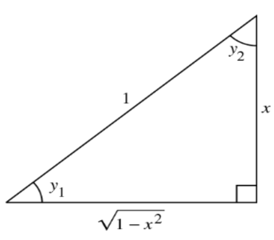
(b) Let $y = \arctan(-x)$. Then,

$$\begin{aligned} \tan y &= -x, -\frac{\pi}{2} < y < \frac{\pi}{2} \\ -\tan y &= x \\ \tan(-y) &= x, -\frac{\pi}{2} < -y < \frac{\pi}{2} \\ \arctan\left(\tan(-y)\right) &= \arctan x \\ -y &= \arctan x \\ y &= -\arctan x \end{aligned}$$

So, $\arctan(-x) = -\arctan(x)$.

(c) Let $y_2 = \frac{\pi}{2} - y_1$.

$$\begin{aligned} \arctan x + \arctan\ \frac{1}{x} &= y_1 + y_2 \\ &= y_1 + \left(\frac{\pi}{2} - y_1\right) \\ &= \frac{\pi}{2} \end{aligned}$$



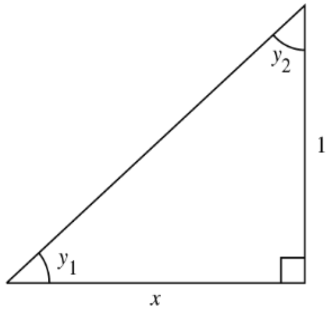
(d) Let $\alpha = \arcsin x$ and $\beta = \arccos x$, then $\sin\alpha = x$ and $\cos\beta = x$. So, $\sin\alpha = \cos\beta$ which implies that α and β are complementary angles and you have

$$\alpha + \beta = \frac{\pi}{2}$$

$$\arcsin x + \arccos x = \frac{\pi}{2}.$$

(e) $\arcsin x = \arcsin \dfrac{x}{1}$

$$= \arctan \frac{x}{\sqrt{1-x^2}}$$



1.7.91.

$$x^2 + 8^2 = 12^2 = 144$$
$$x^2 = 144 - 64 = 80$$
$$x = \sqrt{80} = 4\sqrt{5}$$

1.7.92.

$$x^2 + 3^2 = 5^2$$
$$x^2 = 25 - 9 = 16$$
$$x = 4$$

1.7.93.

$$x^2 + 2^2 = 6^2$$
$$x^2 = 36 - 4 = 32$$
$$x = 4\sqrt{2}$$

1.7.94.

$$x^2 + 9^2 = \left(x+1\right)^2 = x^2 + 2x + 1$$
$$81 = 2x + 1$$
$$80 = 2x$$
$$x = 40$$

1.7.95.

$$\cos\ 40° = \frac{c}{5} \Rightarrow c = 5\ \cos\ 40°$$

1.7.96.

$$\sin\ 40^\circ = \frac{5}{c} \Rightarrow c = \frac{5}{\sin\ 40^\circ}$$

1.7.97.

$$\tan\ 40^\circ = \frac{5}{c} \Rightarrow c = \frac{5}{\tan\ 40^\circ}$$

1.7.98.

$$\tan\ c^\circ = \frac{5}{6}$$

Section 1.8 Applications and Models

1.8.1.

bearing

1.8.2.

harmonic motion

1.8.3.

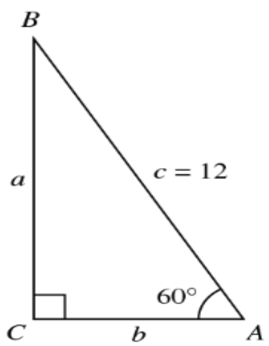
No. $N\,20°\,E$ means a direction first of due north, then $20°$ east of north.

1.8.4.

The frequency is the reciprocal of the period.

1.8.5.

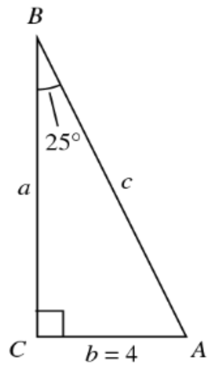
Given: $A = 60°, c = 12$



$$\sin A = \frac{a}{c} \Rightarrow a = c \sin A$$
$$= 12 \sin 60° = \frac{12\sqrt{3}}{2}$$
$$= 6\sqrt{3} \approx 10.39$$
$$\cos A = \frac{b}{c} \Rightarrow b = c \cos A$$
$$= 12 \cos 60° = 12\left(\frac{1}{2}\right) = 6$$
$$B = 90° - 60° = 30°$$

1.8.6.

Given: $B = 25°,\ b = 4$



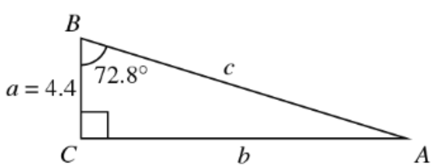
$$A = 90° - B$$
$$= 90° - 25° = 65°$$

$$\sin B = \frac{b}{c} \Rightarrow c = \frac{b}{\sin B}$$
$$= \frac{4}{\sin 25°} \approx 9.46$$

$$\tan B = \frac{b}{a} \Rightarrow a = \frac{b}{\tan B}$$
$$= \frac{4}{\tan 25°} \approx 8.58$$

1.8.7.

Given: $B = 72.8°,\ a = 4.4$



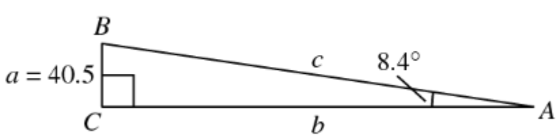
$$\cos B = \frac{a}{c} \Rightarrow c = \frac{a}{\cos B} = \frac{4.4}{\cos 72.8°} \approx 14.88$$

$$\tan B = \frac{b}{a} \Rightarrow b = a \tan B = 4.4 \tan 72.8° \approx 14.21$$

$$A = 90° - 72.8° = 17.2°$$

1.8.8.

Given: $A = 8.4°,\ a = 40.5$



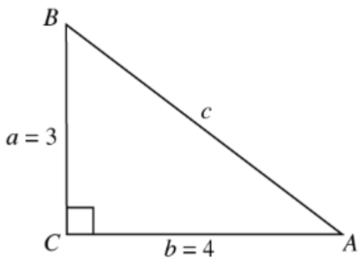
$$B = 90° - A$$
$$= 90° - 8.4° = 81.6°$$

$$\tan A = \frac{a}{b} \Rightarrow b = \frac{a}{\tan A}$$
$$= \frac{40.5}{\tan 8.4°} \approx 274.27$$

$$\sin A = \frac{a}{c} \Rightarrow c = \frac{a}{\sin A} = \frac{40.5}{\sin 8.4°} \approx 277.24$$

1.8.9.

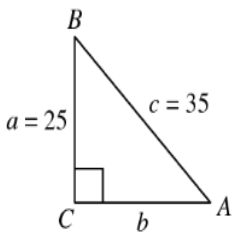
Given: $a = 3, b = 4$



$$a^2 + b^2 = c^2 \Rightarrow c^2 = (3)^2 + (4)^2 \Rightarrow c = 5$$
$$\tan A = \frac{a}{b} \Rightarrow A = \tan^{-1}\left(\frac{a}{b}\right) = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.87°$$
$$B = 90° - 36.87° = 53.13°$$

1.8.10.

Given: $a = 25, c = 35$

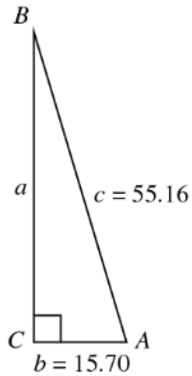


$$b = \sqrt{c^2 - a^2}$$
$$= \sqrt{35^2 - 25^2}$$
$$= \sqrt{600} \approx 24.49$$

$$\sin A = \frac{a}{c} \Rightarrow A = \arcsin\frac{a}{c} = \arcsin\frac{25}{35} \approx 45.58°$$
$$\cos B = \frac{a}{c} \Rightarrow B = \arccos\frac{a}{c} = \arccos\frac{25}{35} \approx 44.42°$$

1.8.11.

Given: $b = 15.70, c = 55.16$



$$a = \sqrt{55.16^2 - 15.7^2} \approx 52.88$$

$$\cos A = \frac{b}{c}$$

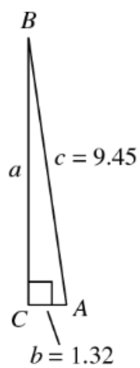
$$\cos A = \frac{15.7}{55.15}$$

$$A = \arccos \frac{15.7}{55.15} \approx 73.46^\circ$$

$$B = 90^\circ - 73.46^\circ \approx 16.54^\circ$$

1.8.12.

Given: $b = 1.32, c = 9.45$



$$a = \sqrt{c^2 - b^2} = \sqrt{87.5601} \approx 9.36$$

$$\cos A = \frac{b}{c} \Rightarrow A = \arccos \frac{b}{c} = \arccos \frac{1.32}{9.45} \approx 81.97^\circ$$

$$\sin B = \frac{b}{c} \Rightarrow B = \arcsin \frac{b}{c}$$

$$= \arcsin \frac{1.32}{9.45}$$

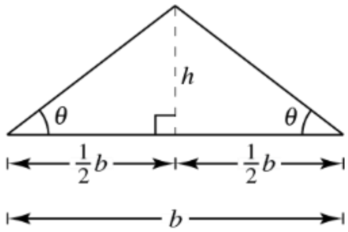
$$\approx 8.03^\circ$$

1.8.13.

$\theta = 45°, b = 6$

$$\tan\theta = \frac{h}{(1/2)b} \Rightarrow h = \frac{1}{2}b\tan\theta$$

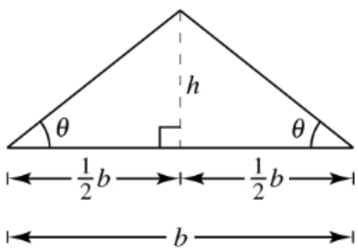
$$h = \frac{1}{2}(6)\tan 45° = 3.00 \text{ units}$$



1.8.14.

$$\tan\theta = \frac{h}{(1/2)b} \Rightarrow h = \frac{1}{2}b\tan\theta$$

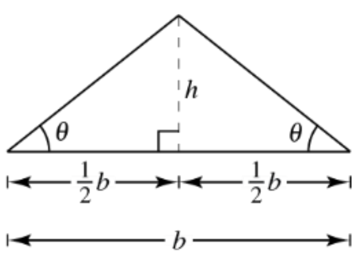
$$h = \frac{1}{2}(14)\tan 22° \approx 2.83 \text{ units}$$



1.8.15.

$$\tan\theta = \frac{h}{(1/2)b} \Rightarrow h = \frac{1}{2}b\tan\theta$$

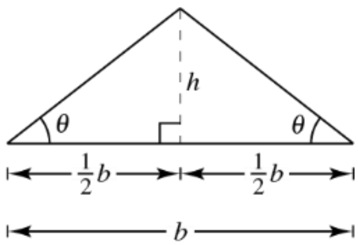
$$h = \frac{1}{2}(8)\tan 32° \approx 2.50 \text{ units}$$



1.8.16.

$$\tan\theta = \frac{h}{(1/2)b} \Rightarrow h = \frac{1}{2}b\tan\theta$$

$$h = \frac{1}{2}(11)\tan 27° \approx 2.80 \text{ units}$$

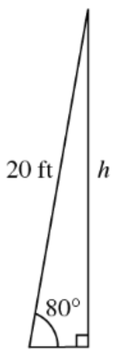


1.8.17.

$$\sin 80° = \frac{h}{20}$$

$$20\sin 80° = h$$

$$h \approx 19.7 \text{ feet}$$

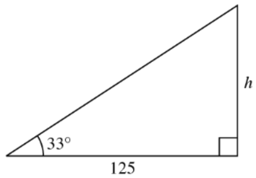


1.8.18.

$$\tan 33° = \frac{h}{125}$$

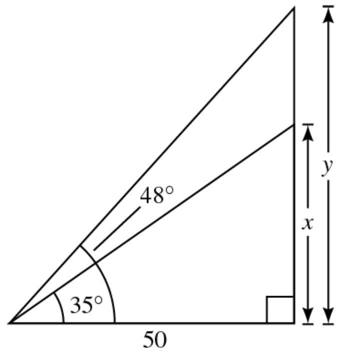
$$h = 125\tan 33$$

$$\approx 81.2 \text{ feet}$$



1.8.19.

Let the height of the castle $= x$ and the height of the castle and tower $= y$.



$\tan 35° = \dfrac{x}{50}$ and $\tan 48° = \dfrac{y}{50}$

$x = 50 \tan 35° \approx 35.01$ and $y = 50 \tan 48° \approx 55.53$

$h = y - x = 55.53 - 35.01 = 20.52.$

$h \approx 20.5$ feet

1.8.20.

$$\tan 57° = \frac{a}{x} \Rightarrow x = a \cot 57°$$

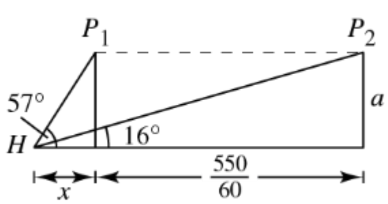
$$\tan 16° = \frac{a}{x + (55/6)}$$

$$\tan 16° = \frac{a}{a \cot 57° + (55/6)}$$

$$\cot 16° = \frac{a \cot 57° + (55/6)}{a}$$

$$a \cot 16° - a \cot 57° = \frac{55}{6} \Rightarrow a \approx 3.23 \text{ miles}$$

$$\approx 17{,}054 \text{ feet}$$



1.8.21.

(a) $\sin \theta = \dfrac{\text{opp}}{\text{hyp}}$

$\sin 60° = \dfrac{h}{30}$

$h = 15\sqrt{3} \approx 25.98$ feet

(b) $\tan\theta = \dfrac{25.981}{d}$

$$\theta = \arctan\left(\frac{25.981}{d}\right)$$

(c) When $\theta = 25°$,

$$\tan 25° = \frac{25.981}{d}$$

$$d = \frac{25.981}{\tan 25°}$$

$$d \approx 55.72 \text{ feet.}$$

When $\theta = 30°$,

$$\tan 30° = \frac{25.981}{d}$$

$$d = \frac{25.981}{\tan 30°}$$

$$d \approx 45.00$$

$45 \text{ ft} \le d \le 55.7$ feet.

1.8.22.

(a) $l^2 = (200)^2 + (150)^2$

$$l = 250 \text{ feet}$$

$$\tan A = \frac{150}{200} \Rightarrow A = \arctan\left(\frac{150}{200}\right) \approx 36.87°$$

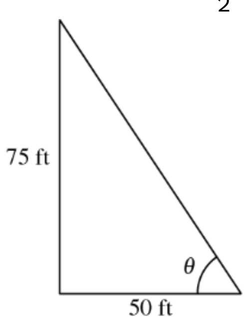
$$\tan B = \frac{200}{150} \Rightarrow B = \arctan\left(\frac{200}{150}\right) \approx 53.13°$$

(b) $250 \text{ ft} \times \dfrac{\text{mile}}{5280 \text{ ft}} \times \dfrac{\text{hour}}{35 \text{ miles}} \times \dfrac{3600 \text{ sec}}{\text{hour}} \approx 4.87$ seconds

1.8.23.

$\tan\theta = \frac{75}{50}$

$$\theta = \arctan\tfrac{3}{2} \approx 56.3°$$



1.8.24.

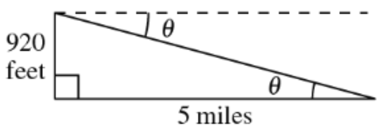
$$1200 \text{ feet} + 120 \text{ feet} - 400 \text{ feet} = 920 \text{ feet}$$

$$5 \text{ miles} = 5 \text{ miles}\left(\frac{5280 \text{ feet}}{1 \text{ mile}}\right) = 26,400 \text{ feet}$$

$$\tan\theta = \frac{920}{26,400}$$

$$\theta = \arctan\left(\frac{920}{26,400}\right) \approx 2.0^\circ$$

Not drawn to scale



1.8.25.

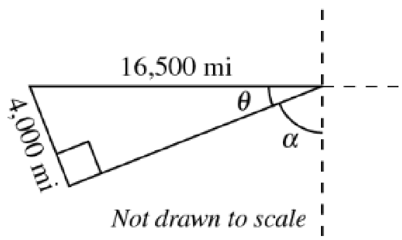
$$12,500 + 4000 = 16,500$$

$$\sin\theta = \frac{4000}{16,500}$$

$$\theta = \arcsin\left(\frac{4000}{16,500}\right)$$

$$\theta \approx 14.03^\circ$$

Angle of depression $= \alpha \approx 90^\circ - 14.03^\circ = 75.97^\circ$



1.8.26.

(a) $l^2 = (h + 17)^2 + 100^2$

$$l = \sqrt{(h + 17)^2 + 10,000}$$

$$= \sqrt{h^2 + 34h + 10,289}$$

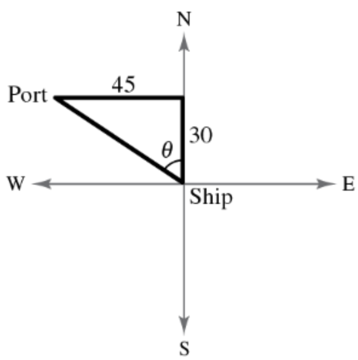
(b) $\cos\theta = \dfrac{100}{l}$

$$\theta = \arccos\left(\frac{100}{l}\right)$$

(c) $\cos\theta = \dfrac{100}{l}$

$$\cos 35^\circ = \frac{100}{l}$$
$$l \approx 122.077$$
$$l^2 = 100^2 + (h+17)^2$$
$$l^2 = h^2 + 34h + 10.289$$
$$0 = h^2 + 34h - 4613.794$$
$$h \approx 53.02 \text{ feet}$$

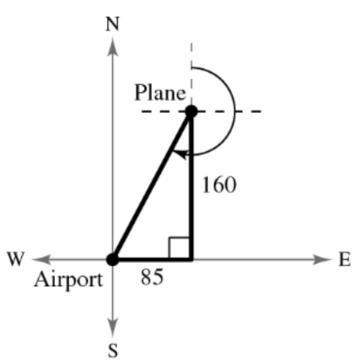
1.8.27.



$\tan\theta = \frac{45}{30} \Rightarrow \theta \approx 56.3^\circ$

Bearing: N 56.31°

1.8.28.

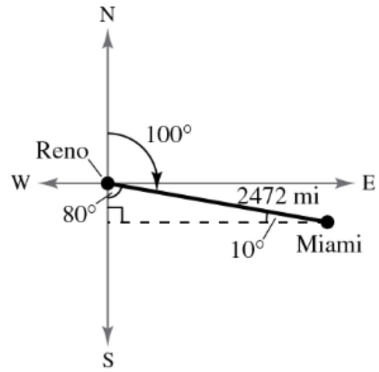


$$\text{Bearing} = 180^\circ + \arctan\left(\tfrac{85}{160}\right)$$
$$= 208.0^\circ$$

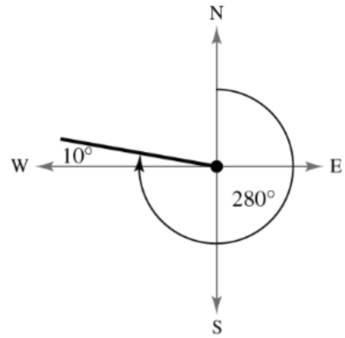
1.8.29.

(a) Reno is $2472 \sin 10^\circ = 429.26$ miles north of Miami.

Reno is $2472 \cos 10^\circ = 2434.44$ miles west of Miami.

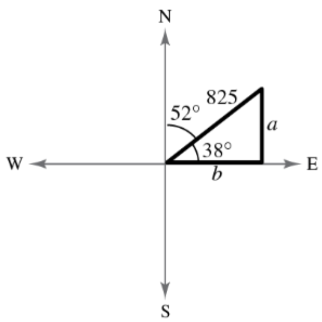


(b) The return heading is $280°$.



1.8.30.

The plane has traveled $1.5(550) = 825$ miles.



$$\sin 38° = \frac{a}{825} \Rightarrow a \approx 508 \text{ miles north}$$

$$\cos 38° = \frac{b}{825} \Rightarrow b \approx 650 \text{ miles east}$$

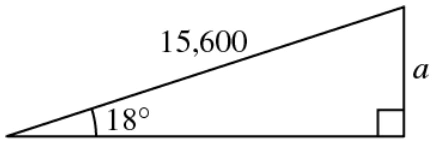
1.8.31.

(a) Because the airplane speed is

$$\left(260\frac{\text{ft}}{\text{sec}}\right)\left(60\frac{\text{sec}}{\text{min}}\right) = 15{,}600\frac{\text{ft}}{\text{min}},$$

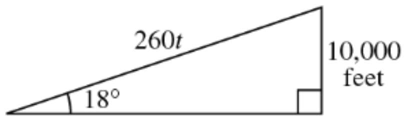
after one minute its distance travelled is 15,600 feet.

$$\sin 18^\circ = \frac{a}{15,600}$$
$$a = 15,600 \sin 18^\circ \approx 4820.7 \text{ ft}$$

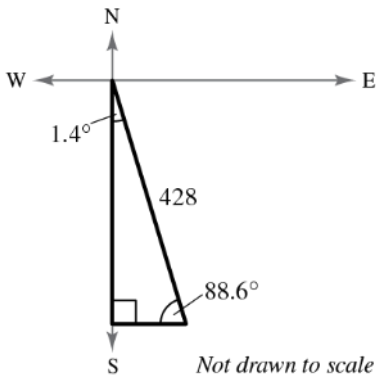


(b) Let t be the time for the plane to reach 10,000 ft.

$$\sin 18^\circ = \frac{10,000}{260t}$$
$$s = \frac{10,000}{260(\sin 18^\circ)} \approx 124.5 \text{ seconds}$$



1.8.32.



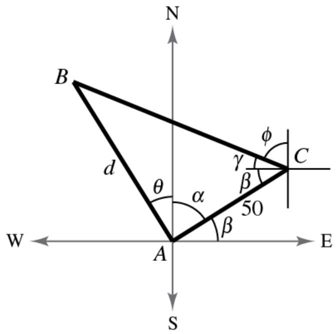
(a) $t = \frac{428}{20} = 21.4$ hours

(b) After 12 hours, the yacht will have traveled 240 nautical miles.
$240 \sin 1.4^\circ \approx 5.86$ miles east
$240 \cos 1.4^\circ \approx 239.93$ miles south

(c) Bearing from N is 178.6°.

1.8.33.

$\theta = 32°, \phi = 68°$



(a) $\alpha = 90° - 32° = 58°$

Bearing from A to C: N 58° E

(b) $\beta = \theta = 32°$

$\gamma = 90° - \phi = 22°$

$C = \beta + \gamma = 54°$

$$\tan C = \frac{d}{50} \Rightarrow \tan 54°$$

$$= \frac{d}{50} \Rightarrow d \approx 68.82 \text{ meters}$$

1.8.34.

$$\tan 14° = \frac{d}{x} \Rightarrow x = d \cot 14°$$

$$\tan 34° = \frac{d}{y} = \frac{d}{30 - x}$$

$$= \frac{d}{30 - d \cot 14°}$$

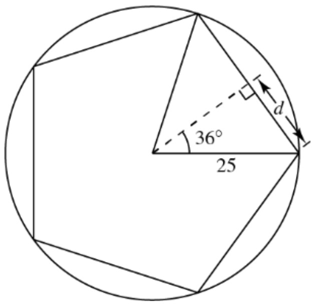
$$\cot 34° = \frac{30 - d \cot 14°}{d}$$

$$d \cot 34° = 30 - d \cot 14°$$

$$d = \frac{30}{\cot 34° + \cot 14°}$$

$$\approx 5.46 \text{ kilometers}$$

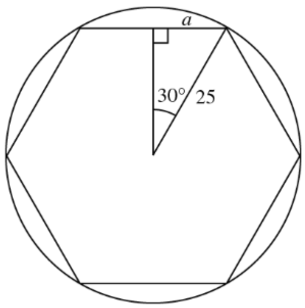
1.8.35.



$\sin 36° = \dfrac{d}{25} \Rightarrow d \approx 14.69$

Length of side: $2d \approx 29.4$ inches

1.8.36.



$\sin 30° = \dfrac{a}{25}$

$a = 25 \sin 30° = 12.5$

$$\begin{aligned}\text{Length of side} &= 2a = 2(12.5)\\ &= 25 \text{ inches}\end{aligned}$$

1.8.37.

Use $d = a \sin \omega t$ because $d = 0$ when $t = 0$.

Period: $\dfrac{2\pi}{\omega} = 2 \Rightarrow \omega = \pi$

So, $d = 4 \sin(\pi t)$.

1.8.38.

Use $d = a \sin \omega t$ because $d = 0$ when $t = 0$.

Amplitude: $|a| = 3$

Period: $\dfrac{2\pi}{\omega} = 6 \Rightarrow \omega = \dfrac{\pi}{3}$

So, $d = 3 \sin\left(\frac{\pi t}{3}\right)$.

1.8.39.

Use $d = a \cos \omega t$ because $d = 3$ when $t = 0$.

Period: $\frac{2\pi}{\omega} = 1.5 \Rightarrow \omega = \frac{4\pi}{3}$

So, $d = 3 \cos\left(\frac{4\pi}{3} t\right) = 3 \cos\left(\frac{4\pi t}{3}\right)$.

1.8.40.

Use $d = a \cos \omega t$ because $d = 2$ when $t = 0$.

Amplitude: $|a| = 2$

Period: $\frac{2\pi}{\omega} = 10 \Rightarrow \omega = \frac{\pi}{5}$

So, $d = 2 \cos\left(\frac{\pi t}{5}\right)$.

1.8.41.

$$d = a \sin \omega t$$

$$\text{Frequency} = \frac{\omega}{2\pi}$$

$$262 = \frac{\omega}{2\pi}$$

$$\omega = 2\pi(262) = 524\pi$$

1.8.42.

At $t = 0$, buoy is at its high point $\Rightarrow d = a \cos \omega t$.

Distance from high to low $= 2|a| = 3.5$

$$|a| = \frac{7}{4}$$

Returns to high point every 10 seconds:

Period: $\frac{2\pi}{\omega} = 10 \Rightarrow \omega = \frac{\pi}{5}$

$$d = \frac{7}{4} \cos \frac{\pi t}{5}$$

1.8.43.

$$d = 9 \cos \frac{6\pi}{5} t$$

(a) Maximum displacement $=$ amplitude $= 9$

(b) Frequency $= \dfrac{\omega}{2\pi} = \dfrac{\frac{6\pi}{5}}{2\pi}$

$= \dfrac{3}{5}$ cycle per unit of time

(c) $d = 9\cos\dfrac{6\pi}{5}(5) = 9$

(d)
$$\begin{aligned} 9\cos\frac{6\pi}{5}t &= 0 \\ \cos\frac{6\pi}{5}t &= 0 \\ \frac{6\pi}{5}t &= \arccos 0 \\ \frac{6\pi}{5}t &= \frac{\pi}{2} \\ t &= \frac{5}{12} \end{aligned}$$

1.8.44.

$d = \dfrac{1}{2}\cos 20\pi t$

(a) Maximum displacement: $|a| = \left|\dfrac{1}{2}\right| = \dfrac{1}{2}$

(b) Frequency: $\dfrac{\omega}{2\pi} = \dfrac{20\pi}{2\pi} = 10$ cycles per unit of time

(c) $d = \dfrac{1}{2}\cos 100\pi \approx \dfrac{1}{2}$

(d)
$$\begin{aligned} \frac{1}{2}\cos 20\pi t &= 0 \\ \cos 20\pi t &= 0 \\ 20\pi t &= \arccos 0 \\ 20\pi t &= \frac{\pi}{2} \\ t &= \frac{\pi}{2}\cdot\frac{1}{20\pi} = \frac{1}{40} \end{aligned}$$

1.8.45.

$d = \dfrac{1}{4}\sin 6\pi t$

(a) Maximum displacement $=$ amplitude $= \dfrac{1}{4}$

(b) Frequency $= \dfrac{\omega}{2\pi} = \dfrac{6\pi}{2\pi}$

$= 3$ cycles per unit of time

(c) $d = \dfrac{1}{4}\sin 30\pi \approx 0$

(d) $$\frac{1}{4}\sin 6\pi t = 0$$
$$\sin 6\pi t = 0$$
$$6\pi t = \arcsin 0$$
$$6\pi t = \pi$$
$$t = \frac{1}{6}$$

1.8.46.

$$d = \frac{1}{64}\sin 792\pi t$$

(a) Maximum displacement: $|a| = \left|\dfrac{1}{64}\right| = \dfrac{1}{64}$

(b) Frequency:

$$\frac{\omega}{2\pi} = \frac{792\pi}{2\pi} = 396 \text{ cycles per unit of time}$$

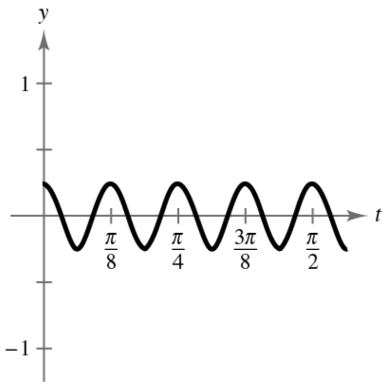
(c) $d = \dfrac{1}{64}\sin(3960\pi) = 0$

(d) $$\frac{1}{64}\sin 792\pi t = 0$$
$$\sin 792\pi t = 0$$
$$792\pi t = \arcsin 0$$
$$792\pi t = \pi$$
$$t = \frac{\pi}{792\pi} = \frac{1}{792}$$

1.8.47.

$$y = \frac{1}{4}\cos 16t,\ t > 0$$

(a)

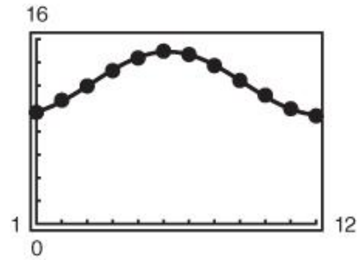


(b) Period: $\dfrac{2\pi}{16} = \dfrac{\pi}{8}$

(c) $\dfrac{1}{4}\cos 16t = 0$ when $16t = \dfrac{\pi}{2} \Rightarrow t = \dfrac{\pi}{32}$

1.8.48.

(a)



(b) $H(t) = 12.19 + 2.73\sin\left(\dfrac{\pi t}{6} - 1.66\right)$

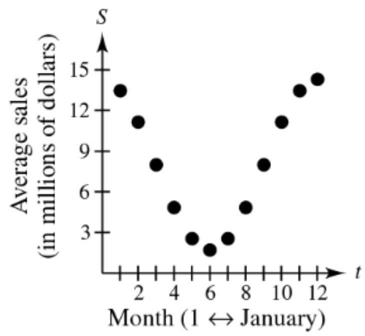
$\text{Period} = \dfrac{2\pi}{b} = \dfrac{2\pi}{\pi/6} = 12$

The period is 12. Yes, there are 12 months in a year.

(c) The amplitude is $a = |2.73| = 2.73.$ The amplitude represents the maximum change from the mean number of hours of daylight.

1.8.49.

(a)



(b) $a = \frac{1}{2}(14.3 - 1.7) = 6.3$

$\frac{2\pi}{b} = 12 \Rightarrow b = \frac{\pi}{6}$

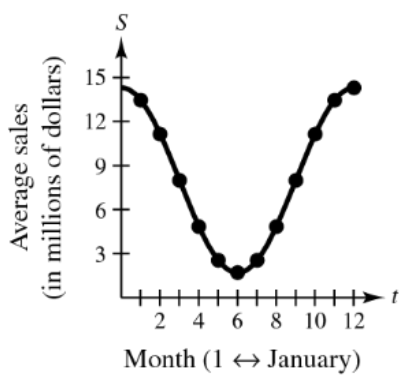
Shift: $d = 14.3 - 6.3$

$= 8$

$S = d + a \cos bt$

$S = 8 + 6.3 \cos\left(\frac{\pi t}{6}\right)$

Note: Another model is $S = 8 + 6.3 \sin\left(\frac{\pi t}{6} + \frac{\pi}{2}\right)$.



The model is a good fit.

(c) The period is $\frac{2\pi}{(\pi/6)} = 12.$ Yes, sales of outwear are seasonal.

(d) The amplitude is the maximum displacement from average sales of $8 million.

1.8.50.

False. N $24°$E means 24 degrees east of north.

1.8.51.

False. The tower isn't vertical and so the triangle formed is not a right triangle.

1.8.52.

(a) The maximum displacement from $d = 0$ is 4, so the amplitude is 4 centimeters.

(b) The period is the time for the graph to complete one cycle is 3 seconds.

(c) Because the spring is at maximum displacement when $t = 0$, the equation is of the form $d = a\cos wt$.

1.8.53.

$$6t^3 - 2t + 10t^2 = 2t\left(3t^2 + 5t - 1\right)$$

1.8.54.

$$d^2 + 7d - 18 = \left(d + 9\right)\left(d - 2\right)$$

1.8.55.

$$10y^2 - 13y - 3 = \left(5y + 1\right)\left(2y - 3\right)$$

1.8.56.

$$\begin{aligned} -3x^2 + 22x - 24 &= -\left(3x^2 - 2x + 24\right) \\ &= -\left(x - 6\right)\left(3x - 4\right) \end{aligned}$$

1.8.57.

$$25z^4 - x^2 = \left(5z^2 - x\right)\left(5z^2 + x\right)$$

1.8.58.

$$w^3 + 125v^3 = \left(w + 5v\right)\left(w^2 - 5wv + 25v^2\right)$$

1.8.59.

$$\sin\theta = \frac{1}{4}$$

(a) $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \left(\frac{1}{4}\right)^2 = \frac{15}{16}$

$$\Rightarrow \cos\theta = \frac{\sqrt{15}}{4}$$

(b) $\cot\theta + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\left(\dfrac{\sqrt{15}}{4}\right)}{\left(\dfrac{1}{4}\right)} = \sqrt{15}$

(c) $\csc\theta = \dfrac{1}{\sin\theta} = 4$

(d) $\tan(90° - \theta) = \cot\theta = \sqrt{15}$

1.8.60.

$\csc\alpha = 6$

(a) $\sin\alpha = \dfrac{1}{\csc\alpha} = \dfrac{1}{6}$

(b) $\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\dfrac{1}{36}} = \sqrt{\dfrac{35}{36}}$

$= \dfrac{\sqrt{35}}{6} \Rightarrow \sec\alpha = \dfrac{1}{\cos\ \alpha} = \dfrac{1}{\sqrt{35}} = \dfrac{6\sqrt{35}}{35}$

(c) $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\left(\dfrac{1}{6}\right)}{\left(\dfrac{\sqrt{35}}{6}\right)} = \dfrac{1}{\sqrt{35}} = \dfrac{\sqrt{35}}{35}$

(d) $\cos(90° - \alpha) = \sin\alpha = \dfrac{1}{6}$

1.8.61.

$$\begin{aligned}(x-0)^2 + (y-3)^2 &= 7^2\\ x^2 + (y-3)^2 &= 49\end{aligned}$$

1.8.62.

$$\begin{aligned}(x-5)^2 + (y-(-2))^2 &= 2^2\\ (x-5)^2 + (y+2)^2 &= 4\end{aligned}$$

1.8.63.

$$\begin{aligned}(x+2)^2 + (y+1)^2 &= r^2\\ (1+2)^2 + (3+1)^2 &= r^2\\ 9 + 16 &= r^2\\ (x+2)^2 + (y+1)^2 &= 25\end{aligned}$$

1.8.64.

$$\text{Center} = \left(\frac{1+11}{2},\ \frac{12-12}{2}\right) = (6,\ 0)$$

$$(x-6)^2 + y^2 = r^2$$

$$(1,\ 12) \text{ on circle} \Rightarrow (1-6)^2 + 12^2 = r^2 \Rightarrow r^2 = 169$$

$$(x-6)^2 + y^2 = 169$$

REVIEW EXERCISES FOR CHAPTER 1

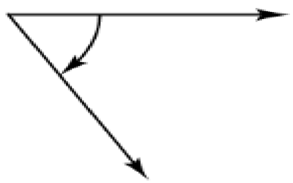
1.R.1.

The angle is approximately 2 radians.

1.R.2.

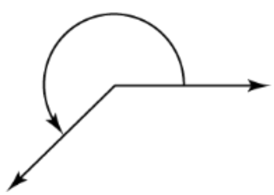
The angle is approximately -4.5 radians.

1.R.3.



The angle shown is $60°$.

1.R.4.

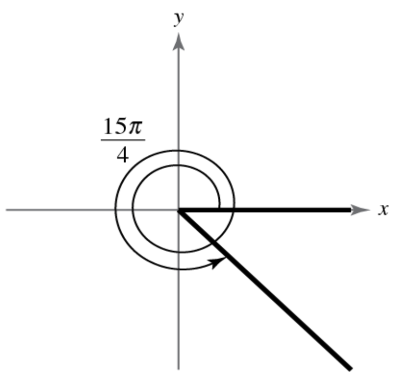


The angle shown is $225°$.

1.R.5.

$\theta = \dfrac{15\pi}{4}$

(a)



(b) Quadrant IV

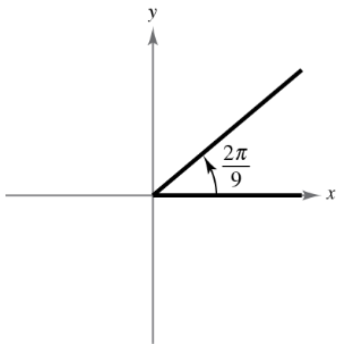
(c) $\dfrac{15\pi}{4} - 2\pi = \dfrac{7\pi}{4}$

$\dfrac{7\pi}{4} - 2\pi = -\dfrac{\pi}{4}$

1.R.6.

$$\theta = \frac{2\pi}{9}$$

(a)



(b) Quadrant I

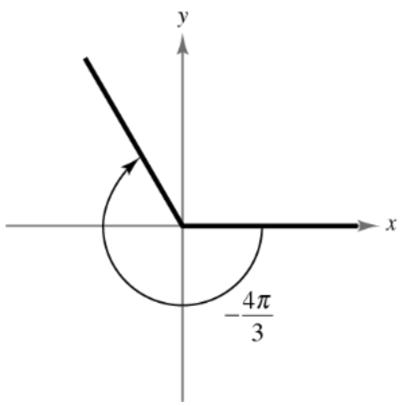
(c) $\dfrac{2\pi}{9} + 2\pi = \dfrac{20\pi}{9}$

$\dfrac{2\pi}{9} - 2\pi = -\dfrac{16\pi}{9}$

1.R.7.

$$\theta = -\frac{4\pi}{3}$$

(a)



(b) Quadrant II

(c) Coterminal angles:

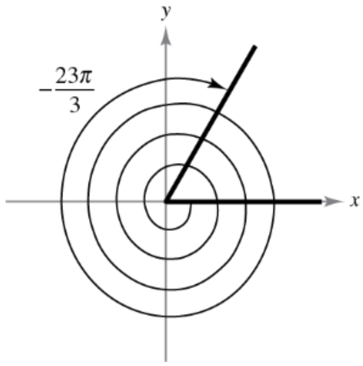
$-\dfrac{4\pi}{3} + 2\pi = \dfrac{2\pi}{3}$

$-\dfrac{4\pi}{3} - 2\pi = -\dfrac{10\pi}{3}$

1.R.8.

$\theta = -\dfrac{23\pi}{3}$

(a)



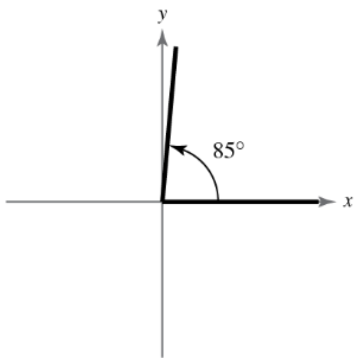
(b) Quadrant I

(c) $-\dfrac{23\pi}{3} + 8\pi = \dfrac{\pi}{3}$

$-\dfrac{23\pi}{3} + 2\pi = -\dfrac{17\pi}{3}$

1.R.9.

(a) $\theta = 85°$



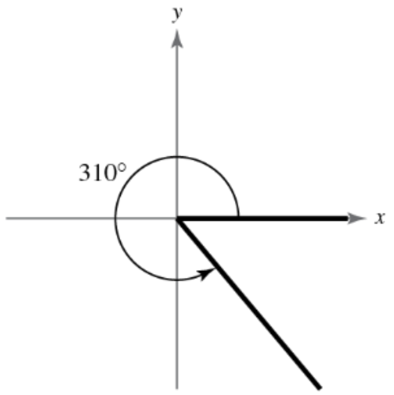
(b) The angle lies in Quadrant I.

(c) Coterminal angles: $85° + 360° = 445°$

$85° - 360° = -275°$

1.R.10.

(a) $\theta = 310°$



(b) The angle lies in Quadrant IV.

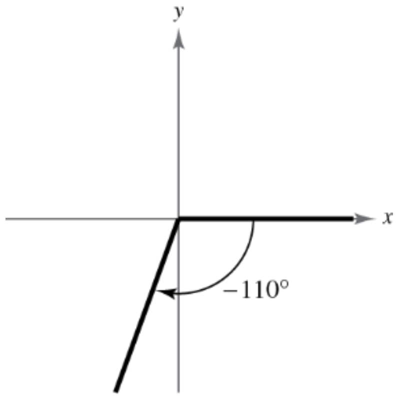
(c) Coterminal angles: $310° + 360° = 670°$

$310° - 360° = -50°$

1.R.11.

$\theta = -110°$

(a)



(b) Quadrant III

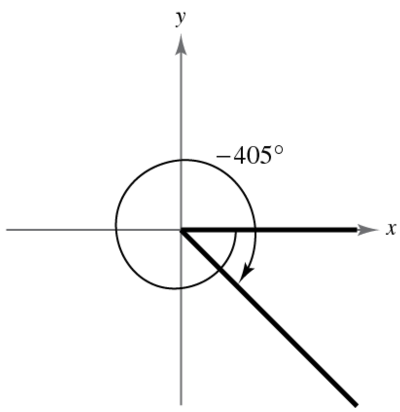
(c) Coterminal angles: $-110° + 360° = 250°$

$-110° - 360° = -470°$

1.R.12.

$\theta = -405°$

(a)



(b) Quadrant IV

(c) $-405° + 720° = 315°$

$-405° + 360° = -45°$

1.R.13.

$$138° = \frac{138\pi}{180} = \frac{23\pi}{30} \text{ radians}$$

$$s = r\theta = 20\left(\frac{23\pi}{30}\right) \approx 48.17 \text{ inches}$$

1.R.14.

(a) $$\text{Angular speed} = \frac{(5000)(2\pi) \text{ radians}}{1 \text{ minute}}$$
$$= 10{,}000\pi \text{ radians per minute}$$

(b) $$\text{Linear speed} = \frac{(6)(10{,}000\pi) \text{ centimeters}}{1 \text{ minute}}$$
$$= 60{,}000\pi \text{ centimeters per minute}$$

1.R.15.

$$150° = \frac{150\pi}{180} = \frac{5\pi}{6} \text{ radians}$$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(20)^2\left(\frac{5\pi}{6}\right) = \frac{500\pi}{3} \approx 523.6 \text{ square inches}$$

1.R.16.

$$A = \frac{1}{2}\theta r^2 = \frac{1}{2}\left(\frac{2\pi}{3}\right)(7.5)^2 = \frac{75\pi}{4} \approx 58.9 \text{ square millimeters}$$

1.R.17.

$t = \dfrac{2\pi}{3}$ corresponds to the point $(x, y) = \left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$.

1.R.18.

$t = \dfrac{7\pi}{4}$ corresponds to the point $(x, y) = \left(\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$.

1.R.19.

$t = \dfrac{7\pi}{6}$ corresponds to the point $(x, y) = \left(-\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right)$.

1.R.20.

$t = -\dfrac{4\pi}{3}$ corresponds to the point $(x, y) = \left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$.

1.R.21.

$t = \dfrac{3\pi}{4}$ corresponds to the point $(x, y) = \left(-\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$.

$$\sin\frac{3\pi}{4} = y = \frac{\sqrt{2}}{2} \qquad \csc\frac{3\pi}{4} = \frac{1}{y} = \sqrt{2}$$

$$\cos\frac{3\pi}{4} = x = -\frac{\sqrt{2}}{2} \qquad \sec\frac{3\pi}{4} = \frac{1}{x} = -\sqrt{2}$$

$$\tan\frac{3\pi}{4} = \frac{y}{x} = -1 \qquad \cot\frac{3\pi}{4} = \frac{x}{y} = -1$$

1.R.22.

$t = -\dfrac{2\pi}{3}$ corresponds to the point $(x, y) = \left(-\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$.

$$\sin\left(-\frac{2\pi}{3}\right) = y = -\frac{\sqrt{3}}{2} \qquad \csc\left(-\frac{2\pi}{3}\right) = \frac{1}{y} = -\frac{2\sqrt{3}}{3}$$
$$\cos\left(-\frac{2\pi}{3}\right) = x = -\frac{1}{2} \qquad \sec\left(-\frac{2\pi}{3}\right) = \frac{1}{x} = -2$$
$$\tan\left(-\frac{2\pi}{3}\right) = \frac{y}{x} = \sqrt{3} \qquad \cot\left(-\frac{2\pi}{3}\right) = \frac{x}{y} = \frac{\sqrt{3}}{3}$$

1.R.23.

$$\sin\frac{11\pi}{4} = \sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$$

1.R.24.

$$\cos 4\pi = \cos 0 = 1$$

1.R.25.

$$\cos\left(-\frac{17\pi}{6}\right) = \cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$$

1.R.26.

$$\sin\left(-\frac{13\pi}{3}\right) = \sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$$

1.R.27.

$$\text{opp} = 4,\ \text{adj} = 5,\ \text{hyp} = \sqrt{4^2 + 5^2} = \sqrt{41}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{41}}{4}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{41}}{5}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{5} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{5}{4}$$

1.R.28.

$$\text{adj} = 4,\ \text{hyp} = 8,\ \text{opp} = \sqrt{8^2 - 4^2} = \sqrt{48} = 4\sqrt{3}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{8}{4\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{8} = \frac{1}{2} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{8}{4} = 2$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4\sqrt{3}}{4} = \sqrt{3} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{4\sqrt{3}} = \frac{\sqrt{3}}{3}$$

1.R.29.

$$\tan 33° \approx 0.6494$$

1.R.30.

$$\sec 79.3° = \frac{1}{\cos 79.3°} \approx 5.3860$$

1.R.31.

$$\cot 15°14' = \frac{1}{\tan\left(15 + \frac{14}{60}\right)} \approx 3.6722$$

1.R.32.

$$\cos 78°\ 11'\ 58'' = \cos\left(78 + \tfrac{11}{60} + \tfrac{58}{3600}\right)° \approx 0.2045$$

1.R.33.

$$\sin\theta = \frac{1}{3}$$

(a) $\csc\theta = \dfrac{1}{\sin\theta} = 3$

(b)
$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1\\
\left(\frac{1}{3}\right)^2 + \cos^2\theta &= 1\\
\cos^2\theta &= 1 - \frac{1}{9}\\
\cos^2\theta &= \frac{8}{9}\\
\cos\theta &= \sqrt{\frac{8}{9}}\\
\cos\theta &= \frac{2\sqrt{2}}{3}
\end{aligned}$$

(c) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{3}{2\sqrt{2}} = \dfrac{3\sqrt{2}}{4}$

(d) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{1/3}{\left(2\sqrt{2}\right)/3} = \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$

1.R.34.

$\csc\theta = 5$

(a) $\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{5}$

(b) $\cot\theta = \sqrt{\csc^2\theta - 1} = \sqrt{25-1} = 2\sqrt{6}$

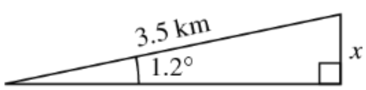
(c) $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{1}{2\sqrt{6}} = \dfrac{\sqrt{6}}{12}$

(d) $\sec\left(90^\circ - \theta\right) = \csc\theta = 5$

1.R.35.

$\sin 1.2^\circ = \dfrac{x}{3.5}$

$x = 3.5 \sin 1.2^\circ \approx 0.0733$ kilometer or 73.3 meters

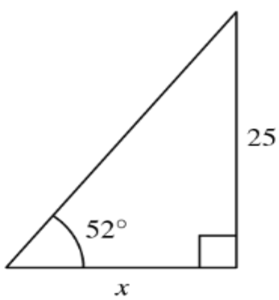


Not drawn to scale

1.R.36.

$\tan 52^\circ = \dfrac{25}{x}$

$x = \dfrac{25}{\tan 52^\circ} \approx 19.5$ feet



1.R.37.

$$x = 12,\ y = 16,\ r = \sqrt{144 + 256} = \sqrt{400} = 20$$

$$\sin\theta = \frac{y}{r} = \frac{4}{5} \qquad \csc\theta = \frac{r}{y} = \frac{5}{4}$$

$$\cos\theta = \frac{x}{r} = \frac{3}{5} \qquad \sec\theta = \frac{r}{x} = \frac{5}{3}$$

$$\tan\theta = \frac{y}{x} = \frac{4}{3} \qquad \cot\theta = \frac{x}{y} = \frac{3}{4}$$

1.R.38.

$$x = 3,\ y = -4$$

$$r = \sqrt{3^2 + (-4)^2} = 5$$

$$\sin\theta = \frac{y}{r} = -\frac{4}{5} \qquad \csc\theta = \frac{r}{y} = -\frac{5}{4}$$

$$\cos\theta = \frac{x}{r} = \frac{3}{5} \qquad \sec\theta = \frac{r}{x} = \frac{5}{3}$$

$$\tan\theta = \frac{y}{x} = -\frac{4}{3} \qquad \cot\theta = \frac{x}{y} = -\frac{3}{4}$$

1.R.39.

$$x = 0.3,\ y = 0.4$$

$$r = \sqrt{(0.3)^2 + (0.4)^2} = 0.5$$

$$\sin\theta = \frac{y}{r} = \frac{0.4}{0.5} = \frac{4}{5} = 0.8 \qquad \csc\theta = \frac{r}{y} = \frac{0.5}{0.4} = \frac{5}{4} = 1.25$$

$$\cos\theta = \frac{x}{r} = \frac{0.3}{0.5} = \frac{3}{5} = 0.6 \qquad \sec\theta = \frac{r}{x} = \frac{0.5}{0.3} = \frac{5}{3} \approx 1.67$$

$$\tan\theta = \frac{y}{x} = \frac{0.4}{0.3} = \frac{4}{3} \approx 1.33 \qquad \cot\theta = \frac{x}{y} = \frac{0.3}{0.4} = \frac{3}{4} = 0.75$$

1.R.40.

$$x = -\frac{10}{3},\ y = -\frac{2}{3}$$

$$r = \sqrt{\left(-\frac{10}{3}\right)^2 + \left(-\frac{2}{3}\right)^2} = \frac{2\sqrt{26}}{3}$$

$$\sin\theta = \frac{y}{r} = \frac{-2/3}{\left(2\sqrt{26}\right)/3} = -\frac{\sqrt{26}}{26} \qquad \csc\theta = \frac{r}{y} = \frac{\left(2\sqrt{26}\right)/3}{-2/3} = -\sqrt{26}$$

$$\cos\theta = \frac{x}{r} = \frac{-10/3}{\left(2\sqrt{26}\right)/3} = -\frac{5\sqrt{26}}{26} \qquad \sec\theta = \frac{r}{x} = \frac{\left(2\sqrt{26}\right)/3}{-10/3} = -\frac{\sqrt{26}}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{-2/3}{-10/3} = \frac{1}{5} \qquad \cot\theta = \frac{x}{y} = \frac{-10/3}{-2/3} = 5$$

1.R.41.

$\sec\theta = \frac{6}{5}$, $\tan\theta < 0 \Rightarrow \theta$ is in Quadrant IV.

$r = 6,\ x = 5,\ y = -\sqrt{36-25} = -\sqrt{11}$

$$\sin\theta = \frac{y}{r} = -\frac{\sqrt{11}}{6}$$

$$\cos\theta = \frac{x}{r} = \frac{5}{6}$$

$$\tan\theta = \frac{y}{x} = -\frac{\sqrt{11}}{5}$$

$$\csc\theta = \frac{r}{y} = -\frac{6\sqrt{11}}{11}$$

$$\cot\theta = -\frac{5\sqrt{11}}{11}$$

1.R.42.

$\csc\theta = \frac{3}{2}$, $\cos\theta < 0 \Rightarrow \theta$ is in Quadrant II.

$$\sin\theta = \frac{1}{\csc\theta} = \frac{2}{3}$$

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\frac{\sqrt{5}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{2\sqrt{5}}{5}$$

$$\sec\theta = \frac{1}{\cos\theta} = -\frac{3\sqrt{5}}{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = -\frac{\sqrt{5}}{2}$$

1.R.43.

$$\cos\theta = \frac{x}{r} = \frac{-2}{5} \Rightarrow y^2 = 21$$

$$\sin\theta > 0 \Rightarrow \theta \text{ is in Quadrant II } \Rightarrow y = \sqrt{21}$$

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{21}}{5}$$

$$\tan\theta = \frac{y}{x} = -\frac{\sqrt{21}}{2}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$$

$$\sec\theta = \frac{r}{x} = \frac{5}{-2} = -\frac{5}{2}$$

$$\cot\theta = \frac{x}{y} = \frac{-2}{\sqrt{21}} = -\frac{2\sqrt{21}}{21}$$

1.R.44.

$$\sin\theta = -\frac{1}{2}, \cos\theta > 0$$

θ is in Quadrant IV.

$$\csc\theta = \frac{1}{\sin\theta} = -2$$

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{1}{4}\right)} = \frac{\sqrt{3}}{2}$$

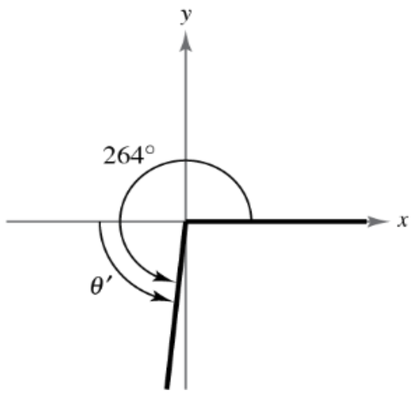
$$\sec\theta = \frac{1}{\cos\theta} = \frac{2\sqrt{3}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{\sqrt{3}}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = -\sqrt{3}$$

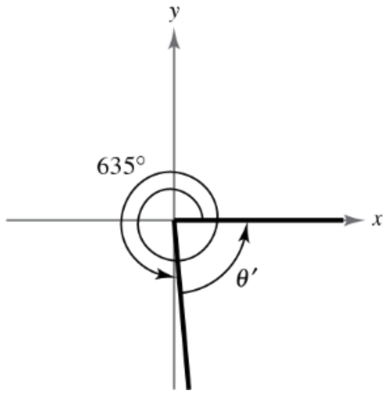
1.R.45.

$$\theta = 264°$$
$$= 264° - 180° = 84°$$



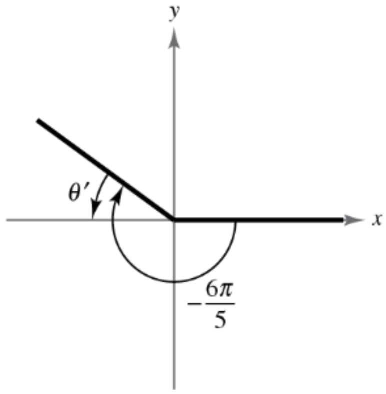
1.R.46.

$$\theta = 635° = 720° - 85°$$
$$\theta' = 85°$$



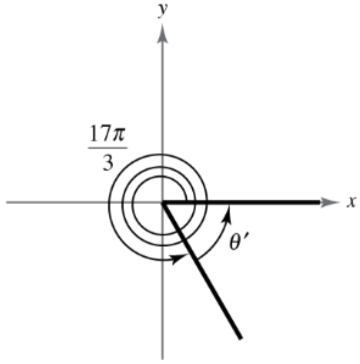
1.R.47.

$$\theta = -\frac{6\pi}{5}$$
$$-\frac{6\pi}{5} + 2\pi = \frac{4\pi}{5}$$
$$\theta' = \pi - \frac{4\pi}{5} = \frac{\pi}{5}$$



1.R.48.

$$\theta = \frac{17\pi}{3} = \frac{18\pi}{3} - \frac{\pi}{3}$$
$$= 6\pi - \frac{\pi}{3}$$
$$\theta' = \frac{\pi}{3}$$



1.R.49.

$$\sin(-150°) = -\frac{1}{2}$$
$$\cos(-150°) = -\frac{\sqrt{3}}{2}$$
$$\tan(-150°) = \frac{-1/2}{-\sqrt{3}/2} = \frac{\sqrt{3}}{3}$$

1.R.50.

$$\sin 495° = \sin 45° = \frac{\sqrt{2}}{2}$$
$$\cos 495° = -\cos 45° = -\frac{\sqrt{2}}{2}$$
$$\tan 495° = -\tan 45° = -1$$

1.R.51.

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$
$$\cos \frac{\pi}{3} = \frac{1}{2}$$
$$\tan \frac{\pi}{3} = \sqrt{3}$$

1.R.52.

$$\sin\left(-\frac{5\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos\left(-\frac{5\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

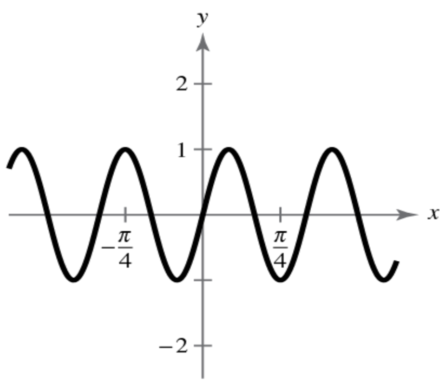
$$\tan\left(-\frac{5\pi}{4}\right) = -\tan\frac{\pi}{4} = \frac{2}{-\sqrt{2}/2} = -1$$

1.R.53.

$y = \sin 6x$

Amplitude: 1

Period: $\dfrac{2\pi}{6} = \dfrac{\pi}{3}$

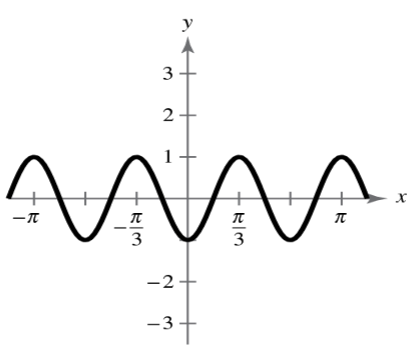


1.R.54.

$y = -\cos 3x$

Amplitude: 1

Period: $\dfrac{2\pi}{3}$

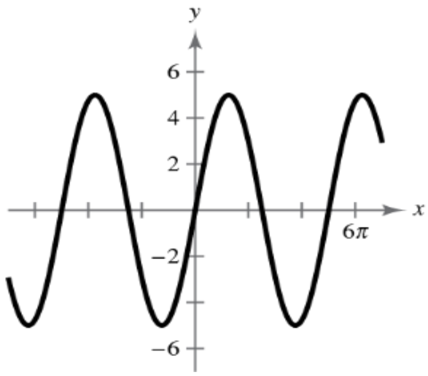


1.R.55.

$f(x) = 5 \sin \frac{2x}{5}$

Amplitude: 5

Period: $\frac{2\pi}{2/5} = 5\pi$

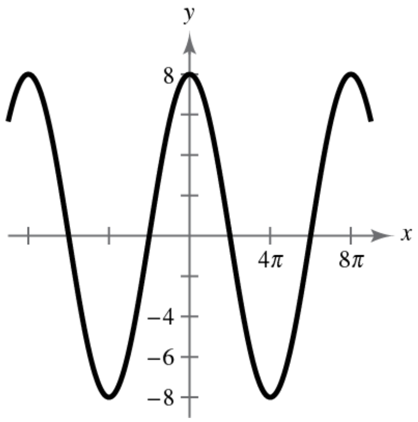


1.R.56.

$f(x) = 8 \cos\left(-\frac{x}{4}\right)$

Amplitude: 8

Period: $\frac{2\pi}{1/4} = 8\pi$

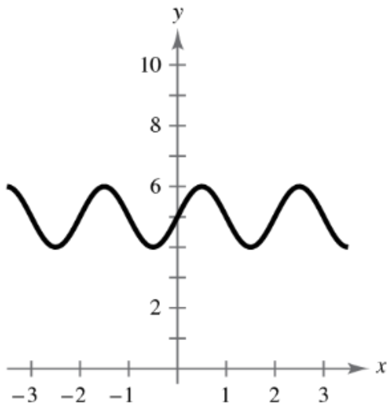


1.R.57.

$y = 5 + \sin \pi x$

Amplitude: 1

Period: $\frac{2\pi}{\pi} = 2$

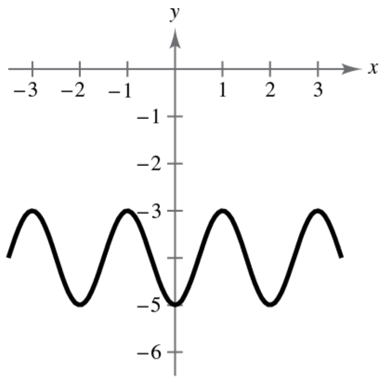


1.R.58.

$y = -4 - \cos \pi x$

Amplitude: 1

Period: $\dfrac{2\pi}{\pi} = 2$

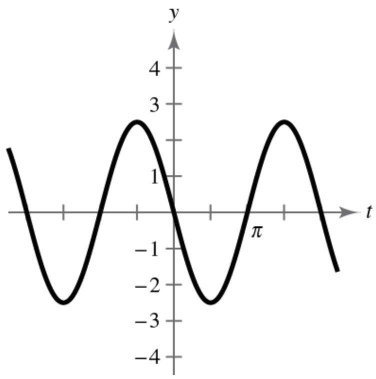


1.R.59.

$g(t) = \frac{5}{2}\sin(t - \pi)$

Amplitude: $\frac{5}{2}$

Period: 2π

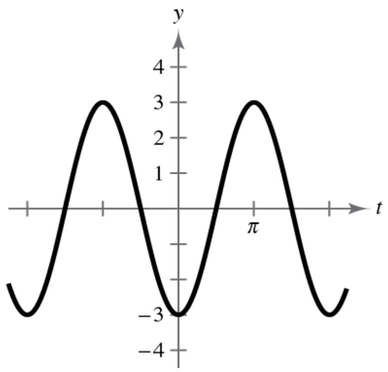


1.R.60.

$g(t) = 3\cos(t + \pi)$

Amplitude: 3

Period: 2π



1.R.61.

$y = a \sin bx$

(a) $a = 2$,

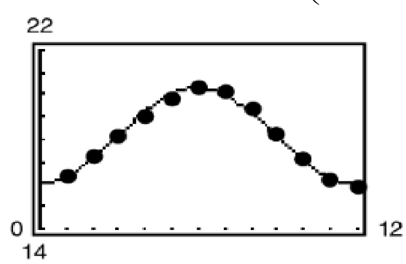
$\frac{2\pi}{b} = \frac{1}{264} \Rightarrow b = 528\pi$

$y = 2\sin 528\pi x$

(b) $f = \frac{1}{1/264} = 264$ cycles per second

1.R.62.

(a) $S(t) = 18.10 - 1.41\sin\left(\frac{\pi t}{6} + 1.55\right)$



(b) Period $= \frac{2\pi}{\pi/6} = (2)(6) = 12$

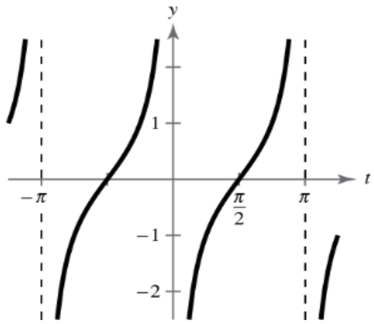
12 months $=$ 1 year, so this is expected.

(c) Amplitude: 1.41

The amplitude represents the maximum change in time of sunset from the average time $(d = 18.10)$.

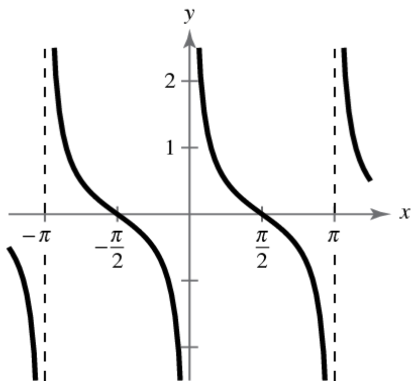
1.R.63.

$$f(t) = \tan\left(t + \frac{\pi}{2}\right)$$



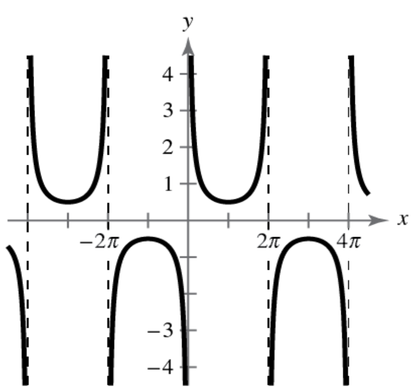
1.R.64.

$$f(x) = \tfrac{1}{2}\cot x$$



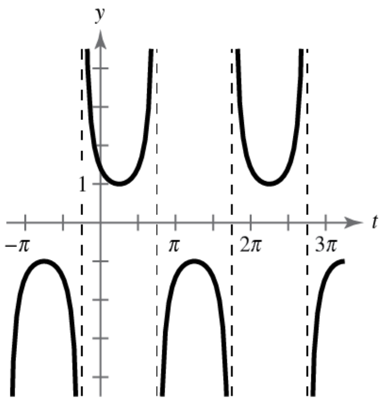
1.R.65.

$$f(x) = \frac{1}{2}\csc\frac{x}{2}$$



1.R.66.

$$h(t) = \sec\left(t - \frac{\pi}{4}\right)$$

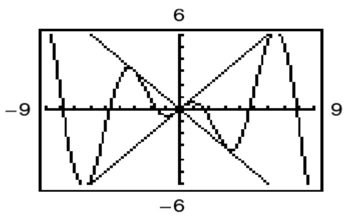


1.R.67.

$f(x) = x \cos x$

Damping factor: x

As $x \to \infty, f(x)$ oscillates.

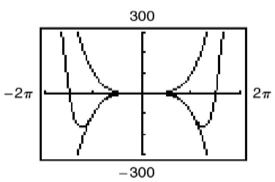


1.R.68.

$g(x) = x^4 \cos x$

Damping factor: x^4

As $x \to +\infty, f(x)$ oscillates.



1.R.69.

$$\arcsin(-1) = -\frac{\pi}{2}$$

1.R.70.

$$\cos^{-1}(1) = 0$$

1.R.71.

$$\operatorname{arccot}\sqrt{3} = \frac{\pi}{6}$$

1.R.72.

$$\operatorname{arcsec}\left(-\sqrt{2}\right) = \frac{3\pi}{4}$$

1.R.73.

The graph of g is a transformation of the graph of f.

$$g(x) = \arcsin(x+3) = f(x+3)$$

The graph of g can be obtained by shifting the graph of f three units to the left.

1.R.74.

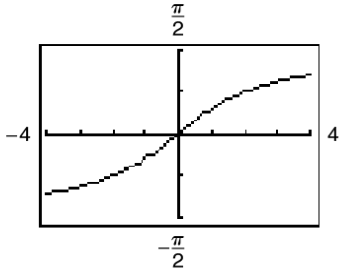
The graph of g is a transformation of the graph of f.

$$g(x) = \arccos(x+1) = f(x+1)$$

The graph of g can be obtained by shifting the graph of f one unit to the left.

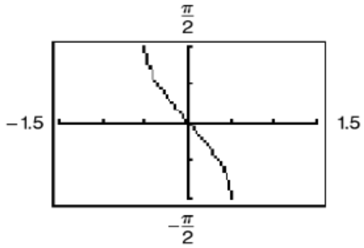
1.R.75.

$$f(x) = \arctan\left(\frac{x}{2}\right) = \tan^{-1}\left(\frac{x}{2}\right)$$



1.R.76.

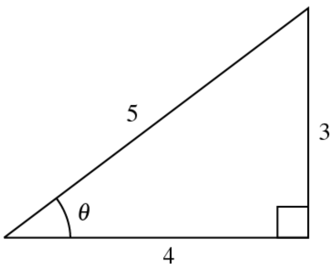
$$f(x) = -\arcsin 2x$$



1.R.77.

Let $u = \arctan \frac{3}{4}$ then $\tan u = \frac{3}{4}$.

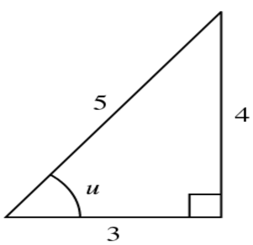
$\cos\left(\arctan \frac{3}{4}\right) = \frac{4}{5}$



1.R.78.

Let $u = \arccos \frac{3}{5}$.

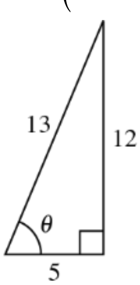
$\tan\left(\arccos \frac{3}{5}\right) = \tan u = \frac{4}{3}$



1.R.79.

Let $u = \arctan \frac{12}{5}$, then $\tan u = \frac{12}{5}$.

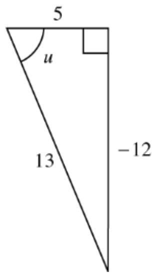
$\sec\left(\arctan \frac{12}{5}\right) = \frac{13}{5}$



1.R.80.

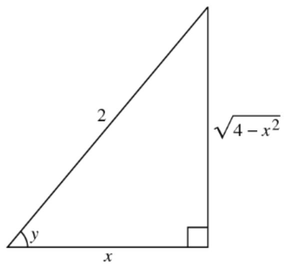
Let $u = \arcsin\left(-\frac{12}{13}\right)$.

$\cot\left[\arcsin\left(-\frac{12}{13}\right)\right] = \cot u = -\frac{5}{12}$



1.R.81.

Let $y = \arccos\left(\dfrac{x}{2}\right)$. Then $\cos y = \dfrac{x}{2}$ and $\tan y = \tan\left(\arccos\left(\dfrac{x}{2}\right)\right) = \dfrac{\sqrt{4-x^2}}{x}$.



1.R.82.

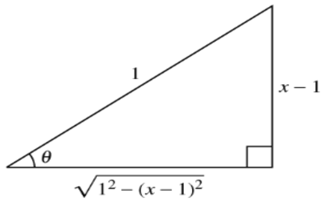
$\sec(\arcsin(x-1))$

$\theta = \arcsin(x-1) \Rightarrow -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$

$\sin\ \theta = x - 1$

$\cos\ \theta = \sqrt{1^2 - (x-1)^2}$

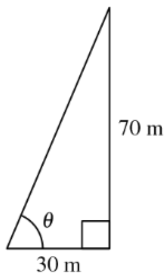
$\sec\ \theta = \dfrac{1}{\sqrt{1-(x-1)^2}}$



1.R.83.

$$\tan\theta = \frac{70}{30}$$

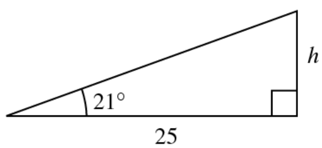
$$\theta = \arctan\left(\frac{70}{30}\right) \approx 66.8^\circ$$



1.R.84.

$$\tan 21^\circ = \frac{h}{25}$$

$$h = 25\tan 21^\circ \approx 9.6 \text{ feet}$$



1.R.85.

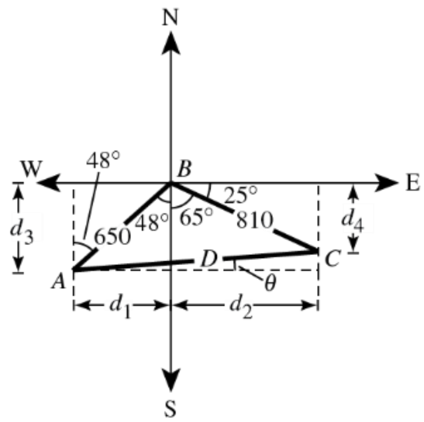
$$\left.\begin{aligned} \sin 48^\circ &= \frac{d_1}{650} \Rightarrow d_1 \approx 483 \\ \cos 25^\circ &= \frac{d_2}{810} \Rightarrow d_2 \approx 734 \end{aligned}\right\} \quad d_1 + d_2 \approx 1217$$

$$\left.\begin{aligned} \cos 48^\circ &= \frac{d_3}{650} \Rightarrow d_3 \approx 435 \\ \sin 25^\circ &= \frac{d_4}{810} \Rightarrow d_4 \approx 342 \end{aligned}\right\} \quad d_3 - d_4 \approx 93$$

$$\tan\theta \approx \frac{93}{1217} \Rightarrow \theta \approx 4.4^\circ$$

$$\sec 4.4^\circ \approx \frac{D}{1217} \Rightarrow D \approx 1217\sec 4.4^\circ \approx 1221$$

The distance is 1221 miles and the bearing is 85.6°.



1.R.86.

Amplitude: $\dfrac{1.5}{2} = 0.75$ inch

Period: 3 seconds

$d = a \cos bt$

$a = 0.75$

$b = \dfrac{2\pi}{3}$

$d = 0.75 \cos\left(\dfrac{2\pi t}{3}\right)$

1.R.87.

False. For each θ there corresponds exactly one value of y.

1.R.88.

False. $\dfrac{3\pi}{4}$ is not in the range $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$ of the arctangent function. However, the reference angle for $\dfrac{3\pi}{4}$ is $\pi - \dfrac{3\pi}{4} = \dfrac{\pi}{4}$. Because tangent is negative in Quadrants II and IV, you have $\tan^{-1}(-1) = -\dfrac{\pi}{4}$.

1.R.89.

$f(\theta) = \sec\theta$ is undefined at the zeros of $g(\theta) = \cos\theta$ because $\sec\theta = \dfrac{1}{\cos\ \theta}$.

1.R.90.

(a)

θ	0.1	0.4	0.7	1.0	1.3
$\tan\left(\theta - \frac{\pi}{2}\right)$	−9.9666	−2.3652	−1.1872	−0.6421	−0.2776
$-\cot\theta$	−9.9666	−2.3652	−1.1872	−0.6421	−0.2776

(b) $\tan\left(\theta - \frac{\pi}{2}\right) = -\cot\theta$

PROBLEM SOLVING FOR CHAPTER 1

1.PS.1.

(a) 8:57 − 6:45 = 2 hours 12 minutes = 132 minutes

$\frac{132}{48} = \frac{11}{4}$ revolutions

$\theta = \left(\frac{11}{4}\right)(2\pi) = \frac{11\pi}{2}$ radians or 990°

(b) $s = r\theta = 47.25(5.5\pi) \approx 816.42$ feet

1.PS.2.

If you alter the model so that $h = 1$ when $t = 0$, you can use either a sine or a cosine model.

$a = \frac{1}{2}[\max - \min] = \frac{1}{2}[101 - 1] = 50$

$d = \frac{1}{2}[\max + \min] = \frac{1}{2}[101 + 1] = 51$

$b = 8\pi$

Cosine model: $h = 51 - 50\cos(8\pi t)$

Sine model: $h = 51 - 50\sin\left(8\pi t + \frac{\pi}{2}\right)$

Notice that you needed the horizontal shift so that the sine value was one when $t = 0$.

Another model would be: $h = 51 + 50\sin\left(8\pi t + \frac{3\pi}{2}\right)$

Here you wanted the sine value to be 1 when $t = 0$.

1.PS.3.

Given that f is periodic with period $c \to f(t + c) = f(t)$

(a) $f(t - 2c) = f(t - 2c + 2c) = f(t)$

True. $f(t - 2c) = f(t)$

(b) $f\left(t + \frac{1}{2}c\right)$ has a period of c, while $f\left(\frac{1}{2}t\right)$ has a period of $\frac{c}{\frac{1}{2}} = 2c$.

False. $f\left(t + \frac{1}{2}c\right) \neq f\left(\frac{1}{2}t\right)$.

(c) $f\left[\frac{1}{2}(t+c)\right] = f\left(\frac{1}{2}t + \frac{1}{2}c\right)$ and $f\left(\frac{1}{2}t\right)$ both have a period of $\frac{c}{\frac{1}{2}} = 2c$, but

$$f\left[\frac{1}{2}(t+c)\right] = f\left(\frac{1}{2}t + \frac{1}{2}c - 2c\right) \neq f\left(\frac{1}{2}t\right)$$

False. $f\left[\frac{1}{2}(t+c)\right] \neq f\left(\frac{1}{2}t\right)$.

(d) $f\left[\frac{1}{2}(t+4c)\right] = f\left(\frac{1}{2}t + 2c\right)$ and $f\left(\frac{1}{2}t\right)$ both have a period of $\frac{c}{\frac{1}{2}} = 2c$ and

$$f\left[\frac{1}{2}(t+4c)\right] = f\left(\frac{1}{2}t + 2c - 2c\right) = f\left(\frac{1}{2}t\right)$$

True. $f\left[\frac{1}{2}(t+4c)\right] = f\left(\frac{1}{2}t\right)$.

1.PS.4.

(a) $\sin 39^\circ = \frac{3000}{d}$

$$d = \frac{3000}{\sin 39^\circ} \approx 4767 \text{ feet}$$

(b) $\tan 39^\circ = \frac{3000}{x}$

$$x = \frac{3000}{\tan 39^\circ} \approx 3705 \text{ feet}$$

(c)
$$\tan 63^\circ = \frac{w + 3705}{3000}$$
$$3000 \tan 63^\circ = w + 3705$$
$$w = 3000 \tan 63^\circ - 3705 \approx 2183 \text{ feet}$$

1.PS.5.

(a) ΔABC, ΔADE, and ΔAFG are all similar triangles because they all have the same angles. $\angle A$ is part of all three triangles and $\angle C = \angle E = \angle G = 90^\circ$. So, $\angle B = \angle D = \angle F$.

(b) Because the triangles are similar, the ratios of corresponding sides are equal.

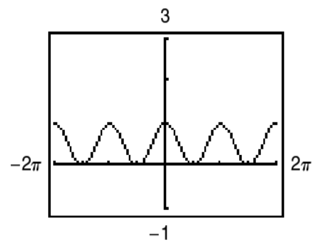
$$\frac{BC}{AB} = \frac{DE}{AD} = \frac{FG}{AF}$$

(c) Because the ratios: $\dfrac{\text{opp}}{\text{hyp}} = \dfrac{BC}{AB} = \dfrac{DE}{AD} = \dfrac{FG}{AF} = \sin A$ it does not matter which triangle is used to calculate sin *A*. Any triangle similar to these three triangles could be used to find sin *A*. The value of sin *A* would not change.

(d) Because the values of all six trigonometric functions can be found by taking the ratios of the sides of a right triangle, similar triangles would yield the same values.

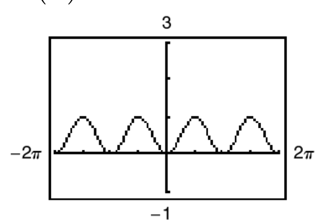
1.PS.6.

(a) $h(x) = \cos^2 x$



h is even.

(b) $h(x) = \sin^2 x$



h is even.

1.PS.7.

Given: *f* is an even function and *g* is an odd function.

(a) $h(x) = \left[f(x)\right]^2$

$$\begin{aligned} h(-x) &= \left[f(-x)\right]^2 \\ &= \left[f(x)\right]^2 \quad \text{since } f \text{ is even} \\ &= h(x) \end{aligned}$$

So, *h* is an even function.

(b) $h(x) = [g(x)]^2$

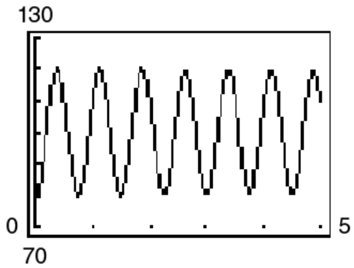
$$\begin{aligned} h(-x) &= [g(-x)]^2 \\ &= [-g(x)]^2 \quad \text{since } g \text{ is odd} \\ &= [g(x)]^2 \\ &= h(x) \end{aligned}$$

So, h is an even function.

Conjecture: The square of either an even function or an odd function is an even function.

1.PS.8.

$$P = 100 - 20\cos\left(\frac{8\pi}{3}t\right)$$

(a) 

(b) Period $= \dfrac{2\pi}{8\pi/3} = \dfrac{6}{8} = \dfrac{3}{4}$ sec

This is the time between heartbeats.

(c) Amplitude: 20

The blood pressure ranges between $100 - 20 = 80$ and $100 + 20 = 120$.

(d) Pulse rate $= \dfrac{60 \text{ sec/min}}{\frac{3}{4} \text{ sec/beat}} = 80$ beats/min

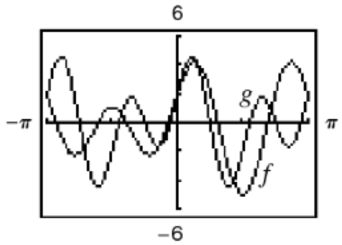
(e) Period $= \dfrac{60}{64} = \dfrac{15}{16}$ sec

$$64 = \frac{60}{2\pi/b} \Rightarrow b = \frac{64}{60} \cdot 2\pi = \frac{32}{15}\pi$$

1.PS.9.

$$f(x) = 2\cos 2x + 3\sin 3x$$
$$g(x) = 2\cos 2x + 3\sin 4x$$

(a)



(b) The period of $f(x)$ is 2π.

The period of $g(x)$ is π.

(c) $h(x) = A\cos\alpha x + B\sin\beta x$ is periodic because the sine and cosine functions are periodic.

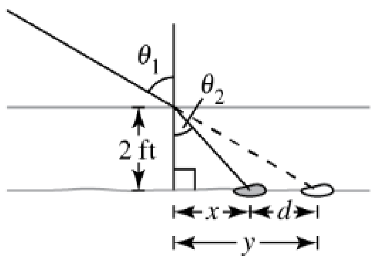
1.PS.10.

(a) Both graphs have a period of 2 and intersect when $x = 5.35$. They should also intersect when $x = 5.35 - 2 = 3.35$ and $x = 5.35 + 2 = 7.35$.

(b) The graphs intersect when $x = 5.35 - 3(2) = -0.65$.

(c) Because $13.35 = 5.35 + 4(2)$ and $-4.65 = 5.35 - 5(2)$ the graphs will intersect again at these values. So, $f(13.35) = g(-4.65)$.

1.PS.11.



(a) $\dfrac{\sin\theta_1}{\sin\theta_2} = 1.333$

$$\sin\theta_2 = \frac{\sin\theta_1}{1.333} = \frac{\sin 60°}{1.333} \approx 0.6497$$

$$\theta_2 = 40.5°$$

(b) $\tan\theta_2 = \dfrac{x}{2} \Rightarrow x = 2\tan 40.52° \approx 1.71$ feet

$\tan\theta_1 = \dfrac{y}{2} \Rightarrow y = 2\tan 60° \approx 3.46$ feet

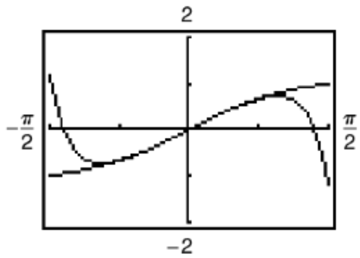
(c) $d = y - x = 3.46 - 1.71 = 1.75$ feet

(d) As you move closer to the rock, θ_1 decreases, which causes y to decrease, which in turn causes d to decrease.

1.PS.12.

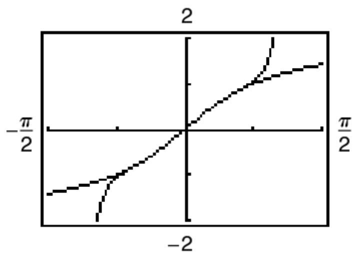
$$\arctan x \approx x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$$

(a)



The graphs are nearly the same for $-1 \le x \le 1$.

(b)



The accuracy of the approximation improved slightly by adding the next term $\left(x^9/9\right)$.